Developed by Jenifer Bohart
Scottsdale Community College

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About this Workbook

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Unit 1: Arithmetic Review

Section 1.1: Order of Operations

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Section 1.5: Decimals and Rounding

Section 1.6: Percents

Section 1.7: Applications of Percents

UNIT 1 LEARNING OBJECTIVES

• Simplify fractions
• Convert improper fractions to mixed numbers and vice versa
• Plot integers on a number line
• Perform operations on fractions
• Perform operations on integers
• Implement the Order of Operations on integers, fractions, and decimals
• Evaluate absolute value
• Round real numbers to an indicated place value
• Convert between fraction, decimal, and percent
• Solve application problems involving percents

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
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<td>Order of Operations</td>
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<td>Absolute Value</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
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<tr>
<td>Rounding</td>
<td></td>
</tr>
<tr>
<td>Percent</td>
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</tr>
</tbody>
</table>
Unit 1: Media Lesson

Section 1.1: Order of Operations

PEMDAS

If we are working with a mathematical expression that contains more than one operation, then we need to understand how to simplify. The acronym **PEMDAS** stands for Parentheses, Exponents, Multiplication, Division, Addition, Subtraction.

- **P** Terms inside parenthesis ( ) or brackets [ ]
- **E** Exponents and roots
- **MD** Multiplication and division (from Left to Right).
- **AS** Addition and subtraction (from Left to Right).

Use the order of operations to evaluate each of the following expressions. Use your calculator to check your answers.

**Example 1:**

\[
(2 \cdot 5)^2 \\
2 \cdot 5^2
\]

\[
10 - 7 + 1 \\
10 - (7 + 1)
\]

**Example 2:**

\[
24 \div (4 - 2)^3
\]
Example 3: \[ 4 + 5(1 + 12 ÷ 6)^2 \]

Example 4: \[ \frac{15-3}{1+5} \]

Section 1.1: You Try

Use the order of operations to evaluate each of the following expressions. Show all steps as in the media examples. Use your calculator to check your answers.

a. \[ 11 + 3(7 - 2)^2 \]

b. \[ \frac{6+8}{4-2} \]
Section 1.2: Fractions

### Improper Fractions and Mixed Numbers

**Converting a mixed number to an improper fraction:**
1. Multiply the denominator and the whole number
2. Add the numerator
3. Write the result over the denominator

**Example 1:** Express as an improper fraction.

\[
3 \frac{2}{7} \quad 12 \frac{1}{3}
\]

**Converting an improper fraction to a mixed number:**
1. Divide the numerator by the denominator
2. The quotient becomes the whole number part of the mixed number
3. Write the remainder over the denominator

**Example 2:** Express an improper fraction as a mixed number.

\[
\frac{42}{5} \quad \frac{53}{9}
\]
**Example 3:** Find two fractions equivalent to \( \frac{2}{7} \).

**Fractions in Simplest Form**

**Example 4:** Write the following fractions in simplest form.

\[
\frac{3}{18} \quad \frac{42}{54}
\]

**ONE and ZERO**

**Example 5:**

\[
\frac{1}{4} = \frac{4}{16} = \frac{4}{4} = \frac{0}{4} = \frac{4}{0}
\]
Section 1.2 – You Try

Complete the problems below. Show all steps as in the media examples.

a. Reduce the fraction \( \frac{24}{36} \) to lowest terms.

b. Rewrite the mixed number \( 4 \frac{1}{5} \) as an improper fraction.

c. Rewrite the improper fraction \( \frac{35}{11} \) as a mixed number.
Section 1.3: Operations on Fractions

Addition and Subtraction of Fractions

**Adding and Subtracting Fractions:**
1. Rewrite mixed numbers and whole numbers as improper fractions.
2. Find a common denominator
3. Rewrite the fractions as equivalent fractions with the common denominator
4. Add or subtract the numerators
5. Be sure to reduce your answer to simplest form!

**Example 1:** Perform the indicated operations

a. \[ \frac{1}{2} + \frac{1}{3} \]

b. \[ \frac{11}{15} - \frac{5}{12} \]

c. \[ 4\frac{3}{5} - 1\frac{5}{6} \]

d. \[ 2 - \frac{8}{5} \]
Multiplying Fractions:
1. Rewrite mixed numbers and whole numbers as improper fractions.
2. Multiply straight across (Multiply the numerators with the numerators, and the denominators with the denominators) NOTE: There is no need to find a common denominator when multiplying.
3. Be sure to reduce your answer to simplest form!

Example 2: Multiply. Write your answers in simplest form

\[ \frac{2}{3} \times \frac{3}{4} \hspace{2cm} \frac{12}{25} \times \frac{35}{48} \]

\[ \frac{7}{8} \times 5 \hspace{2cm} 3 \frac{1}{5} \times 1 \frac{1}{9} \]

Dividing Fractions:
1. Rewrite mixed numbers and whole numbers as improper fractions. NOTE: There is no need to find a common denominator when dividing.
2. Change the second fraction (the divisor) to its reciprocal
3. Multiply
4. Be sure to reduce your answer to simplest form!

Example 3: Divide. Write your answers in simplest form.

\[ \frac{1}{2} \div \frac{3}{5} \hspace{2cm} 8 \div \frac{4}{5} \]
**Order of Operations with Fractions**

**Example 4:** Perform the indicated operations. \( \frac{1}{2} + \frac{3}{2} \times \frac{2}{5} \)

---

**Section 1.3 – You Try**

Perform the indicated operations. Show all steps as in the media examples. Each answer must be written as a **reduced** fraction. Where appropriate, write your answer as **both** a mixed number and an improper fraction.

a. \( \frac{3}{5} + \frac{2}{3} \)

b. \( \frac{3}{5} \left( \frac{2}{3} \right) \)

c. \( \frac{3}{5} ÷ \frac{2}{3} \)

d. \( 3 - 2\frac{2}{5} \)

e. \( \frac{3}{7} ÷ 5 \)

f. \( \frac{3}{4} ÷ \frac{4}{5} \times \frac{5}{6} \)
Section 1.4: Signed Numbers

The Number Line

Absolute Value

The ABSOLUTE VALUE of a number is the distance that number is from 0 on the number line.

Example 1: Find the absolute value:

a. \(|-3|\)  
b. \(|3|\)  
c. \(-|-3|\)  
d. \(|0|\)

Mathematical Operations with Signed Numbers

Some hints for working with signed numbers:

- Use ( ) to separate numbers with negative signs
- When two signs are given together, use these rules to resolve the signs:
  \((-)(-) = + \quad (-)(+) = - \quad (+)(-) = - \quad (+)(+) = +\)
- Use the number line to add and subtract

Example 2: Perform the indicated operations.

a. \(3 + (-2)\)  
b. \(-3 + 2\)  
c. \(-3 - (-2)\)  
d. \(-3 + (-2)\)

Example 3: Multiply and divide.

a. \((-5)(-6)\)  
b. \(3(-4)\)  
c. \(-\frac{24}{8}\)  
d. \(\frac{2}{3}(-\frac{1}{5})\)

Example 4: Evaluate the following exponents:

\((-5)^2\)  
\(-5^2\)  
\((-5)^3\)  
\(-5^3\)
Example 5: Perform the indicated operations.

\[-8 \div (-2)^3 - (-3) - 5^2\]

Example 6: Evaluate.

\[-4(6)(-2) = \]
\[7(-3)(-2)(-4) = \]

The product or an odd number of negatives is negative.
The product or an even number of negatives is positive

SIMPLIFIED FORM FOR A SIGNED FRACTION

The following fractions are all equivalent (meaning they have the same value):

\[-\frac{1}{2} = \frac{1}{-2} = -\frac{1}{2}\]

Notice that only the placement of the negative sign is different.
HOWEVER, only the last one, \(-\frac{1}{2}\) is considered to be in simplest form.

Section 1.4 – You Try

Complete the problems below. Show all steps as in the media examples. Use your calculator to check your answers.

a. Find the absolute value: \(|-5| = _____ \quad |5| = _____ \quad |0| = _____

b. \((-2)^3 - 2^3\)

c. \(-\frac{8}{3} \left(-\frac{15}{2}\right)\)
Section 1.5: Decimals, Place Value, and Rounding

<table>
<thead>
<tr>
<th>BILLIONS</th>
<th>MILLIONS</th>
<th>THOUSANDS</th>
<th>ONES</th>
<th>DECIMALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 1: What place does the DIGIT 4 occupy in each number?

a. 324, 231.17
b. 256.134
c. 0.04
d. 1.4671

To round a number means to approximate that number by replacing it with another number that is “close” in value. Rounding is often used when estimating. For example, if I wanted to add 41 and 37, I could round each number to the nearest ten (40 and 40) then add to estimate the sum at 80.

When rounding, the analogy of a road may help you decide which number you are closer to. See the image below. The numbers 43, 45, and 46 are all rounded to the nearest tens place. Note that a number in the middle of the “road” is rounded up.

Example 2:

a. Round 40,963 to the nearest tens place.

b. Round 40,963 to the nearest hundreds place.

c. Round 40,963 to the nearest thousand

d. Round 40,963 to the nearest ten thousand
Decimal rounding is similar to whole number rounding, however, the decimal place values have different names and locations.

**Example 3:** Round each of the following numbers to the indicated place value.

a. 42.3456 to the nearest tenths place

b. 42.3999 to the nearest hundredths place

c. 42.3456 to the nearest thousandths place

**NOTE:** When working with money, the “dollar” is the ones place, and the “cent” or “penny” is the hundredths place.

**Worked Example 4:** Round each of the following numbers to the indicated place value.

a. $42.3456 to the nearest dollar: $42

b. $42.3456 to the nearest cent: $42.35

c. $42.3456 to the nearest penny: $42.35

**Section 1.5 – You Try

Round each of the following numbers to the indicated place value.

a. 358.4528 to the nearest ten

b. 358.4528 to the nearest thousandth

c. $358.4528 to the nearest cent

d. $358.4528 to the nearest dollar
### Example 1: Converting Fractions, Decimals, and Percents

Complete the missing parts of the table. Round to the nearest thousandth (three decimal places) as needed. Simplify all fractions. Show all work.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{4})</td>
<td>0.750</td>
<td>75%</td>
</tr>
<tr>
<td>(1\frac{3}{7})</td>
<td>1.429</td>
<td>142.857%</td>
</tr>
</tbody>
</table>
Complete the missing parts of the table. Round to the nearest thousandth (three decimal places) as needed. Simplify all fractions. Show all work.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{1}{8} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td>3%</td>
</tr>
</tbody>
</table>
Section 1.7: Applications of Percents

Example 1: Trader Joe’s sold 4,872 bags of tortilla chips last month. If 1,643 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest percent.

Example 2: A salesperson receives a 6.9% commission on her sales. If her total sales for the month are $4,100, what is her commission?

Example 3: You and your friends spend $56 at a restaurant. If you want to tip 15%, what is the tip amount? What is the total bill amount?

Example 4: A shirt was originally priced at $36. The store is having a 20% off sale. How much will you pay for the shirt after the discount (not including tax)?
Section 1.7 – You Try

Complete the following problems using the methods shown in this Media Lesson. Show all steps as in the media examples.

a. To win the election as president of the United States of America, a person must obtain 270 out of 538 possible votes from the Electoral College. What percentage of the overall electoral votes is this? Round your answer to the nearest tenth of a percent.

b. The sales tax rate for the state of Washington was 9.3%. Round your answers to the nearest cent as needed.

What is the sales tax on a $5,300 car in Washington?

What is the final cost of a $5,300 car in Washington, including tax?

c. A violin was originally priced at $350. The store is having a 40% off sale. How much will you pay for the violin after the discount is applied (not including tax)? Round your answer to the nearest cent.
Unit 1: Answers to You-Try Problems

Section 1.1:  a.  86  
              b.  7

Section 1.2:  a. \(\frac{2}{3}\)  
              b. \(\frac{21}{5}\)  
              c. \(3\frac{2}{11}\)

Section 1.3:  a. \(\frac{19}{15} = 1\frac{4}{15}\)  
              b. \(\frac{2}{5}\)  
              c. \(\frac{9}{10}\)  
              d. \(\frac{3}{5}\)  
              e. \(\frac{3}{35}\)  
              f. \(\frac{25}{32}\)

Section 1.4:  a. \(|-5| = 5\)  
              b. \(|5| = 5\)  
              c. \(|0| = 0\)  
              b. -16  
              c.  20

Section 1.5:  a.  360  
              b.  358.453  
              c. $358.45$  
              d. $358$

Section 1.6:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 1/8</td>
<td>.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>b. 2/5</td>
<td>0.4</td>
<td>40%</td>
</tr>
<tr>
<td>c. 3/100</td>
<td>0.03</td>
<td>3%</td>
</tr>
</tbody>
</table>

Section 1.7:  a.  50.2%  
              b. Tax: $492.90, Total cost: $5,792.90  
              c. $210
Unit 1: Practice Problems

1. Evaluate using the correct order of operations. Show all of your work. Use your calculator to check your answer. Write your answers as integers or reduced fractions.
   a. \( 4 + 2(9 - 5)^2 \)  
   b. \( 24 ÷ (1 + 2)^3 \)  
   c. \( 20 - (8 - 2) ÷ 3 \cdot 4 \)
   d. \( 10 \times 3^2 + \frac{15-3}{3\times2} \)  
   e. \( \left(\frac{8+2}{7-2}\right)^2 \)  
   f. \( 2 + 4 \times 8 - (2 + 3)^2 \)

2. Express the following fractions as improper fractions. Write your answer in simplest form.
   a. \( \frac{3}{8} \)  
   b. \( -\frac{3}{4} \)  
   c. \( \frac{2}{6} \)

3. Express the following fractions as mixed numbers. Write your answer in simplest form.
   a. \( \frac{43}{8} \)  
   b. \( \frac{38}{12} \)  
   c. \( \frac{70}{6} \)

4. For each of the following pairs, circle the larger number.
   a. \( \frac{5}{7} \)  \( \frac{5}{8} \)  
   b. \( \frac{5}{7} \)  \( \frac{7}{5} \)  
   c. \( \frac{5}{7} \)  \( \frac{6}{7} \)  
   d. \( \frac{4}{7} \)  \( \frac{1}{2} \)  
   e. \( \frac{5}{6} \)  \( \frac{6}{7} \)  
   f. \( \frac{1}{7} \)  \( \frac{7}{1} \)

5. Write each of the following in simplest form.
   a. \( \frac{54}{72} \)  
   b. \( \frac{165}{345} \)  
   c. \( \frac{4\frac{12}{28}}{} \)

6. Show the each step involved in evaluating each of the following. Write your answers in simplest form.
   a. \( \frac{1}{6} + \frac{2}{9} \)  
   b. \( \frac{5}{8} - \frac{6}{12} \)  
   c. \( \frac{1}{3} + \frac{2}{7} \)  
   d. \( \frac{8}{9} - \frac{3}{6} \)  
   e. \( \frac{2}{3} - \frac{3}{4} + \frac{1}{6} \)  
   f. \( 2\frac{2}{5} - 1\frac{1}{3} \)

7. Evaluate each of the following. Show all steps. Write your answers in simplest form.
   a. \( \frac{24}{3} \times \frac{27}{8} \)  
   b. \( 8 \times \frac{3}{24} \)  
   c. \( \frac{1}{4} \times \frac{3}{5} \times \frac{2}{9} \)
   d. \( \frac{24}{3} ÷ \frac{8}{3} \)  
   e. \( \frac{3}{5} ÷ \frac{9}{15} \)  
   f. \( 2\frac{1}{3} ÷ 1\frac{1}{2} \)

8. Evaluate using the correct order of operations. Show all of your work. Use your graphing calculator to check your answer
   a. \( (-2)^2 - 2^2 \)  
   b. \( 2(-3)^3 \times 8 ÷ 4 \)  
   c. \( -\frac{2}{3} - \frac{8}{3} \times \frac{3}{2} \)
   d. \( \frac{2}{3}(\frac{3}{8} - 1)^2 \)  
   e. \( (-4)^2 - 12 ÷ 3 \times 9 \)  
   f. \( \frac{8-(1+3)^2}{4-(-5)} \)
9. Round the number represented below.

Rounded to the nearest whole number: _______

Rounded to one decimal place: _______

Rounded to the nearest tenth: _______

10. Round the number represented below.

Rounded to the nearest whole number: _______

Rounded to one decimal place: _______

Rounded to the nearest hundredth: _______

11. Complete the table below.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
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<tr>
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<td></td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td></td>
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</tr>
</tbody>
</table>

12. Sam takes out a $25,000 student loan to pay his expenses while he is in college. After graduation, he will begin making payments of $167.68 per month for the next 20 years to pay off the loan. How much more will Sam end up paying for the loan than the original value of $25,000?

13. Abie makes $39,000 a year, and spends about $250 each month on entertainment. What fraction of her annual income is spent on entertainment?
14. Last year, the daily high temperatures in northern Washington for the first week of January were \(-8^\circ, -5^\circ, -4^\circ, 0^\circ, 8^\circ, 7^\circ, -5^\circ\) Fahrenheit. What was the average daily high temperature for that week?

15. Michelle wants to make cupcakes for her daughter’s birthday. The recipe calls for \(\frac{3}{4}\) cup of brown sugar, \(1\frac{1}{2}\) cups of white sugar, and 2 cups of powdered sugar, and will make 12 cupcakes. How much sugar will be in each cupcake?

16. Judy took Jen and Bill to the casino. Bill and Jen each won $100 playing the nickel slots. To say thanks, Jen gave Judy \(\frac{1}{4}\)th of her winnings and Bill gave Judy \(\frac{1}{5}\)th of his winnings. Who gave Judy more money? How much more?

17. So far this season, a hockey team has won 8 games and lost 4 games. This team has won what fraction of the games that it has played?

18. Sara buys a bag of candy. In the bag, \(\frac{1}{2}\) of the candies are red, \(\frac{1}{5}\) are green, and the remainder are white. What fraction of the candies are white?

19. The national debt is also constantly changing. The website \www.usdebtclock.org\ shows real time estimates of the national debt. At one point, the estimate of the debt was $17,882,815,724,883. Round this number to the nearest billion dollars.

20. Round the results of the application problems so that it makes sense in the context of the problem.
   a) Amy is buying ribbon for an art project. She estimates that she will need 3.34 meters of ribbon. The store only sells ribbon by the tenth of a meter. How many meters should she buy?
   b) John is catering a luncheon and needs 12.37 pounds of sugar. If sugar is only sold in one pound bags, how many bags should John buy?
   c) Shelly is buying shoes online and computes that she has enough money to buy 2.78 pairs of shoes. How many pairs of shoes can she buy?
   d) Tia is making a work bench for her art studio. She measures the space and needs 8.24 meters of plywood. The store only sells plywood by the tenth of a meter. How many meters should Tia buy?
   e) Jamie is running a booth at the local fair. She computes that she needs to sell 86.25 snow cones that day to make a profit. Since she can only sell a whole number of snow cones, how many does she need to sell to make a profit?
   f) Crystal is buying Halloween candy at the store. She has $20 and wants to buy as many bags of candy as possible. She computes that she has enough to buy 6.91 bags of candy. How many bags of candy can she buy?
21. In a survey, a group of people were asked if they had run a red light in the last year. 69% responded "yes". What percentage responded "no"?

22. In a recent poll, 38% of the 750 individuals polled indicated that they would vote purely Democratic in the next election. How many of the individuals would vote a straight Democratic ticket?

23. What is a 15% tip on a bill of $35.20? What is the total amount paid (including tip)?

24. Tiffany paid $165 for an item that was originally priced at $500. What percent of the original price did Tiffany pay? Round your answer to the nearest tenth of a percent.

25. 84% of the questions on a student's test were correct. There were 50 questions. How many of the questions were correct? How many were incorrect?

26. What is the sales tax on a suit priced at $1562 if the sales tax is 9.78%?

27. Suppose your school costs for this term were $7100 and financial aid covered 80% of that amount. How much did financial aid cover? How much do you still have to pay?

28. A TV was originally priced at $590. The store is having a 35% off sale. How much will you pay for the TV after the discount (not including tax)? Round your answer to the nearest cent.

29. A company wants to decrease their energy use by 14%. If their electric bill is currently $1,300 a month, what will their bill be if they are successful? Round your answer to the nearest dollar.

30. In the news, you hear "tuition is expected to increase by 4% next year." If tuition this year was $1,500 per semester, what will it be next year?

31. Jenelle bought a home for $170,000, paying 10% as a down payment, and financing the rest.
   a. How much money did Jenelle pay as a down payment?
   b. What was the amount financed (after making the down payment)?

32. Donna is going to swim 25 laps. She has completed 10 laps.
   a. What fraction of laps has she completed? What percent of laps has she completed?
   b. What fraction of her swim remains? What percent of her swim remains?

33. Out of 250 racers who started the marathon, 222 completed the race, 18 gave up, and 10 were disqualified. What percentage did not complete the marathon? Round your answer to the nearest tenth of a percent.
34. On a survey, 130 people out of 610 state that they like dogs. What percent is this?

35. In a given bag of M & M’s, 18 were yellow, 13 were green, 14 were brown, and 17 were red.
   a. How many M&M’s were in the bag?
   b. What percent of the M&M’s in the bag were yellow? Green? Red?

36. If you decrease your daily intake of calories from 2500 to 1750, by what percent do your daily calories decrease?

37. On a recent trip to Walmart, you bought $75.25 worth of goods and paid a total of $82.02. What was the rate of sales tax that you paid?

38. The value of a car dropped from $38,000 to $34,600 over the last year. Determine the absolute and relative change in this situation.

39. Joyce paid $99.00 for an item at the store that was 45% off the original price. What was the original price? Round your answer to the nearest cent.

40. Sara had a party for her parent’s anniversary. Fifty-six people attended. This was approximately 72% of the people she invited. How many people did Sara invite? (Round to the nearest person)

41. Amy decreased her restaurant spending from $287 a month to $54 a month. What percent decrease is this?

42. Jose spent $136.25 on a video game including 9% sales tax. What was the cost of the video game without tax?

43. In the U.S. Civil War, 750,000 people were estimated to have died. If that number represented 2.5% of the U.S. population of the day, how many people lived in the U.S. during the Civil War? If a war of that scale happened today and the same percentage of people died, how many people would be killed (assume U.S. population of 314,721,724 people). [Source: Smithsonian Magazine, November 2012, page 48]

44. Fred and Wilma purchase a home for $180,000. Determine the value of the house 10 years after its purchase, assuming that the value:
   a) Decreases by $1,500 per year  
   b) Decreases by 2% per year  
   c) Increases by $3,300 per year  
   d) Increases by 6% per year
45. Working with square roots.
   a. **Without using your calculator**, fill in the blanks below.
      \[ \sqrt{0} = \ldots \quad \sqrt{1} = \ldots \quad \sqrt{4} = \ldots \quad \sqrt{9} = \ldots \]
      \[ \sqrt{16} = \ldots \quad \sqrt{25} = \ldots \quad \sqrt{49} = \ldots \]
      \[ \sqrt{64} = \ldots \quad \sqrt{81} = \ldots \]
   b. **Without using your calculator**, place each of the following on the number line below.
      \[ \sqrt{2} \quad \sqrt{11} \quad \sqrt{40} \quad \sqrt{60} \quad \sqrt{99} \]
      \[ \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \]
   c. Now use your calculator to evaluate each of the following. Round your answers to the nearest hundredth.
      \[ \sqrt{2} \approx \ldots \quad \sqrt{11} \approx \ldots \quad \sqrt{40} \approx \ldots \]
      \[ \sqrt{60} \approx \ldots \quad \sqrt{99} \approx \ldots \]

46. Find the reciprocal of each of the numbers below.
   a. \( \frac{2}{3} \)  
   b. \( -\frac{7}{9} \)  
   c. 8
   d. -8  
   e. \( \frac{1}{5} \)  
   f. \( 5 \frac{1}{2} \)
   g. Why does zero not have a reciprocal?
Unit 1: Review

1. Write each of the following in simplest form. Write DNE if the answer does not exist.
   \[
   \frac{8}{1} = \underline{\text{________}} \quad \frac{8}{0} = \underline{\text{________}} \quad \frac{-36}{-84} = \underline{\text{________}} \quad \frac{0}{2} = \underline{\text{________}}
   \]

2. Evaluate:  \(|74| = \underline{\text{________}} \quad |-32| = \underline{\text{________}} \quad |0| = \underline{\text{________}}

3. Perform the indicated operations. Be sure to leave your answers in simplest (reduced) form. If applicable, write your answer as both an improper fraction and a mixed number. Show all steps and box your answers.
   a. \(\frac{3}{4} - \frac{5}{6}\)
   b. \(\frac{4}{7} \div \frac{3}{8}\)
   c. \(\left(\frac{3}{7}\right)^2\)

4. Evaluate and simplify completely. Show all steps and box your answers.
   a. \(1 + 5(4 - 6)^3\)
   b. \(\frac{3}{4} - \frac{3}{5} \left(\frac{1}{2}\right)\)

5. Donna is going to swim 22 laps. She has completed 14 laps. What fraction of her swim remains?

6. In a given bag of candies, 14 were yellow, 11 were green, 15 were blue, and 20 were red. What fraction of the candies in the bag were red?

7. Danny takes out a $16,400 student loan to pay his expenses while he is in college. After graduation, he will begin making payments of $88 per month for the next 30 years to pay off the loan. How much more will Danny end up paying for the loan than the original value of $16,400? Write your answer in a complete sentence.
8. While training for the barrel racing competition at the upcoming rodeo, Sara records her times for each barrel run. Today her times were 16.22, 15.45, and 16.08 seconds. Determine her average time, rounded to the nearest hundredth of a second. Write your answer in a complete sentence.

9. Carlos ordered 3 items online. He is charged $3.47 per pound per shipping. The items weighed 1.4 lbs., 0.6 lbs., and 5.3 lbs. How much will he be charged for shipping? Write your answer in a complete sentence. Round your answer to the nearest cent.

10. Trader Joe's sold 9763 bags of tortilla chips last month. If 7782 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest percent.

11. A salesperson receives a 7.7% commission on her sales. If her total sales for the month are $1600, what is her commission?

12. The sales tax rate for the state of Washington was 7.2%.
   a. What is the state sales tax on a $14,000 car in Washington?
   b. What is the final cost of the car, including tax?

13. A TV was originally priced at $860. The store is having a 25% off sale. How much will you pay for the TV after the discount (not including tax)? Round your answer to the nearest cent.
Unit 2: Introduction to Variables

Section 2.1: Writing Algebraic Expressions
Section 2.2: The Story of “x”
Section 2.3: Evaluating Algebraic Expressions
Section 2.4: Applications
Section 2.5: Geometric Formulas
Section 2.6: Some Vocabulary
Section 2.7: Like Terms
Section 2.8: The Distributive Property
Section 2.9: Simplifying Algebraic Expressions

UNIT 2 LEARNING OBJECTIVES

- Evaluate algebraic expressions given fractions, integers, or decimals
- Use knowledge of order of operations to tell the story of x in an algebraic expression
- Assign a variable to an unknown or varying quantity and construct an algebraic expression based on reasonableness
- Construct and evaluate algebraic expressions in application problems, and interpret results
- Define the basic components of an algebraic expression
- Identify and combine like terms
- Apply the distributive property to simplify expressions

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
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<tbody>
<tr>
<td>Variable</td>
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<td>Algebraic Expression</td>
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<tr>
<td>Evaluate an Algebraic Expression</td>
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<tr>
<td>Term</td>
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<td>-----------------------------</td>
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<tr>
<td>Commutative Property</td>
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<tr>
<td>Exact Form</td>
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<td>Approximate Form</td>
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<td>Constant Term</td>
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<td>Factors</td>
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<td>Coefficient</td>
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<tr>
<td>Like Terms</td>
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<td>Combining Like Terms</td>
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<td>Distributive Property</td>
<td></td>
</tr>
<tr>
<td>Simplifying an Algebraic Expression</td>
<td></td>
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</tbody>
</table>
Section 2.1: Writing Algebraic Expressions

Definitions

A **variable**, usually represented by a letter or symbol, can be defined as:
- A quantity that may change within the context of a mathematical problem.
- A placeholder for a specific value.

An **algebraic expression** is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc...).

---

**Example 1:** Juan is 6 inches taller than Niko. Let $N$ represent Niko’s height in inches. Write an algebraic expression to represent Juan’s height.

**Example 2:** Juan is 6 inches taller than Niko. Let $J$ represent Juan’s height in inches. Write an algebraic expression to represent Niko’s height.

**Example 3:** Suppose sales tax in your town is currently 9.8%. Write an algebraic expression representing the sales tax for an item that costs $D$ dollars.
Example 4: You started this year with $362 saved and you continue to save an additional $30 per month. Write an algebraic expression to represent the total amount saved after $m$ months.

Example 5: Movie tickets cost $8 for adults and $5.50 for children. Write an algebraic expression to represent the total cost for $A$ adults and $C$ children to go to a movie.

Section 2.1 – You Try

Complete the following problems. Show all steps as in the media examples.

a. There are about 80 calories in one chocolate chip cookie. If we let $n$ be the number of chocolate chip cookies eaten, write an algebraic expression for the number of calories consumed.

b. Brendan recently hired a contractor to do some necessary repair work. The contractor gave a quote of $450 for materials and supplies plus $38 an hour for labor. Write an algebraic expression to represent the total cost for the repairs if the contractor works for $h$ hours.

c. A concession stand charges $3.50 for a slice of pizza and $1.50 for a soda. Write an algebraic expression to represent the total cost for $p$ slices of pizza and $s$ sodas.
Section 2.2: The Story of “x”

**Example 1:** Tell the story of \( x \) in each of the following expressions.

a. \( x - 5 \)  
b. \( 5 - x \)

c. \( 2x \)  
d. \( x^2 \)

**Example 2:** Tell the story of \( x \) in each of the following expressions.

a. \( 2x + 4 \)  
b. \( 2(x + 4) \)

c. \( 5(x - 3)^2 - 2 \)
**Example 3:** Write an algebraic expression that summarizes the stories below.

a. Step 1: Add 3 to $x$
   Step 2: Divide by 2
b. Step 1: Divide $x$ by 2
   Step 2: Add 3

**Example 4:** Write an algebraic expression that summarizes the story below.

Step 1: Subtract $x$ from 7
Step 2: Raise to the third power
Step 3: Multiply by 3
Step 4: Add 1

---

**Section 2.2 — You Try**

Complete the following problems.

a. Tell the story of $x$ in the expression $\frac{x-3}{5}$

b. Write an algebraic expression that summarizes the story below:
   
   Step 1: Multiply $x$ by 2
   Step 2: Add 5
   Step 3: Raise to the second power.
Section 2.3: Evaluating Algebraic Expressions

**Example 1:** Find the value of each expression when $w = 2$. Simplify your answers.

\[
\begin{align*}
w - 6 & \quad 6 - w & \quad 5w - 3 \\
\end{align*}
\]

\[
\begin{align*}
w^3 & \quad 3w^2 & \quad (3w)^2 \\
\end{align*}
\]

\[
\begin{align*}
\frac{4}{5w} & \quad \frac{5w}{4} & \quad 3^w \\
\end{align*}
\]

**Example 2:** Evaluate $ab + c$ given $a = -5$, $b = 7$, and $c = -3$
Example 3: Evaluate \( a^2 - b^2 \) given \( a = -5 \) and \( b = -3 \)

Example 4: A local window washing company charges $11.92 for each window plus a reservation fee of $7.

a. Write an algebraic expression to represent the total cost from the window washing company for washing \( w \) windows.

b. Use this expression to determine the total cost for washing 17 windows.

Section 2.3 – You Try

Evaluate \( b^2 - 4ac \) given \( a = 5, \ b = -1, \ c = -2 \). Show all steps as in the media examples.
Section 2.4: Applications

Example 1: The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you get the most benefits and reduce the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate (pulse) is about 80 percent of your maximum heart rate. The formula \( M = 0.8(220 - A) \), gives the recommended maximum heart rate, \( M \), in beats per minute, for a person who is \( A \) years of age. What is the recommended maximum heart rate for a person who is 40 years old?

Example 2: A golfer strikes a golf ball. The height, \( H \) (in feet), of the ball above the ground after \( t \) seconds is given by the equation \( H = -16t^2 + 80t \). Determine the height of the ball after 3 seconds. Show all of your work, and write your answer in a complete sentence.

Example 3: Simple interest is given by the formula \( A = P + Prt \). Where \( A \) is the accrued value of the investment after \( t \) years, and \( P \) is the starting principal invested at an annual percentage rate of \( r \), expressed as a decimal. Sally buys a $1,000 savings bond that pays 4% simple interest each year. How much will the bond be worth after 5 years?
Example 4: The formula $P = 266(1.009)^t$ estimates the population of the United States (in millions of people), $t$ years after 1995.

a. Use this formula to estimate the U.S. population in 1995. Round your answer to the nearest million.

b. Use this formula to estimate the U.S. population in 2016. Round your answer to the nearest million.

Section 2.4 – You Try

Paul is planning to sell bottled water at the local carnival. He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul’s profit, $P$ in dollars, from selling $b$ bottles of water is given by the formula $P = 1.5b - 360$. Determine Paul’s profit if he sells all 2000 bottles of water. Show all of your work, and write your answer in a complete sentence.
Section 2.5: Geometric Formulas

Example 1: The circumference of a circle with radius $r$ is given by the formula $C = 2\pi r$. Determine the circumference of a circle with radius 32 cm. Write your answer in exact form (in terms of $\pi$) and in approximate form, rounded to the nearest hundredth.

Example 2: The formula for the volume of a cone of base radius $r$ and height $h$ is $V = \frac{1}{3} \pi r^2 h$. Determine the volume of a cone with base radius 5 inches and height 12 inches. Write your answer in exact form (in terms of $\pi$) and in approximate form, rounded to the nearest hundredth.

The Pythagorean Theorem

The Pythagorean Theorem states that given any right triangle with legs $a$ and $b$, and hypotenuse $c$ as below, the following relationship is always true: $a^2 + b^2 = c^2$. Consequently, if the lengths of two sides are known, the length of the third side can be found using the formulas below:

\[ a = \sqrt{c^2 - b^2} \]
\[ b = \sqrt{c^2 - a^2} \]
\[ c = \sqrt{a^2 + b^2} \]
Example 3: Find the length of the leg $x$ of the right triangle shown below. Write your answer in **exact form** and in **approximate form**, rounded to the nearest thousandth.

![Right triangle with sides 8 cm, 14 cm, and variable leg $x$.]

Section 2.5 – You Try

Complete the following problems. Show all steps as in the media examples.

a. The formula for the volume, $V$, of a cylinder of radius $r$ and height $h$ is $V = \pi r^2 h$. Determine the volume of a cylinder with radius 4 inches and height 10 inches. Write your answer in **exact form** (in terms of $\pi$) and in **approximate form**, rounded to the nearest hundredth. Include appropriate units in your answers.

Exact Form: $V =$ _______________

Approximate Form: $V \approx$ _______________

b. Use the Pythagorean Theorem to find the length of side $x$ of the right triangle shown below. Write your answer in **exact form** and in **approximate form**, rounded to the nearest thousandth. Include appropriate units in your answers.

![Right triangle with sides 5 ft, 7 ft, and variable leg $x$.]

Exact Form: $x =$ _______________

Approximate Form: $x \approx$ _______________
Section 2.6: Some Vocabulary

<table>
<thead>
<tr>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Terms</strong>: Parts of an algebraic expression separated by addition or subtraction symbols.</td>
</tr>
<tr>
<td><strong>Constant Term</strong>: A number with no variable factors. A term whose value never changes.</td>
</tr>
</tbody>
</table>

**Example 1**: Consider the algebraic expression $4x^5 + 3x^4 - 22x^2 - x + 17$

a. List the terms. ________________________________________________________

b. Identify the constant term. ___________________

<table>
<thead>
<tr>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factors</strong>: Numbers or variables that are multiplied together</td>
</tr>
<tr>
<td><strong>Coefficient</strong>: The number that multiplies the variable.</td>
</tr>
</tbody>
</table>

**Example 2**: Complete the table below.

<table>
<thead>
<tr>
<th></th>
<th>$-4m$</th>
<th>$-x$</th>
<th>$\frac{1}{2}bh$</th>
<th>$\frac{2r}{5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>List the Factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the Coefficient</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 3: Consider the algebraic expression $5y^4 - 8y^3 + y^2 - \frac{y}{4} - 7$

a. How many terms are there? ____________

b. Identify the constant term. ______________

c. What is the coefficient of the first term? ____________

d. What is the coefficient of the second term? ____________

e. What is the coefficient of the third term? ____________

f. List the factors of the fourth term. ___________________________

Section 2.6 – You Try

Consider the algebraic expression $2m^3 + m^2 - \frac{2m}{5} - 8$

a. How many terms are there? ____________

b. Identify the constant term. ______________

c. What is the coefficient of the first term? ____________

d. What is the coefficient of the second term? ____________

e. What is the coefficient of the third term? ____________
Section 2.7: Like Terms

<table>
<thead>
<tr>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>Terms whose variable factors (letters and exponents) are exactly the same are called LIKE TERMS.</td>
</tr>
</tbody>
</table>

**Example 1:** Identify the like terms in each of the following expressions

\[ 3a - 6a + 10a - a \quad 5x - 10y + 6z - 3x \quad 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3 \]

**Example 2:** Combine the like terms.

\[ 3a - 6a + 10a - a \]

\[ 5x - 10y + 6z - 3x \]

\[ 7n + 3n^2 - 2n^3 + 8n^2 + n - n^3 \]

---

**Section 2.7 – You Try**

Combine the like terms. Show all steps as in the media examples.

a. \[ 3x - 12x + 2x - x \]  
   b. \[ -5 + 2a^2 - 4a + a^2 + 7 \]
Section 2.8: The Distributive Property

Use the Distributive Property to Expand Each of the Following Expressions

Example 1:  \( 5(2x + 4) \)

Example 2:  \( -3(x^2 - 2x + 7) \)

Example 3:  \( -(5x^4 - 8) \)

Example 4:  \( \frac{2}{5} \left( \frac{x}{4} - \frac{1}{3} \right) \)
Section 2.8 – You Try

Use the Distributive Property to expand the algebraic expression. Show all steps as in the media examples.

a. \(-5(3x^2 - 2x + 8)\)

b. \(\frac{2}{3} \left( 6x + \frac{1}{2} \right)\)
Section 2.9: Simplifying Algebraic Expressions

<table>
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<td>Step 1: Simplify within parentheses</td>
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<tr>
<td>Step 2: Use distributive property to eliminate parentheses</td>
</tr>
<tr>
<td>Step 3: Combine like terms.</td>
</tr>
</tbody>
</table>

Example 1: Simplify the following algebraic expressions. Show all possible steps.

a. \(-3(2x - 4) - (3x + 8)\) 

b. \(3[2 - (x - 5)] - (4x - 10)\)

c. \(\frac{8 - 5x}{2}\) 

d. \(\frac{9 - 3(2x - 5)}{-6}\)
Simplify completely. Show all steps as in the media examples.

a. \(2(7x^2 + 3x + 2) - (8x^2 - 7)\)

b. \(\frac{2x - 1}{2}\)
Unit 2: Answers to You-Try Problems

Section 2.1:  a. 80n  b. 450 + 38h or 38h + 450  c. 3.5p + 1.5s

Section 2.2:  a. Step 1: Subtract 3 from x. Step 2: Divide by 5  b. (2x + 5)^2

Section 2.3:  41

Section 2.4:  If Paul sells all 2000 bottles, his profit will be $2640.

Section 2.5:  a. Exact form: 160π cubic inches
                   Approximate form: (using 3.14 for π) 502.4 cubic inches
                   Approximate form: (using the π key on your calculator) 502.65 cubic inches
                   b. Exact form: \sqrt{74} feet
                      Approximate form: 8.602 feet

Section 2.6:  a. 4  b. –8  c. 2  d. 1  e. \(-\frac{2}{5}\)

Section 2.7:  a. –8x  b. 3a^2 − 4a + 2

Section 2.8:  a. \(-15x^2 + 10x − 40\)  b. \(4x + \frac{1}{3}\)

Section 2.9:  a. 6x^2 + 6x + 11  b. \(x − \frac{1}{2}\)
Unit 2: Practice Problems

Skills Practice

1. Tell the story of $x$ in each of the following expressions.
   
   a. $x - 11$
   b. $x + 5$
   c. $5x$
   d. $x^5$
   e. $x^3$
   f. $2 - x$
   g. $2x - 3$
   h. $8x^2$
   i. $(2x)^2$
   j. $7 - 2x$
   k. $5(7 - x)^3$
   l. $\left(\frac{3x - 8}{5}\right)^3$

2. Write an algebraic expression that summarizes the stories below.
   
   a. Step 1: Add 8 to $x$
      Step 2: Raise to the third power
   b. Step 1: Divide $x$ by 8
      Step 2: Subtract 5
   c. Step 1: Subtract 3 from $x$
      Step 2: Multiply by 7
   d. Step 1: Multiply $x$ by 10
      Step 2: Raise to the $3^{rd}$ power
      Step 3: Multiply by 2
   e. Step 1: Add 5 to $x$
      Step 2: Divide by 2
      Step 3: Raise to the second power
      Step 4: Add 8
   f. Step 1: Raise $x$ to the second power
      Step 2: Multiply by 5
      Step 3: Subtract from 9
   g. Step 1: Subtract $x$ from 2
      Step 2: Multiply by -8
      Step 3: Raise to the third power
      Step 4: Add 1
      Step 5: Divide by 3
   h. Step 1: Multiply $x$ by -4
      Step 2: Add 9
      Step 3: Divide by 2
      Step 4: Raise to the fifth power

3. Find the value of each expression when $b = -8$. Simplify your answers.
   
   a. $b - 11$
   b. $b + 5$
   c. $5b$
   d. $b^2$
   e. $b^3$
   f. $2 - b$

4. Evaluate each of the following given $q = 10$.
   
   a. $2q - 3$
   b. $8q^2$
   c. $(2q)^2$
   d. $\frac{4}{7q}$
   e. $7 - 2q$
   f. $2^q$
5. Find the value of each expression when $c = \frac{2}{3}$. Write your answers as proper fractions or mixed numbers in simplest form.
   a. $c - 5$ 
   b. $c + \frac{3}{5}$ 
   c. $\frac{2}{3}c$ 
   d. $c^2$ 
   e. $c^3$ 
   f. $\frac{2}{c}$

6. Evaluate the following expressions for the given values. Simplify your answers.
   a. $\frac{-b}{2a}$ for $a = 6$ and $b = 4$ 
   b. $\frac{4x+8}{5+x}$ for $x = 3$ 
   c. $\frac{3}{5}ab$ for $a = 8$ and $b = 1 \frac{2}{3}$ 
   d. $3x^2 + 2x - 1$ for $x = -1$ 
   e. $x^2 - y^2$ for $x = -3$ and $y = -2$ 
   f. $2x - 7y$ for $x = 3.81$ and $y = 6.22$ 
   g. $\sqrt{c^2 - a^2}$ for $a = 3$ and $c = 5$ 
   h. $\sqrt{b^2 - 4ac}$ for $a = -1, b = -5, c = 6$

7. Complete the table below.

<table>
<thead>
<tr>
<th></th>
<th>$5t$</th>
<th>$-3abc$</th>
<th>$-y$</th>
<th>$x$</th>
<th>$\frac{3}{5}x$</th>
<th>$\pi d$</th>
<th>$\frac{4x}{7}$</th>
<th>$\frac{m}{5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the Coefficient</td>
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</table>

8. Consider the algebraic expression $5n^8 - n^5 + n^2 + \frac{n}{8} - 1$
   a. How many terms are there? ____________ 
   b. Identify the constant term. ____________ 
   c. What is the coefficient of the first term? ____________ 
   d. What is the coefficient of the second term? ____________ 
   e. What is the coefficient of the third term? ____________ 
   f. List the factors of the fourth term. ___________________________

9. Consider the algebraic expression $w^3 - w^2 - \frac{2w}{3} + 3$
   a. How many terms are there? ____________ 
   b. Identify the constant term. ____________ 
   c. What is the coefficient of the first term? ____________ 
   d. What is the coefficient of the second term? ____________ 
   e. What is the coefficient of the third term? ____________
10. Identify and combine the Like Terms.
   a. \(3d - 5d + d - 7d\)  
   b. \(3x^2 + 3x^3 - 9x^2 + x - x^3\)
   c. \(a - 2b + 4a + b - (-2b)\)  
   d. \(\frac{2}{5}r - \frac{2}{3}r + r\)
   e. \(3.82x - 5.6x + 8.91\)  
   f. \(M + 2.74M\)

11. Apply the distributive property to expand the following expressions.
   a. \(6(4x - 8)\)  
   b. \(-5(6w^2 - 3w + 1)\)
   c. \(-(4y^2 + 3y - 8)\)  
   d. \(\frac{3}{4}\left(\frac{2}{5}x + \frac{7}{12}\right)\)
   e. \(\frac{1}{3}\left(\frac{3}{4}b - 5\right)\)  
   f. \(4(6.3d - 5.8)\)

12. Simplify by using the distributive property and combining like terms. Show all steps.
   a. \((5x^2 + 3x - 6) - (3x + 6)\)  
   b. \(3(2x^2 - x + 3) + 2\)
   c. \(2a + 3ab - 5a + 8ab + 3b\)  
   d. \(12 + 3x^2 + 4x - 2x^2 - x - 6\)
   e. \(5(2x + 3) + 4(3x - 7)\)  
   f. \(-2(4x^2 + 3x - 2) - (x^2 - 6)\)

13. Simplify completely. Show all steps.
   a. \(\frac{12 - 9x}{3}\)  
   b. \(\frac{21m - 18}{6}\)  
   c. \(\frac{3a - 7}{7}\)
   d. \(\frac{4w + 1}{4}\)  
   e. \(\frac{3(4a - 8) + 2}{2}\)  
   f. \(\frac{3(10x - 4) + 6}{6} + 3x + 1\)

Applications

14. Shea bought \(C\) candy bars for \$1.50 each.
   a. Write an algebraic expression for the total amount Shea spent.
   b. Use this expression to determine the amount Shea will spend for 3 candy bars. Show all of your work and write your answer in a complete sentence.

15. Suppose sales tax in your town is currently 9%.
   a. Write an algebraic expression representing the sales tax for an item that costs \(D\) dollars.
   b. Use this expression to determine the sales tax for an item that costs \$354. Show all of your work and write your answer in a complete sentence.
16. Ben bought $M$ movie tickets for $8.50 each and $B$ bags of popcorn for $3.50 each.
   a. Write an algebraic expression for the total amount Ben spent.
   b. Use this expression to determine the amount Ben will spend if he buys 6 movie tickets and 4 bags of popcorn. Write your answer in a complete sentence.

17. Noelle is 5 inches shorter than Amy. Amy is $A$ inches tall.
   a. Write an algebraic expression for Noelle's height.
   b. Use this expression to determine Noelle’s height if Amy is 5 feet 8 inches tall.

18. Jamaal studied $H$ hours for a big test. Karla studied one fourth as long.
   a. Write an algebraic expression for the length of time that Karla studied.
   b. Use this expression to determine the length of time that Karla studied if Jamaal studied for 5 hours and 20 minutes. Write your answer in a complete sentence.

19. A caterer charges a delivery fee of $45 plus $6.50 per guest.
   a. Write an algebraic expression to represent the total catering cost if $G$ guests attend the reception.
   b. Use this expression to determine the total catering cost for if 80 people attend the reception. Write your answer in a complete sentence.

20. Tickets to the museum cost $18 for adults and $12.50 for children.
   a. Write an algebraic expression to represent the cost for $A$ adults and $C$ children to visit the museum.
   b. Use this expression to determine the cost for 4 adults and 6 children to attend the museum. Write your answer in a complete sentence.

21. Irena invested money into two mutual funds. During the first year, Fund A earned 2% interest and Fund B earned 3% interest.
   a. Let $A$ represent the amount of money invested in Fund A, and let $B$ represent the amount of money invested in Fund B. Write an algebraic expression for the total amount of interest Irena earned from both accounts during the first year.
   b. Use this expression to determine the total amount of interest earned during the first year if she invested $4,500 in Fund A and $1,200 in Fund B. Write your answer in a complete sentence.

22. Ivan invested money into two mutual funds. Fund A earned 8% profit during the first year while Fund B suffered a 3% loss.
   a. Let $A$ represent the amount of money invested in Fund A, and let $B$ represent the amount of money invested in Fund B. Write an algebraic expression for the total profit Ivan earned from both mutual funds during the first year.
   b. Use this expression to determine the total amount of profit earned during the first year if he invested $3,800 in Fund A and $2,400 in Fund B. Write your answer in a complete sentence.
23. The formula to convert from Fahrenheit to Celsius is \[ C = \frac{5}{9}(F - 32). \]

The temperature on a summer day in Phoenix, Arizona is 115ºF. What would this temperature be in degrees Celsius? Round your answer to the nearest tenth of a degree.

24. Isabel has a headache, and takes 500mg of Tylenol. The amount, \( A \), of Tylenol (measured in mg) remaining in her body after \( n \) hours is given by the formula \( A = 500(0.882)^n \). How much of the Tylenol remains in her body after 4 hours? Show all work, and round your answer to the nearest hundredth. Write your answer in a complete sentence.

25. A person’s Body Mass Index (BMI) is given by the formula \[ BMI = \frac{703W}{H^2}, \] where \( W \) is the weight of the person in pounds, and \( H \) is the person’s height, measured in inches. If a person is 5 feet 7 inches tall, and weighs 142 pounds, what is that person’s BMI? Show all of your work. Round your answer to the nearest tenth. Write your answer in a complete sentence.

26. The formula for the volume, \( V \), of a cylinder of radius \( r \) and height \( h \) is \[ V = \pi r^2 h. \]

Determine the volume of a cylinder with radius 3 inches and height 8 inches. Write your answer in exact form (in terms of \( \pi \)) and in approximate form, rounded to the nearest hundredth. Include appropriate units in your answer.

27. The formula \[ A = \frac{1}{2}bh \] gives the area of a triangle with base \( b \) and height \( h \).

Determine the area of a triangle with base 4cm and height \( 2\frac{2}{3} \) cm. Write your answer as a proper fraction or mixed number in simplest form. Include appropriate units in your answer.

28. The formula \[ V = 9.54 + 0.08m \] represents the value of an investment (in thousands of dollars) after \( m \) months. Determine the value of this investment after two years.

29. The formula \[ E = 3861 - 77.2t \] gives the surface elevation (in feet above sea level) of Lake Powell \( t \) years after 1999. Use this formula to predict the surface elevation of Lake Powell in the year 2016.

30. Simple interest is given by the formula \[ A = P + Prt. \] Where \( A \) is the accrued value of the investment after \( t \) years, and \( P \) is the starting principal invested at an annual percentage rate of \( r \), expressed as a decimal. Sally buys a $5,000 savings bond that pays 2.3% simple interest each year. How much will the bond be worth after 5 years?

31. The formula for compound interest is \[ A = P(1 + r)^t \] where \( A \) is the accrued amount after \( t \) years, \( P \) is the starting principal, and \( r \) is the annual interest rate expressed as a decimal. If you invest $12,000 at an annual interest rate of 1.7% and leave it there for 30 years, what would your ending balance be? Round your answer to the nearest cent.
32. February is a busy time at Charlie’s Chocolate Shoppe! During the week before Valentine’s Day, Charlie advertises that his chocolates will be selling for $1.50 a piece (instead of the usual $2.00 each). The fixed costs to run the Chocolate Shoppe total $650 for the week, and he estimates that each chocolate costs about $0.60 to produce. Write an algebraic expression that represents Charlie’s profit from selling $n$ chocolates during the week before Valentine’s Day. (HINT: Profit = Revenue – Costs) Simplify your answer.

Extension

33. Evaluate \( \frac{-b + \sqrt{b^2 - 4ac}}{2a} \) for \( a = 1 \), \( b = -4 \), and \( c = -3 \). Write your answer in exact form (simplified completely) and in approximate form, rounded to the nearest thousandth.

34. A pebble is dropped into a calm pond, causing ripples in the shape of concentric circles to expand on the surface of the water. The area of the outer ripple is given by the formula \( A = \pi r^2 \), where \( r \) is the radius of the outer ripple measured in inches. The formula \( r = 3t \) gives the radius of the outer ripple after \( t \) seconds. Determine the area of the outer ripple after 5 seconds. Write your answer in exact form (in terms of \( \pi \)) and in approximate form, rounded to the nearest hundredth. Include appropriate units in your answer.

35. The formula for the surface area, \( S \), of a cylinder of radius \( r \) and height \( h \) is \( S = 2\pi r^2 + 2\pi rh \). Determine the surface area of a cylinder with radius 5 inches and height 4 inches. Give the exact answer (with \( \pi \)) and the approximate answer, rounded to the nearest hundredth. Include appropriate units in your answer.

36. Given that \( 2x + 1 = 5 \), evaluate each of the following expressions. Simplify your answers.
   a. \( 8(2x + 1) \)   b. \( \frac{1}{2x+1} \)   c. \( (2x + 1)^2 \)   d. \( (2x + 1) + 6 \)
   e. \( (2x + 1) – 8 \)   f. \( 2x + 5 \)   g. \( 2x – 1 \)   h. \( 2x \)   i. \( 6x + 3 \)

37. The formula when interest is compounded \( n \) times per year is \( A = P \left(1 + \frac{r}{n}\right)^{nt} \) where \( A \) is the accrued amount after \( t \) years, \( P \) is the starting principal, and \( r \) is the interest rate, expressed as a decimal, that is compounded \( n \) times per year. If you invest $1000 at an interest rate of 7%, and leave it there for 30 years, determine your ending balance if the interest is compounded
   a. Once each year   b. Twice each year   c. Monthly   d. Daily
   e. Explain what happens to the ending balance as the number of compoundings increases. Why does this occur?
1. Evaluate the following expressions for $a = -5$, $b = -3$, and $c = 2$.
   
   a. $|a + b|$
   
   b. $\frac{-b}{2a}$
   
   c. $b^2 - 4ac$

2. A caterer charges a delivery fee of $80 plus $6 per guest.
   
   a. Write an algebraic expression to represent the total catering cost if $n$ guests attend the reception.

   b. Determine the total catering cost if 350 guests attend the reception. Show your work and write your answer in a complete sentence.

3. Ivan invested money into two mutual funds. Fund A earned 8% profit during the first year while Fund B suffered a 3% loss.
   
   a. Let $A$ represent the amount of money invested in Fund A, and let $B$ represent the amount of money invested in Fund B. Write an algebraic expression for the total profit Ivan earned from both mutual funds during the first year.

   b. Use this expression to determine the total amount of profit earned during the first year if he invested $3,800 in Fund A and $2,400 in Fund B.

4. A toy rocket is shot straight up into the air. The formula $H = -16t^2 + 118t + 4$ gives the height (in feet) of the rocket after $t$ seconds. Determine the height of the rocket after 3.8 seconds. Round your answer to the nearest tenth.
5. The formula for the volume, \( V \), of a cylinder of radius \( r \) and height \( h \) is \( V = \pi r^2 h \).
Determine the volume of a cylinder with radius 10 inches and height 7 inches. Give your answer in **exact form** (simplified completely) and **approximate form**, rounded to the nearest hundredth. Indicate the correct **units** for your answer.

   Exact answer: ______________
   Approximate answer: ______________
   Units: ______________

6. The formula \( V = 9.54 + 0.08m \) represents the value of an investment (in thousands of dollars) after \( m \) months. Determine the value of this investment after two years.

7. The formula \( E = 3861 - 77.2t \) gives the surface elevation (in feet above sea level) of Lake Powell \( t \) years after 1999. Use this formula to predict the surface elevation of Lake Powell in the year 2016.

8. Simplify completely. When applicable, write your answers in descending order.
   a. \( 3x^2 - 5x - x^3 - 14x^2 + x - 2x^3 \)
   b. \( 5 + 2(3x - 7) \)
   c. \( 2(5x + 3y) - (3x + 6y) \)
   d. \( 3x^2 - 4h - 3(x + h) \)
   e. \( \frac{3}{5} \left( \frac{5}{8}x + \frac{1}{12} \right) \)
   f. \( \frac{5x+1}{5} \)
Unit 3: Solving Equations

Section 3.1: Algebraic Equations

Section 3.2: Solving One-Step Equations

Section 3.3: Solving Two-Step Equations

Section 3.4: Solving Multi-Step Equations

Section 3.5: Solving Equations – Applications

Section 3.6: Writing and Solving Equations

Section 3.7: Percents and Equations

Section 3.8: Equations and the Story of x

Section 3.9: Literal Equations

UNIT 3 LEARNING OBJECTIVES

- Create equations that describe numbers or relationships
- Verify that a given value is a solution to an algebraic equation
- Tell the story of x in an equation and describe solving as a process of undoing each operation
- Use additive and multiplicative identities to solve linear equations
- Apply the process of problem solving to solve applied problems involving a linear equation
- Assign a variable to an unknown or varying quantity and construct an algebraic equation based on reasonableness
- Model, solve and interpret solutions to contextual problems
- Validate solution(s) through a reasonable mathematical defense

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

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**Unit 3: Media Lesson**

**Section 3.1: Algebraic Equations**

<table>
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<th>Definition</th>
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| An algebraic equation is a mathematical sentence stating that an algebraic expression is equal to a specified value, variable, or another expression.  

The solution to an equation is the value, or values, that make the equation true. |

**Example 1:** Verify that $x = -3$ is a solution to the algebraic equation $5x - 2 = 8x + 7$.

**Example 2:** Is $m = -1$ a solution to the algebraic equation $m + 9 = 3m + 5$?

**Example 3:** Is $a = 5$ a solution to the algebraic equation $-4(a + 1) = 6(1 - a)$?
### Definition

**Equivalent equations** are two or more equations that have the same solution.

---

**Example 4:** Verify that \( x = 2 \) is a solution to the following equations.

\[
\begin{align*}
8x - 5 &= x + 9 \\
7x - 5 &= 9 \\
7x &= 14
\end{align*}
\]

---

**Section 3.1 – YOU TRY**

Complete the following problems. Show all steps as in the media examples.

a. Verify that \( p = -9 \) is a solution to the algebraic equation \( p - 4 = 2p + 5 \).

b. Verify that \( x = 2 \) is a solution to the algebraic equation \( 2(5x - 12) = 1 - 5(x - 1) \).
Section 3.2: Solving One-Step Equations

Properties of Equality

The Addition/Subtraction Property of Equality:
If \( a = b \), then \( a + c = b + c \).  
If \( a = b \), then \( a - c = b - c \).

The Multiplication/Division Property of Equality:
If \( a = b \), then \( a \times c = b \times c \).  
If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).

Definition

To solve an equation means to “undo” all the operations of the equation, leaving the variable by itself on one side. This is known as isolating the variable.

Solve for the variable in each of the following equations. Check your answers.

Example 1: \( x + 7 = 18 \)

Example 2: \( r - 4 = -5 \)

Example 3: \( -4 + b = 45 \)

Example 4: \( 3 = 19 + m \)

Example 5: \( -3y = -42 \)

Example 6: \( \frac{x}{6} = -5 \)
Example 7: \( \frac{3}{4} a = 8 \)  

Example 8: \( 17 = -x \)

Section 3.2 – YOU TRY

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

a. \( 12 + x = -40 \)  
b. \( \frac{3}{5} n = -2 \)

c. \( 14 = -x \)  
d. \( -3 = \frac{w}{5} \)
Section 3.3: Solving Two-Step Equations

<table>
<thead>
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<th>STEPS FOR SOLVING A LINEAR TWO-STEP EQUATION</th>
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<td>1. Apply the Addition/Subtraction Property of Equality.</td>
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<td>2. Apply the Multiplication/Division Property of Equality to isolate the variable.</td>
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<td>3. Check by substituting your answer into the original equation.</td>
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Solve for the variable in each of the following equations. Check your answers.

**Example 1:** Solve: \(2b - 4 = 12\)  
Check:

**Example 2:** Solve: \(4 + 3r = 5\)  
Check:

**Example 3:** Solve: \(3 = 19 - 2m\)  
Check:

**Example 4:** Solve: \(11 - y = 32\)  
Check:
Example 5: Solve: $3 + \frac{3}{5}x = 12$  

Section 3.3 – YOU TRY

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

a. Solve: $14 - 3x = -40$  

b. Solve: $\frac{3}{4}w - 8 = -2$  

c. Solve: $14 = 2 - x$
Section 3.4: Solving Multi-Step Equations

<table>
<thead>
<tr>
<th>STEPS FOR SOLVING A LINEAR EQUATION</th>
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<tbody>
<tr>
<td>1. Simplify each side of the equation. Remove parenthesis if necessary. Combine like terms.</td>
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<tr>
<td>2. Add or subtract terms on each side of the equation so that all terms containing the variable are on one side and all constant terms are on the other side.</td>
</tr>
<tr>
<td>3. Simplify each side of the equation by combining like terms.</td>
</tr>
<tr>
<td>4. Apply the Multiplication/Division Property of Equality to isolate the variable.</td>
</tr>
<tr>
<td>5. Check by substituting the solution into the original equation.</td>
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Solve for the variable in each of the following equations. Check your answers.

**Example 1:** Solve \( x - 5 = 4x + 7 \)  

**Example 2:** Solve \( 3(4n - 2) = 5(n + 3) \)
**Example 3:** Solve $4 - (2y - 1) = 2(5y + 9) + y$

**Check:**

---

**Section 3.4 – You Try**

Solve for the variable in each equation and check your answer. Show all steps as in the media examples.

a. Solve $m - 5 = 8m + 2$

   **Check:**

b. Solve $2(5x - 12) = -(5x - 6)$

   **Check:**
Section 3.5: Solving Equations – Applications

For this type of problem, first determine the Givens and the Goal, then form a Strategy, Solve, and Check. Write your answer in a complete sentence.

**Example 1:** The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate (pulse) is about 70% percent of your maximum heart rate. The formula \( T = 0.7(220 - a) \), gives the target heart rate, \( T \), in beats per minute, for a person who is \( a \) years of age. Determine the age of a person whose target heart rate is 135 beats per minute.

**GIVEN:**

**GOAL:**

**STRATEGY:**

**SOLUTION:**

**CHECK:**

**FINAL RESULT AS A COMPLETE SENTENCE:**
Example 2: Tristan got a job selling frozen bananas at the local fairground. He is paid a fixed amount of $160 per week plus $0.67 per item sold. This is represented by the equation \( P = 0.67n + 160 \) where \( P \) is the amount he is paid for the week, and \( n \) is the frozen bananas he sold. Use this equation to answer the following.

a. How much will Tristan be paid if he sells 340 frozen bananas?

b. How many frozen bananas did Tristan sell if he earned $535.20?

Section 3.5 – YOU TRY

The cost of tuition at a local community college is given by the equation \( C = 76n \) where \( C \) represents the total cost of tuition and \( n \) represents the number of credits taken. If you have $800 dollars to spend on tuition, how many credits can you take?
## Section 3.6: Writing and Solving Equations

<table>
<thead>
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<th>Steps for Writing and Solving Equations</th>
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<tr>
<td>1. Read and understand the problem. Underline the givens and circle the goal.</td>
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<tr>
<td>2. Form a strategy to solve the problem.</td>
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<tr>
<td>3. Choose a variable to represent the unknown quantity.</td>
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<tr>
<td>4. Read every word in the problem and translate the given information into an algebraic equation.</td>
</tr>
<tr>
<td>5. Solve the equation</td>
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<tr>
<td>6. Write your answer in a complete sentence</td>
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**Example 1:** The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

**Example 2:** You have just bought a new Sony 55” 3D television set for $1,600. The value of the television set decreases by $250 per year. How long before the television set is worth half of its original value?
Example 3: When purchasing a new cell phone plan you are given two pay options (both include unlimited text). With Plan A, you pay $0.35 per minute. With plan B, you pay $0.15 per minute plus a flat fee of $10.00.

Write an algebraic expression to represent the cost for plan A if you talk for m minutes.

m = ____________________

Cost for Plan A = ____________________

Write an algebraic expression to represent the cost for plan B if you talk for m minutes.

Cost for Plan B = ____________________

How many minutes would you need to use for the two plans to be equal?

Write an equation to represent this situation: ________________________________

SOLVE this equation and use your answer to complete the following sentence. Round your answer to the nearest minute.

The cost for these plans is the same if you talk for _____________ minutes.

Section 3.6 – YOU TRY

Your yard is a mess, and you decide to hire a landscaper. The Garden Pros charges a $50 consultation fee plus $36 per hour for the actual work. If the total cost is $212, how many hours did the landscapers work?

a. Write an equation to represent this situation. Clearly indicate what the variable represents.

b. Solve the equation. Show all work and write your answer in a complete sentence. Your answer must include correct units of measure.
Section 3.7: Percents and Equations

**Example 1:** A $750 watch is on sale for 15% off. Find the sale price.

**Example 2:** A salesman tells you that the $140 earrings are already marked 20% off. What was the original price?

**Example 3:** Tommy’s grandma gave him a $50 gift card to Toys R Us for his birthday. Sales tax is currently 9%. Determine the price of the most expensive toy Tommy can buy with the $50 gift card

---

**Section 3.7 – YOU TRY**

A lender requires a down payment of 20% toward the value of a new home. Rita has $32,700 to use as a down payment. What is the maximum home value that she can afford?
Section 3.8: Equations and the Story of $x$

Not all equations look the same. It is important that you are able to think about solving an equation as a process of undoing the operations on the variable $x$. The following examples will show you another way to think about solving equations.

**Example 1:** Tell the story of $x$ in the expression $x + 7$, and use this to determine the solution to the equation $x + 7 = 18$.

**Example 2:** Tell the story of $x$ in the expression $4x$, and use this to determine the solution to the equation $4x = 28$.

**Example 3:** Tell the story of $x$ in the expression $5x + 1$, and use this to determine the solution to the equation $5x + 1 = 21$. 
Example 4: Tell the story of $x$ in the expression $2(x + 1) - 3$, and use this to determine the solution to the equation $2(x + 1) - 3 = 5$.

Example 5: Tell the story of $x$ in the expression $\frac{3x-4}{2}$, and use this to determine the solution to the equation $\frac{3x-4}{2} = 7$.

Section 3.8 – YOU TRY

Tell the story of $x$ in the expression $\frac{x+6}{3}$ and use this to determine the solution to the equation $\frac{x+6}{3} = 5$. Write all steps of your solution process, as shown in the media examples.
Section 3.9: Literal Equations

What are literal equations? __________________________________________________________

What does it mean to “solve” a literal equation? _______________________________________

**Example 1:** Solve for $b$ in each of the following equations.

\[ 2b = 8 \quad ab = c \]

\[ 5 + b = 9 \quad a + b = c \]

\[ 2b + 1 = 13 \quad ab + c = d \]

**Example 2:** Solve the following equation for $c$: $4abc = 32$

**Example 3:** Solve the following equation for $B$: $A = B + C + D$

**Example 4:** Solve the following equation for $x$: $y = mx + b$
Example 5: Solve the following equation for y: \(3x + 4y = 20\)

Example 6: Solve the following equation for y: \(x - y = 5\)

Example 7: Solve the following equation for C: \(F = \frac{9}{5}C + 32\)

Section 3.9 – You Try

Solve the following equation for y. Simplify your answer. \(3xy = 12\)
Unit 3: Answers to You-Try Problems

Section 3.1:  

a. \( p = -9 \) is a solution to the algebraic equation \( p - 4 = 2p + 5 \).

b. \( x = 2 \) is a solution to the algebraic equation \( 2(5x - 12) = 1 - 5(x - 1) \).

Section 3.2:  

a. \( x = -52 \)  

b. \( n = -\frac{10}{3} = -\frac{1}{3} \)  

c. \( x = -14 \)  

d. \( w = -15 \)

Section 3.3:  

a. \( x = 18 \)  

b. \( w = 8 \)  

c. \( x = -12 \)

Section 3.4:  

a. \( m = -1 \)  

b. \( x = 2 \)

Section 3.5: You can take 10 credits. ($800 is not enough to pay for 11 credits)

Section 3.6:  

a. \( 50 + 36h = 212, \ h= \) the number of hours worked  

b. If the cost is $212, then the landscapers worked for 4.5 hours.

Section 3.7: She can afford a $163,500 home.

Section 3.8: \( x = 9 \)

Section 3.9: \( y = \frac{4}{x} \)
1. Verify that \( a = -1 \) is a solution to \( 4 - a = 6a + 11 \).

2. Verify that \( x = -5 \) is a solution to \( 3(2x + 4) = 8(x + 2) + 6 \).

3. Is \( x = 8 \) a solution to the equation \( -16 = \frac{3}{4}x - 10 \)?

4. Is \( x = -3 \) a solution to the equation \( 3(6 + 2x) = 8 + (x - 5) \)?

5. Tell the story of \( x \) in the expression \( 5x + 7 \), and use this to determine the solution to the equation \( 5x + 7 = 62 \).

6. Tell the story of \( x \) in the expression \( 4(x - 8) + 2 \), and use this to determine the solution to the equation \( 4(x - 8) + 2 = 22 \).

7. Tell the story of \( x \) in the expression \( \frac{4x - 2}{3} \), and use this to determine the solution to the equation \( \frac{4x - 2}{3} = 10 \).

8. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.
   a. \( 8x - 2 = 30 \)
   b. \( 5 - x = 3 \)
   c. \( \frac{1}{2}x - 4 = 8 \)
   d. \( \frac{2}{3}x + 3 = 15 \)
   e. \( 4x - 8 = -x + 7 \)
   f. \( \frac{3}{4}x - \frac{1}{2} = \frac{9}{8}x + \frac{3}{2} \)
   g. \( 6x - 4(-2x + 8) = 10 \)
   h. \( 2(4 - 4x) = 5 - (2x - 3) \)
   i. \( 7 + 4(x + 6) = -(2x + 5) \)

9. Solve for the variable in each of the following equations. Round your answers to the nearest hundredth as needed (TIP: Do not round until your final answer!) Show all steps, and box your answer.
   a. \( 3.24a = 6.39 \)
   b. \( 6.77w - 18.02 = 7.96 \)
   c. \( 3.05(2.2b + 3.97) = 3.88b \)
   d. \( n + 1.08n = 3.57 \)
10. Solve the following equation. Give your answer in exact form (as an integer, proper fraction, or mixed number in simplest form) and in approximate form, rounded to the nearest thousandth. Show your work.

\[7x - 4 = 18\]

11. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.

a. \[\frac{z + 9}{5} = 11\]

b. \[\frac{5x - 8}{2} = 6\]

12. Solve the following equations for the given variable. Show all steps. Simplify your answers.

a. \[I = Prt \quad \text{Solve for } t\]

b. \[2x + 3y = 6 \quad \text{Solve for } y\]

c. \[A = B(C + D) \quad \text{Solve for } D\]

d. \[A = p + prt \quad \text{Solve for } r\]

e. \[6x - y = 11 \quad \text{Solve for } y\]

f. \[A = P(1 + r)^t \quad \text{Solve for } P\]

g. \[r = \frac{C}{2\pi} \quad \text{Solve for } C\]

h. \[3x - 5y = 8 \quad \text{Solve for } y\]

i. \[P = A - B - C \quad \text{Solve for } B\]

j. \[ax^2 + bx + c = 0 \quad \text{Solve for } b\]

Applications

For each of the following, underline the Givens and circle the Goal of the problem. Form a Strategy, Solve, and Check. Show all work, and write your answers in complete sentences.

13. John is a door to door vacuum salesman. His weekly salary, \(S\), is $200 plus $50 for each vacuum he sells. This can be written as \(S = 200 + 50v\), where \(v\) is the number of vacuums sold. If John earns $1000 for a week’s work, how many vacuums did he sell?

14. Paul is planning to sell bottled water at the local Lollapalooza. He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul’s profit, \(P\) in dollars, from selling \(b\) bottles of water is given by the formula \(P = 1.5b - 360\). How many bottles does Paul need to sell in order to break even?

15. Ringo has $100 in the bank and is adding $50 each week in savings. George has $250 in the bank, and is adding $40 each week in savings. Their plan is to wait until their savings are equal and then buy a Magic Yellow Bus and take a road trip. They figure out that the equation can be written as \(50w + 100 = 40w + 250\), where \(w\) is the number of weeks. How long will it take for their savings to be equal?
16. The formula for the area, $A$, of a triangle with base $b$ and height $h$ is $A = \frac{1}{2} bh$. Determine the height of a triangle with a base of 18 inches and area 84.6 square inches. Round your answer to the nearest tenth, and include appropriate units in your answer.

17. Suppose you want to accumulate $1,000,000 for your retirement in 30 years. You decide to put money into an account that earns 3% interest compounded annually. How much should you deposit? The formula for compound interest is $A = P(1 + r)^t$, where $A$ is the accrued amount after $t$ years, $P$ is the starting principal, and $r$ is the annual interest rate expressed as a decimal. Round your answer up to the nearest cent. (TIP: Do not round until your final answer!)

18. Andrew and Andrea want to start a college fund for their baby girl. They decide to put money into an investment that is expected to earn 4.2% simple interest each year. How much would they have to deposit now in order to accumulate $100,000 by the time their newborn goes to college in 18 years? The formula for simple interest is $A = P + Prt$, where $A$ is the accrued value of the investment after $t$ years, $r$ is the interest rate (expressed as a decimal), and $P$ is the starting principal invested. Round your answer up to the nearest cent.

19. February is a busy time at Charlie’s Chocolate Shoppe! During the week before Valentine’s Day, Charlie advertises that his chocolates will be selling for $1.80 a piece (instead of the usual $2.00 each). The fixed costs to run the Chocolate Shoppe total $450 for the week, and he estimates that each chocolate costs about $0.60 to produce. Write an equation to represent Charlie’s profit, $P$, from selling $n$ chocolates during the week before Valentine’s Day. (HINT: Profit = Revenue – Total Costs) use this equation the number of Chocolates Charlie will need to sell in order to break even.

20. A new Sony 55” 3D television set costs $2,499. You are going to pay $600 as a down payment, and pay the rest in equal monthly installments for one year. Write an equation to represent this situation, and use it to determine how much you should pay each month. Clearly indicate what the variable in your equation represents.

21. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $20 consultation fee plus $11 per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $15 per hour for the actual work. Write an equation that will help you determine the number of hours at which the two companies charge the same. Clearly indicate what the variable represents. Solve the equation, and write your answer in a complete sentence.
22. Let $p$ represent the marked price of an item at Toys R Us. Emma’s aunt gave her a $50 gift card to Toys R Us for her birthday. If sales tax is currently 9%, set up an equation to express how much she can spend using her gift card. Solve the equation, and interpret your answer in a complete sentence.

23. Solve the following equations for the given variable. Show all steps. Simplify your answers.
   a. $A = \frac{2}{3}BC$  Solve for $B$  
   b. $A = P + Prt$  Solve for $P$

24. The surface area of a cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$. Solve this equation for $h$. Show your work. Simplify your answer.

25. The maximum heart rate, $M$, is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate is between 60 and 80 percent of your maximum heart rate. Let $T$ represent your target heart rate. Write a compound inequality to represent this situation.

26. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.
   a. $2(4x + 3) = 8x + 1$  
   b. $5(x + 6) - x = 4(x + 7) + 2$

27. Solve the following nonlinear equations.
   a. $x^2 = 25$  
   b. $x^3 = 27$  
   c. $|x| = 3$
   d. $\sqrt{x} = 7$  
   e. $\sqrt[3]{x} = 2$  
   f. $\frac{1}{x} = 4$

28. Write a story problem for the equation shown below. Solve the problem, and write your answer in a complete sentence.
   
   $300 - 50x = 0$
Unit 3: Review

1. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Write your answer as an integer, proper fraction, or mixed number in simplest form. Show all steps.
   a. \(5 - x = 18\)
   b. \(\frac{3}{7}x + 3 = 10\)
   c. \(\frac{3x - 8}{2} = 11\)
   d. \(2(4 - 4x) = 5 - (2x - 3)\)

2. Consider the equation \(y = mx + b\). Simplify your answers.
   a. Solve for \(x\)
   b. Solve for \(b\)

3. Tristan's goal is to find a job that provides an income of $50,000 a year. Esmeralda's Furniture offers him a job paying a base salary of $28,000 a year, plus a commission of 4% of his sales. Determine what Tristan's total sales will need to be for him to have a yearly income of $50,000.
5. Randall got a job selling bottles of water at the local carnival. He is paid a fixed amount of $50 per week plus $0.70 per item sold. This is represented by the equation \( P = 0.7n + 50 \) where \( P \) is the amount he is paid for the week and \( n \) is the number of bottles of water he sold.
   a. How much will Randall get paid if he sells 100 bottles of water?

   b. How many bottles of water does Randall have to sell to get paid at least $100 for the week?

6. The Aquarium charges $21 for adult admission and $11 for each child. The Aquarium bill for a school field trip was $575.
   a. Write an equation to represent this situation. Let \( C \) represent the number of children and \( A \) represent the number of adults attending the field trip.

   b. If 8 adults attended the field trip, how many children were there? Show all of your work and write your answer in a complete sentence.
Unit 4: Inequalities and Absolute Value

Section 4.1: Linear Inequalities
Section 4.2: Compound Inequalities
Section 4.3: Expressing Intervals
Section 4.4: Solving Linear Inequalities
Section 4.5: Solving Inequalities – Applications
Section 4.6: Absolute Value Equations and Inequalities

UNIT 4 LEARNING OBJECTIVES

- Create inequalities that describe numbers or relationships
- Identify values in the solution set of a linear inequality
- Graph the solution set of a linear inequality
- Write inequalities in interval notation
- Use additive and multiplicative identities and properties to solve linear inequalities in one variable

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inequality Symbols</td>
<td></td>
</tr>
<tr>
<td>Algebraic Inequality</td>
<td></td>
</tr>
<tr>
<td>Compound Inequality</td>
<td></td>
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<tr>
<td>Interval Notation</td>
<td></td>
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<tr>
<td>-------------------</td>
<td>---</td>
</tr>
<tr>
<td>Solution to an Algebraic Inequality</td>
<td></td>
</tr>
<tr>
<td>Solution Set</td>
<td></td>
</tr>
</tbody>
</table>
Section 4.1: Inequalities

<table>
<thead>
<tr>
<th>Symbol</th>
<th>In words</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
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<td>≤</td>
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<td>≥</td>
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<tr>
<td>≠</td>
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</tbody>
</table>

Definitions

An algebraic inequality is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign.

A solution to an inequality is a value that makes the inequality true.

Example 1: Determine whether the number 4 is a solution to the following inequalities.

\[ x > 1 \quad x < 1 \quad x \leq 9 \quad x > 4 \quad x \geq 4 \]

THE SOLUTION SET OF A LINEAR INEQUALITY

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &gt; 2 )</td>
<td>![Graph]</td>
<td>(-\infty \quad \rightarrow \infty)</td>
</tr>
<tr>
<td>( x \geq 2 )</td>
<td>![Graph]</td>
<td>(-\infty \quad \rightarrow \infty)</td>
</tr>
<tr>
<td>( x &lt; 2 )</td>
<td>![Graph]</td>
<td>(-\infty \quad \rightarrow \infty)</td>
</tr>
<tr>
<td>( x \leq 2 )</td>
<td>![Graph]</td>
<td>(-\infty \quad \rightarrow \infty)</td>
</tr>
</tbody>
</table>
Example 2: Write an inequality to represent the following situation. Clearly indicate what the variable represents.

a. In order to go on the ride, a child must be more than 48 inches tall.

b. Jordan can spend at most $10 on lunch.

Section 4.1 – You Try

Complete the following problems.

a. Which of the following values are in the solution set for \( n < 5 \) ?

\[
\begin{align*}
n &= -3 & n &= 0 & n &= 4.99 & n &= 5 & n &= 12
\end{align*}
\]

b. Translate the statement into an inequality. Let \( a \) represent the age of a child.

Children age 2 and under are free at Disneyland. Inequality: ______________

c. Complete the table below:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x \geq -3 )</td>
<td>(-\infty ) ( \rightarrow ) ( \infty )</td>
<td>(-\infty, 11] )</td>
</tr>
</tbody>
</table>
# Section 4.2: Compound Inequalities

The solution set of a COMPOUND inequality

<table>
<thead>
<tr>
<th>Inequality Notation</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1 &lt; x &lt; 2$</td>
<td>$\leftarrow -\infty \rightarrow 2$</td>
<td>$-\infty \rightarrow \infty$</td>
</tr>
<tr>
<td>$-1 \leq x \leq 2$</td>
<td>$\leftarrow -\infty \rightarrow 2$</td>
<td>$-\infty \rightarrow \infty$</td>
</tr>
<tr>
<td>$-1 \leq x &lt; 2$</td>
<td>$\leftarrow -\infty \rightarrow 2$</td>
<td>$-\infty \rightarrow \infty$</td>
</tr>
</tbody>
</table>

**Example 1**: Which of the following values are in the solution set for $-3 \leq n < 5$?

- $n = -5$
- $n = -3$
- $n = 0$
- $n = 4.99$
- $n = 5$
- $n = 12$

![Example 1 Graph]

**Example 2**: Write a compound inequality to represent the following situation. Clearly indicate what the variable represents.

a. A number is greater than or equal to 5 but less than 8.

b. My car’s tank can hold a maximum of 20 gallons of gas.
Section 4.2 – You Try

Complete the following problems.

a. Which of the following values are in the solution set for $-8 \leq w < 2$? Show your work.

\[ w = -11 \quad w = -8 \quad w = -5 \quad w = 0 \quad w = 2 \quad w = -8.5 \]

b. Complete the table below:

<table>
<thead>
<tr>
<th>Inequality Notation</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; x &lt; 4$</td>
<td>(-\infty &lt; x &lt; \infty)</td>
<td>((-3, 1])</td>
</tr>
<tr>
<td>$-\infty &lt; x &lt; \infty$</td>
<td>(-\infty &lt; x &lt; \infty)</td>
<td>((-3, 1])</td>
</tr>
<tr>
<td>$\infty &lt; x &lt; \infty$</td>
<td>(-\infty &lt; x &lt; \infty)</td>
<td>((-3, 1])</td>
</tr>
</tbody>
</table>
### Section 4.3: Expressing Intervals

**Example 1:** Complete the table below.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; -5 \text{ or } x \geq -1$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td>$(-\infty,-6] \cup [-4,\infty)$</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
<td>\geq 2$</td>
</tr>
<tr>
<td></td>
<td><img src="image3.png" alt="Graph" /></td>
<td>$(-\infty,0] \cup [0,\infty)$</td>
</tr>
<tr>
<td></td>
<td><img src="image4.png" alt="Graph" /></td>
<td>$(-\infty,2] \cup [2,\infty)$</td>
</tr>
</tbody>
</table>
Consider the graphs. Write your answers in interval notation and in inequality notation.

a.

Interval Notation: ________________________

Inequality notation: _______________________

b.

Interval Notation: ________________________

Inequality notation: _______________________
Section 4.4: Solving Linear Inequalities

**STEPS FOR SOLVING A LINEAR INEQUALITY**

1. Simplify each side of the inequality. Remove parenthesis if necessary. Collect like terms.
2. Add or subtract terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side.
3. Simplify each side of the inequality by combining like terms.
4. Multiply or divide on both sides to isolate the variable. CAUTION!!! *If you multiply or divide both sides of an inequality by a negative number, you have to reverse the inequality sign.*
5. Check by substituting the solution (*endpoint and a value from the solution set*) into the original inequality.

**Example 1:** Solve the inequality, check your answer, and graph the solution on a number line.

\[3x > x + 6\]

Graph:

\[\infty \leftarrow \ldots \rightarrow \infty\]

Interval Notation: ________________

**Example 2:** Solve the inequality and graph the solution on a number line.

\[3 - 5a \leq 2(a + 5)\]

Graph:

\[\infty \leftarrow \ldots \rightarrow \infty\]

Interval Notation: ________________
Example 3: Solve the inequality and graph the solution on a number line.

\[-5(x + 2) \geq -3(x + 4)\]

Graph:

\[-\infty \leftarrow \rightarrow \infty\]

Interval Notation: ________________

Section 4.4 – You Try

Solve the inequality, check your answer, and graph the solution on a number line. Show all steps as in the media examples.

a. \(7 - 4x \geq -5\)

Graph:

\[-\infty \leftarrow \rightarrow \infty\]

Interval Notation: ________________
Section 4.5: Solving Inequalities – Applications

**Example 1:** The cost of tuition is $76 per credit hour. Write an inequality that can be used to determine the number of credit hours a student can take for under $1000. Solve the inequality, and write your answer in a complete sentence.

**Example 2:** Sean owns a business that builds computers. The fixed operating costs for his business are $2,700 per week. In addition to fixed operating costs, each computer costs $600 to produce. Each computer sells for $1,500. Write an inequality that can be used to determine the number of computers Sean needs to sell in order make a profit each week. Solve the inequality, and write your answer in a complete sentence.

**Example 3:** You have taken 4 of 5 tests for your math class. What must you score on the last test to have a mean or average of 90% or higher?

Scores: 87%, 82%, 91%, 95%, %
Complete the following problems.

1. Gasoline costs $2.98 per gallon. Determine how many gallons of fuel can be purchased for under $20.
   a. Write an inequality that can be used to determine how many gallons of fuel can be purchased for under $20. Clearly indicate what the variable represents.
   b. Solve the inequality in part a, and write your answer in a complete sentence. Show all steps as in the media examples. Round your answer to the nearest hundredth.

2. Dustin's goal is to find a job that provides an income of at least $45,000 a year. Nicole's Furniture offers him a job paying a base salary of $23,000 a year, plus a commission of 8% of his sales. Determine what Dustin's total sales will need to be for him to have a yearly income of at least $45,000.
   a. Write an inequality that can be used to determine what Dustin's total sales will need to be for him to have a yearly income of at least $45,000. Clearly indicate what the variable represents.
   b. Solve the inequality in part a, and write your answer in a complete sentence. Show all steps as in the media examples. Round your answer to the nearest dollar.
**Section 4.6: Absolute Value Equations and Inequalities**

### Absolute Value

\[ -\infty \quad \text{____________} \quad \text{____________} \quad \infty \]

**Example 1:** Evaluate the following:  
\[ |2| = \quad |–2| = \]

### Absolute Value Equations

Determine the solution to each of the following equations.

**Example 2:**  
\[ |x| = 2 \quad |x| = 3 \quad |x| = –4 \]

### Absolute Value Inequalities

<table>
<thead>
<tr>
<th>Inequality</th>
<th>List some values in the solution set: _____________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>x</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
</tr>
</tbody>
</table>
Complete the following problems.

a. Determine the solutions to the following absolute value equations. Write DNE if the solution does not exist.

\[ |x| = 8 \quad |m| = -9 \quad |a| = 0 \]

b. Absolute Value Inequalities: For each of the following, list some values in the solution set, then draw the graph on the number line provided. Write the solution set in interval notation.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>List some values in the solution set:</th>
<th>Interval Notation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>x</td>
<td>&lt; 3 )</td>
</tr>
</tbody>
</table>
Unit 4: Answers to You-Try Problems

Section 4.1:  

| a. \( n = -3, \ 0, \ 4.99 \)
| b. \( a \leq 2 \)
| c. 
| \( x \geq -3 \) & Use “]” or “●” for the endpoint at -3 & \([-3, \infty)\) 
| \( x \leq 11 \) & Use “[” or “●” for the endpoint at 11 & \((-\infty, 11]\) 
| \( x < 2 \) & \((-\infty, 2)\) 

Section 4.2:  

| a. \( w = -8 \) \( w = -5 \) \( w = 0 \)
| b. 
| \( 0 < x < 4 \) & \((-\infty, 4)\) 
| \(-3 < x \leq 1 \) & \((-3, 1]\) 
| \(-1 \leq x < 4 \) & \([-1, 4)\) 

Section 4.3:  

| a. Interval Notation: \((-\infty,-3]\cup(0,\infty)\) Inequality notation: \( x \leq -3 \) or \( x > 0 \)
| b. Interval Notation: \((-\infty,-2)\cup(-2,\infty)\) Inequality notation: \( x < -2 \) or \( x > -2 \), \( x \neq -2 \)

Section 4.4:  

| a. \( x \leq 3 \), Interval Notation: \((-\infty, 3]\) 

Section 4.5:  

1. \( 2.98g < 20 \), where \( g \) = the number of gallons pumped  
   b. Up to 6.71 gallons of gasoline can be purchased for under $20 
2. \( x = \) Dustin’s sales \( 23000 + 0.08x \geq 45000 \) or \( 45000 \leq 23000 + 0.08x \)  
   b. \( x \geq 275000 \) Dustin will need to make a minimum of $275,000 in sales in order to have a yearly income of at least $45,000
Section 4.6:  

a. $x = \{-8, 8\}$  \quad m DNE \quad a = 0

b.  

| $|x| < 3$    | Values in solution set: $-2, -1, 0, 1, 2$ Answers will vary |
|-----------|-----------------------------------------------------------------|
| $|x| \geq 3$ | Values in solution set: $-5, -4, 4, 5$ Answers will vary       |

Interval Notation: $(-\infty, 3)$

Interval Notation: $(-\infty, -3] \cup [3, \infty)$
1. For each of the following, circle all correct answers.

a. Which of the given values are in the solution set for $x < 3$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]

b. Which of the given values are in the solution set for $x \geq -1$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]

c. Which of the given values are in the interval $[-2, \infty)$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]

d. Which of the given values are in the interval $(-\infty, -1)$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]

e. Which of the given values are in the interval $(-1, 5]$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]

f. Which of the given values are in the interval $-5 < x \leq 3$?
   \[x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \quad x = 5 \quad x = -\frac{5}{3}\]
2. Complete the table below:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; 8$</td>
<td><img src="image1" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td>$x \leq -1$</td>
<td><img src="image2" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td>$8 \leq x &lt; 12$</td>
<td><img src="image3" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image4" alt="Graph" /></td>
<td>$(-2, \infty)$</td>
</tr>
<tr>
<td></td>
<td><img src="image5" alt="Graph" /></td>
<td>$(-\infty, 6]$</td>
</tr>
<tr>
<td></td>
<td><img src="image6" alt="Graph" /></td>
<td>$(-10, -7]$</td>
</tr>
<tr>
<td></td>
<td><img src="image7" alt="Graph" /></td>
<td>$(-\infty, 2)\cup(5, \infty)$</td>
</tr>
<tr>
<td></td>
<td><img src="image8" alt="Graph" /></td>
<td>$(-\infty, 8)\cup[10, \infty)$</td>
</tr>
<tr>
<td></td>
<td><img src="image9" alt="Graph" /></td>
<td>$(-\infty, 1)\cup(1, \infty)$</td>
</tr>
</tbody>
</table>
3. Solve the inequalities, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.
   a. \(4x \leq 2x + 12\) 
   b. \(14m + 8 > 6m - 8\) 
   c. \(5(-2a - 8) \leq -9a + 4\)

4. Solve the following absolute value equations.
   a. \(|x| = 1\) 
   b. \(|x| = 13\) 
   c. \(|x| = -4\) 
   d. \(|x| = 0\)

5. ABSOLUTE VALUE. For each of the following, circle all correct answers.
   a. Which of the given values are in the solution set for \(|x| = 5\)?
      \[x = 0 \quad x = -1 \quad x = -5 \quad x = -7 \quad x = 3 \quad x = 5 \quad x = 9\]
   b. Which of the given values are in the solution set for \(|x| \geq 5|?\)
      \[x = 0 \quad x = -1 \quad x = -5 \quad x = -7 \quad x = 3 \quad x = 5 \quad x = 9\]
   c. Which of the given values are in the solution set for \(|x| < 5|?\)
      \[x = 0 \quad x = -1 \quad x = -5 \quad x = -7 \quad x = 3 \quad x = 5 \quad x = 9\]

6. Graph the solution set for the inequalities shown below.
   a. \(|x| < 1\)
      \[\infty \begin{array}{c|cccccccc} \mid \end{array} \infty \]
   b. \(|x| \geq 4\)
      \[\infty \begin{array}{c|cccccccc} \mid \end{array} \infty \]

Applications

7. Translate each of the given statements into an algebraic inequality.
   a. You must be at least 13 years of age in order to view a PG-13 movie. Let \(a\) represent your age.
   b. Your car’s gas tank can hold up to 25 gallons of gas. Let \(g\) represent the number of gallons in your gas tank.
   c. A company must sell more than 850 items in order to make a positive profit. Let \(n\) represent the number of items sold.
8. You have $1200 for your trip to the beach. You estimate that it will cost $160 a day for food, entertainment and hotel, plus $230 round trip air fair.
   a. Write an inequality that can be used to determine the maximum number of days you can stay at the beach. Clearly indicate what the variable represents.
   b. Solve the inequality, and interpret your answer in a complete sentence.

9. Let \( p \) represent the marked price of an item at Toys R Us. Bella’s aunt gave her a $100 gift card to Toys R Us for her birthday.
   a. If sales tax is currently 9%, set up an algebraic inequality to express how much she can spend using her gift card. Clearly indicate what the variable represents.
   b. Solve the inequality, and interpret your answer in a complete sentence.

10. Your car is worth $1000 at most. It is old. You find out that it needs repairs to pass inspection. The auto shop tells you that the parts cost a total of $520, and the labor cost is $68 per hour. If the repairs are more than the car is worth, you are going to donate the car to charity.
    a. Write an inequality that can be used to determine the maximum number of hours the mechanic can spend working on your car to help you decide to repair it or donate it. Clearly indicate what the variable represents.
    b. Solve the inequality, and interpret your answer in a complete sentence.

**Extension**

11. Solve the compound inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

   \[
   1 < x + 3 \leq 7 \quad -12 < 4n < 20 \quad 3 \leq 2v - 5 < 11
   \]

   \[
   -27 \leq -3x \leq 30 \quad 2 < 6 - 4m \leq 6
   \]

12. Solve the following absolute value equations, showing all steps.

   a. \(|x - 7| = 8\)
   b. \(3|x| - 6 = 11\)
   c. \(|5x + 3| + 4 = 22\)

   d. \(|4x| + 9 = 1\)
   e. \(|2x - 8| + 2 = 2\)
   f. \(2|x - 1| + 7 = 12\)
Unit 4: Review

1. Which of the given values are in the interval (–1, ∞)? Circle all that apply.
   
   \[ x = 0 \quad x = -1 \quad x = -5 \quad x = 3 \]

2. Which of the given values are in the interval (–3, 5]? Circle all that apply.
   
   \[ x = -8 \quad x = -2 \quad x = -3 \quad x = 5 \]

3. You have $1400 for your trip to the beach. You estimate that it will cost $250 a day for food, entertainment and hotel, plus $198 for round trip air fair.
   
   a. Write an inequality that can be used to determine the maximum number of full days you can stay at the beach. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.

4. Leah is trying to qualify for the local bowling championship. To qualify, a person must bowl five games and have a combined average score of at least 175. In the first four games, Leah’s scores were 180, 177, 155 and 195.

   a. Write an inequality that can be used to determine the minimum score that Leah must bowl in her fifth game to qualify for the championship. Clearly indicate what the variable represents.

   b. Solve the inequality, and interpret your answer in a complete sentence.
5. Complete the table below.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 0$</td>
<td>$\infty$</td>
<td>$(-\infty, 0)$</td>
</tr>
<tr>
<td>$-2 &lt; x \leq 1$</td>
<td>$\infty$</td>
<td>$[-3, 1]$</td>
</tr>
<tr>
<td>$x &lt; 1 \text{ or } x \geq 4$</td>
<td>$\infty$</td>
<td>$(-\infty, 1) \cup [4, \infty)$</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
<td>&lt; 4$</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
<td>\geq 1$</td>
</tr>
</tbody>
</table>

6. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

$$1 - 3x > 14 - (4 - 6x)$$

Interval Notation: $(-\infty, \infty)$

Graph:
Unit 5: Graphs and Functions

Section 5.1: Characteristics of Graphs
Section 5.2: Constructing a Graph from Data
Section 5.3: Equations and Graphs
Section 5.4: Using Your Graphing Calculator
Section 5.5: Relations and Functions
Section 5.6: Function Notation
Section 5.7: Formulas in Function Notation
Section 5.8: Domain and Range
Section 5.9: Applications

UNIT 5 LEARNING OBJECTIVES

- Recognize and describe the meaning of each entry in an ordered pair: (input, output)
- Recognize that on a graph an ordered pair represents a horizontal and vertical distance from the origin
- Define and identify horizontal and vertical intercepts from a graph
- Identify the behavior of a graph: increasing, decreasing, constant
- Construct a good graph from a given data set
- Draw the graph of an equation by plotting points
- Define Function
- Represent functions in multiple ways: tables, graphs, ordered pairs
- Identify the behavior of a function given its graph: increasing, decreasing, constant
- Apply the Vertical Line Test to determine if a relation is a function
- Use and interpret function notation in terms of input and output
- Evaluate and solve functions given in function notation
- Use and interpret function notation in terms of input and output in contextual situations

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian Coordinate System</td>
<td></td>
</tr>
<tr>
<td>Ordered Pair</td>
<td></td>
</tr>
<tr>
<td>Vertical Intercept</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td></td>
</tr>
<tr>
<td>Relation</td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td></td>
</tr>
<tr>
<td>Vertical Line Test</td>
<td></td>
</tr>
<tr>
<td>Dependent Variable</td>
<td></td>
</tr>
<tr>
<td>Independent Variable</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
<tr>
<td>Practical Domain</td>
<td></td>
</tr>
<tr>
<td>Practical Range</td>
<td></td>
</tr>
</tbody>
</table>
In this chapter, we will begin looking at the relationships between two variables. Typically one variable is considered to be the **INPUT**, and the other is called the **OUTPUT**. The input is the value that is considered first, and the output is the value that corresponds to or is matched with the input. The input/output designation may represent a cause/effect relationship, but that is not always the case.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Ordered Pairs (input, output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>−3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>(0, −4)</td>
</tr>
</tbody>
</table>

**Example 2:** The Rectangular Coordinate System (Cartesian Coordinate System)

---

**Example 1:** Ordered Pairs (input value, corresponding output value)

Ordered Pairs (input, output)

(0, −4)

(−2, 6)
Vertical and Horizontal Intercepts

The **vertical intercept** is the point at which the graph crosses the vertical axis.

The input value of the vertical intercept is always __________

The coordinates of the vertical intercept will be __________

The **horizontal intercept** is the point at which the graph crosses the horizontal axis.

The output value of the horizontal intercept is always __________

The coordinates of the horizontal intercept will be __________

**Example 3:** Identify the vertical and horizontal intercepts of the graph below.
Behavior of Graphs

A graph is **increasing** if as the inputs increase, the outputs increase.

A graph is **decreasing** if as the inputs increase, the outputs decrease.

A graph is **constant** if as the inputs increase, the outputs do not change.

<table>
<thead>
<tr>
<th>Increasing</th>
<th>Decreasing</th>
<th>Constant</th>
</tr>
</thead>
</table>

**Example 4**: On the graph below, use a highlighter to identify where the graph is **increasing**.

![Graph example](image)

**Section 5.1 – You Try**

Consider the graph below.

a. Identify the vertical and horizontal intercepts of the graph. Intercepts must always be written as ordered pairs.

b. Use a highlighter to show where the graph is **decreasing**.

![Graph example](image)
Section 5.2: Constructing a Graph from Data

### Criteria for a Good Graph

1. The horizontal axis should be properly labeled with the name and units of the input variable.
2. The vertical axis should be properly labeled with the name and units of the output variable.
3. Use an appropriate scale.
   - Start at or just below the lowest value.
   - End at or just above the highest value.
   - Scale the graph so the adjacent tick marks are equal distance apart.
   - Use numbers that make sense for the given data set.
   - The axes must meet at (0,0). Use a “/” between the origin and the first tick mark if the scale does not begin at 0.
4. All points should be plotted correctly, and the graph should make use of the available space.

---

**Example 1:** The table below shows the total distance (including reaction time and deceleration time) it takes a car traveling at various speeds to come to a complete stop.

<table>
<thead>
<tr>
<th>Speed (miles per hour)</th>
<th>15</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>50</th>
<th>60</th>
<th>75</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stopping Distance (ft)</td>
<td>44</td>
<td>85</td>
<td>135</td>
<td>196</td>
<td>229</td>
<td>304</td>
<td>433</td>
<td>481</td>
</tr>
</tbody>
</table>

Input: ___________________
Lowest Value: __________
Highest Value: __________

Output: ___________________
Lowest Value: __________
Highest Value: __________
Section 5.2 – You Try

The table below shows the height of a toy rocket during the time it is in flight.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (feet)</td>
<td>0</td>
<td>70</td>
<td>118</td>
<td>135</td>
<td>118</td>
<td>70</td>
<td>0</td>
</tr>
</tbody>
</table>

a. What is the input variable? _____________________________
   What are the units of the input variable? _____________________

b. In a complete sentence, interpret the meaning of the ordered pair (6, 0).

c. Construct a good graph of this data set.
Section 5.3: Equations and Graphs

<table>
<thead>
<tr>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>The <strong>graph of an equation</strong> is the set of all points for which the equation is true.</td>
</tr>
</tbody>
</table>

**Example 1:** Verify that the ordered pairs below satisfy the equation $y = 2x + 3$.

$(-2, -1)$ $ (0, 3)$ $ (1, 5)$

**Example 2:** Verify that the ordered pairs below satisfy the equation $3x + 2y = 6$.

$(-2, 6)$ $ (0, 3)$ $ (2, 0)$
Example 3: Use the equation \( y = -x^2 + 5 \) to complete the table below. Graph your results.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Section 5.3 – You Try

Use the equation \( y = |x - 2| \) to complete the table below. Graph your results.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 5.4: Using Your Graphing Calculator

Example 1: Linear Equation: \( A = 8 - 2n \)

a) Use the TABLE feature of your graphing calculator to complete the table below.

<table>
<thead>
<tr>
<th>( n )</th>
<th>-5</th>
<th>0</th>
<th>2</th>
<th>7</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A  )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Use your graphing calculator to sketch the graph of \( A = 8 - 2n \). Use the indicated viewing windows. Draw what you see on your calculator screen.

- Standard Viewing Window
  - \( \text{xmin} = -10, \text{xmax} = 10, \text{ymin} = -10, \text{ymax} = 10 \)
  - \( \text{xmin} = 0, \text{xmax} = 5, \text{ymin} = -2, \text{ymax} = 8 \)

Example 2: Exponential Equation: \( P = 28(1.17)^t \)

a) Use the TABLE feature of your graphing calculator to complete the table below. Round to the nearest tenth as needed.

<table>
<thead>
<tr>
<th>( t )</th>
<th>-15</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P  )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Use your graphing calculator to sketch the graph of \( P = 28(1.17)^t \). Use the indicated viewing windows. Draw what you see on your calculator screen.

- Standard Viewing Window
  - \( \text{xmin} = -10, \text{xmax} = 10, \text{ymin} = -10, \text{ymax} = 10 \)
  - \( \text{xmin} = -15, \text{xmax} = 15, \text{ymin} = 0, \text{ymax} = 300 \)
Common Calculator Errors

**SYNTAX** – Check your previous entry for syntax errors. Look for extra (or missing) parentheses. Look to see if you may have typed in a “-” instead of a “–” or vice versa.

**WINDOW RANGE** – Check your window settings. You may have an xmin that is larger than the xmax or a ymin that is larger than the ymax.

**DIM MISMATCH** – Check your lists (in STAT). They may be blank, or one list is longer than the other.

**NO SIGN CHNG** – In order to use the intersection method for solving equations, the intersection point must appear on your calculator screen. Adjust your window settings so that the intersection point appears on the screen.

Section 5.4 – You Try

📝 Consider the Quadratic Equation: \( H = -5w^2 - 8w + 52 \)

a) Use the TABLE feature of your graphing calculator to complete the chart below.

<table>
<thead>
<tr>
<th>( w )</th>
<th>-4</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Use your graphing calculator to sketch the graph of \( H = -5w^2 - 8w + 52 \). Use the indicated viewing windows. Draw what you see on your calculator screen.

Standard Viewing Window
\[
\begin{align*}
\text{xmin} &= -10, \text{ xmax} = 10, \\
\text{ymin} &= -10, \text{ ymax} = 10
\end{align*}
\]

\[
\begin{align*}
\text{xmin} &= -4, \text{ xmax} = 3, \\
\text{ymin} &= -20, \text{ ymax} = 60
\end{align*}
\]
Section 5.5: Relations and Functions

### Definitions

A **RELATION** is any set of ordered pairs.

A **FUNCTION** is a relation in which every input value is paired with exactly one output value.

### Table of Values

One way to represent the relationship between the input and output variables in a relation or function is by means of a table of values.

**Example 1:** Which of the following tables represent functions?

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>–9</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>–5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>–3</td>
<td>–87</td>
</tr>
</tbody>
</table>

Yes  No  Yes  No  Yes  No

### Ordered Pairs

A relations and functions can also be represented as a set of points or ordered pairs.

**Example 2:** Which of the following sets of ordered pairs represent functions?

A = {(0, –2), (1,4), (–3,3), (5,0)}

B = {(-4,0), (2, –3), (2, –5)}

C = {(-5,1), (2,1), (–3,1), (0,1)}

D = {(3, –4), (3, –2), (0, 1), (2, –1)}

E = {(1,3)}
Example 3: On the graphs below, plot the points for A, B, C, and D from Example 2, then circle the “problem points”

The Vertical Line Test

- If all vertical lines intersect the graph of a relation at no more than one point, the relation is also a function. One and only one output value exists for each input value.

- If any vertical line intersects the graph of a relation at more than one point, the relation “fails” the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Example 4: Use the Vertical Line Test to determine which of the following graphs are functions.
In general, we say that the output depends on the input.

Output variable = **Dependent Variable**

Input Variable = **Independent Variable**

If the relation is a function, then we say that the output is a function of the input.

<table>
<thead>
<tr>
<th>Yes or No</th>
<th>Yes or No</th>
<th>Yes or No</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Table" /></td>
<td>(2, −3), (−5, 2), (−3, 1)</td>
</tr>
</tbody>
</table>
Section 5.6: Function Notation: f (input) = output

If a relation is a function, we say that the output is a function of the input.

Function Notation: f (input) = output

Example: If y is a function of x, then we can write f (x) = y.

Example 1: The function V(m) represents value of an investment (in thousands of dollars) after m months. Explain the meaning of V(36) = 17.4.

Example 2: Ordered Pair

<table>
<thead>
<tr>
<th>Ordered Pair (input, output)</th>
<th>Function Notation f (input) = output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 3)</td>
<td>f (2) = 3</td>
</tr>
<tr>
<td>(−4, 6)</td>
<td>f (___) = ___</td>
</tr>
<tr>
<td>(___, ___)</td>
<td>f (5) = −1</td>
</tr>
</tbody>
</table>

Example 3: Consider the function: f = {(2, −4), (5, 7), (8, 0), (11, 23)}

f(5) = ___

f(____) = 0

Example 4: The function B(t) is defined by the table below.

<table>
<thead>
<tr>
<th>T</th>
<th>1</th>
<th>3</th>
<th>12</th>
<th>18</th>
<th>22</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(t)</td>
<td>70</td>
<td>64</td>
<td>50</td>
<td>39</td>
<td>25</td>
<td>18</td>
</tr>
</tbody>
</table>

B(12) = _____________________________

B(t) = 18 when t = ____________________
Example 5: Consider the graph $g(x)$ of shown below

![Graph Image]

$g(2) = \underline{\hspace{2cm}}$ \hspace{1cm} $g(\underline{\hspace{2cm}}) = 2$

Ordered pair: \underline{\hspace{2cm}} \hspace{1cm} Ordered pair: \underline{\hspace{2cm}}

$g(0) = \underline{\hspace{2cm}}$ \hspace{1cm} $g(\underline{\hspace{2cm}}) = 1$

Ordered pair: \underline{\hspace{2cm}} \hspace{1cm}Ordered pair: \underline{\hspace{2cm}}

Section 5.6 – You Try

Complete the problems below.

a. The function $k(x)$ is defined by the following table

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k(x)$</td>
<td>8</td>
<td>2</td>
<td>-9</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$k(2) = \underline{\hspace{2cm}}$ \hspace{1cm} $k(x) = 1$ when $x = \underline{\hspace{2cm}}$

Ordered Pair: \underline{\hspace{2cm}} \hspace{1cm} Ordered Pair: \underline{\hspace{2cm}}

b. At an ice cream factory, the total cost production is a function of the number of gallons of ice cream produced. The function $C(g)$, gives the cost, in dollars, to produce $g$ gallons of ice cream. Explain the meaning of $C(580)=126$ in terms of ice cream production.
Section 5.7: Formulas in Function Notation

**Example 1:** Let \( f(x) = x^2 - 2x + 11 \)

a. Determine \( f(-3) \)

b. Determine \( f(0) \)

**Example 2:** Let \( h(x) = 2x - 5 \)

a. Determine \( h(4) \)

b. For what value of \( x \) is \( h(x) = 17 \)?

**Example 3:** Let \( g(x) = 71 \)

a. Determine \( g(5) \).

b. Determine \( g(-40) \).

**Example 4:** Let \( f(x) = -4x + 2 \)

a. Determine \( f(x - 2) \).

b. Determine \( f(x + h) \).
1. Let \( g(x) = 4 - 5x \). Show all steps as in the media examples. Write each answer using function notation \textbf{and} as an ordered pair.

   a. Determine \( g(-2) \).

      \[ \text{Function Notation: } \quad \quad \text{Ordered Pair: } \quad \quad \]

   b. For what value of \( x \) is \( g(x) = 19 \)?

      \[ \text{Function Notation: } \quad \quad \text{Ordered Pair: } \quad \quad \]

2. Let \( g(x) = 4 - 5x \). Show all steps as in the media examples.
   a. Determine \( g(x + 8) \)
   b. Determine \( g(x + h) \)
Section 5.8: Domain and Range

DEFINITIONS

The **DOMAIN** of a function is the set of all possible values for the **input** variable.

The **RANGE** of a function is the set of all possible values for the **output** variable.

DOMAIN AND RANGE

**Example 1**: Consider the function below

<table>
<thead>
<tr>
<th>x</th>
<th>–2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>k(x)</td>
<td>3</td>
<td>–7</td>
<td>11</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Input values ___________________________________
Domain: {___________________________}
Output values:__________________________________
Range:  {___________________________}

**Example 2**: Consider the function: $B = \{(2, –4), (5, 7), (8, 0), (11, 23)\}$

Input values ___________________________________
Domain: {___________________________}
Output values:__________________________________
Range:  {___________________________}

**Example 3**: Consider the graph of $f(x)$ shown below

![Graph of f(x)](image)

Domain: ______________ ≤ x ≤ ______________
Range: ______________ ≤ f(x) ≤ ______________
Example 4: Determine the Domain and Range of each of the following graphs:

A(x)  

B(x)  

C(x)  

Domain  Domain  Domain

Range  Range  Range

Example 5: Consider the graph of h(x) shown below. Write your answers in interval notation.

Domain: ___________________

Range: ___________________

The Practical Domain of a function is the set of all possible values for the input variable that make sense in a given situation.

The Practical Range of a function is the set of all possible values for the output variable that make sense in a given situation.
Example 6: The gas station is currently charging $3.83 per gallon for gas. The cost, \( C(n) \), in dollars, to fill up your car depends on the number of gallons, \( n \), that you pump. Your car’s tank can hold a maximum of 20 gallons of gas.

- In this situation, the input variable is \_______________.
- The practical domain of this function is \_______________.
- The output variable in this situation is \_______________.
- The practical range of this function is \_______________.

Section 5.8 – You Try

Complete the following problems.

1. The graph of \( f(x) \) is shown below

   Domain in Inequality Notation: \_______________.
   Domain in Interval Notation: \_______________.
   Range in Inequality Notation: \_______________.
   Range in Interval Notation: \_______________.

2. The platform for the high dive is 35 feet above the water. A diver jumps from the platform and lands in the water after 1.5 seconds. The function \( H(s) \) represents the height of the diver after \( s \) seconds. Determine the practical domain and practical range in this situation. Use inequality notation and include units.

   Practical Domain: \_______________.
   Practical Range: \_______________.

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Section 5.9: Applications

**Example 1:** Consider the graph of the function $H(t)$ shown below.

![Graph of function H(t)](image)

Input Variable: _______________

Units of Input Variable: __________

Output Variable: _______________

Units of Output Variable: __________

a. Interpret the meaning of the statement $H(5)=82$.

b. Determine $H(7)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Determine the maximum height of the rocket.

e. Determine the practical domain for $H(t)$.

f. Determine the practical range for $H(t)$. 
Example 2: Grace is selling snow cones at a local carnival. Her profit, in dollars, from selling \( x \) snow cones is given by the function \( P(x) = 2.5x - 30 \).

a. Write a complete sentence to explain the meaning of \( P(30) = 45 \) in words.

b. Determine \( P(10) \). Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

c. Determine \( P(0) \). Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

d. Determine \( x \) when \( P(x) = 100 \). Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________

e. Determine \( x \) when \( P(x) = 0 \). Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

Ordered Pair: ____________
The graph of $A(m)$ below shows the amount of water in a play pool.

Input Variable: ________________
Units of Input Variable: ________________
Output Variable: ________________
Units of Output Variable: ________________

a. Interpret the meaning of the statement $A(3)=28$.

b. Determine $A(5)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

c. Determine $t$ when $A(m) = 0$. Write it as an ordered pair and interpret its meaning in a complete sentence.

d. Describe what is happening to the water in the pool. (Is the pool being filled or drained?)

e. Determine the practical domain for $A(m)$. Use inequality notation and include units.

f. Determine the practical range for $A(m)$. Use inequality notation and include units.
Unit 5: Answers to You-Try Problems

Section 5.1: Vertical Intercept: (0, -2)
   Horizontal Intercepts: (-1, 0), (1, 0), (3, 0)

Section 5.2: a. Elapsed time; Seconds; Height of Rocket; Feet
   b. After 6 seconds, the rocket is 0 feet above the ground (the rocket lands).

Section 5.3:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>8</td>
<td>(-6,8)</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
<td>(-4,6)</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td>(-2,4)</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>(0,2)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(2,0)</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>(4,2)</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>(6,4)</td>
</tr>
</tbody>
</table>

Section 5.4: Table

<table>
<thead>
<tr>
<th>w</th>
<th>-4</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4</td>
<td>55</td>
<td>52</td>
<td>39</td>
<td>-17</td>
</tr>
</tbody>
</table>

Section 5.5: a. Yes    b. Yes    c. Yes

Section 5.6: a. \( k(2) = 6 \) \( (2, 6) \) \( k(x) = 1 \) when \( x = 3 \) \( (3, 1) \)
   b. It costs $126 to produce 580 gallons of ice cream.
Section 5.7:  

1. a. \( g(-2) = 14 \) \((-2,14)\) 
   b. \( g(-3) = 19 \) \((-3,19)\) 

2. a. \( g(x + 8) = -5x - 36 \) 
   b. \( g(x + h) = -5x - 5h + 4 \) 

Section 5.8: 

a. Domain: \(-1 \leq x < 3\); \([-1, 3)\) 
   Range: \(-5 < f(x) \leq 4\); \((-5, 4]\) 

b. Practical Domain: \(0 \leq s \leq 1.5\) seconds 
   Practical Range: \(0 \leq H(s) \leq 35\) feet 

Section 5.9: 

Input variable: Time 
Units of input variable: minutes 

Output variable: Amount of Water 
Units of output variable: gallons 

a. After 3 minutes, there are 28 gallons of water in the pool. 

b. \((5, 20)\) After 5 minutes, there are 20 gallons of water in the pool. 

c. \((10, 0)\) After 10 minutes, the pool is empty. 

d. Drained 

e. Practical Domain: \(0 \leq m \leq 10\) minutes 

f. Practical Range: \(0 \leq A(m) \leq 40\) gallons
Unit 5: Practice Problems

Skills Practice

1. Which of the following ordered pairs satisfy the equation \( y = -2x - 4 \)? Circle all that apply.
   (9, -22)  \hspace{1cm} (6, -5)  \hspace{1cm} (-9, 14)  \hspace{1cm} (2, 0)  \hspace{1cm} (-4, 0)

2. Graph the equation \( y = -4x + 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

3. Graph the equation \( y = \frac{2}{5}x - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Graph the equation \( y = 5 - x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Graph the equation $y = |x + 2|$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Use the equation $y = 3^x$ to complete the table below. Graph your results.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Identify the vertical and horizontal intercepts of each of the graphs below. Write the intercepts as ordered pairs.
8. For each of the graphs below, use a highlighter to indicate the intervals where the graph is decreasing.

9. Linear Equation: \( A = 1200 - 450n \)
   a) Use the TABLE feature of your graphing calculator to complete the table below.
   
<table>
<thead>
<tr>
<th>( n )</th>
<th>-4</th>
<th>-1</th>
<th>0</th>
<th>3</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) Use your graphing calculator to sketch the graph of \( A = 1200 - 450n \). Use the indicated viewing windows.
   
   \( \text{xmin} = -10, \text{xmax} = 10, \text{ymin} = -10, \text{ymax} = 10 \)
   \( \text{xmin} = -4, \text{xmax} = 7, \text{ymin} = -2000, \text{ymax} = 3000 \)

10. Exponential Equation: \( P = 567(0.73)^t \)
    a) Use the TABLE feature of your graphing calculator to complete the table below.
    
    | \( t \) | -5  | -2  | 0  | 2  | 5  | 10 |
    |---|---|---|---|---|---|---|
    | \( P \) |     |     |     |     |     |     |

    b) Use your graphing calculator to sketch the graph of \( P = 567(0.73)^t \). Use the indicated viewing windows.
    
    \( \text{xmin} = -10, \text{xmax} =10, \text{ymin} = -10, \text{ymax} = 10 \)
    \( \text{xmin} = -5, \text{xmax} = 10, \text{ymin} = 0, \text{ymax} = 3000 \)

11. Which of the following tables represent functions?

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>-9</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>-3</td>
<td>-87</td>
</tr>
</tbody>
</table>
12. Are these functions? Circle yes or no.
   a. \{(2, –4), (6, –4), (0, 0), (5, 0)\} Yes No
   b. \{(1, 1), (2, 2), (3, 3), (4, 4)\} Yes No
   c. \{(1, –8), (5, 2), (1, 6), (7, –3)\} Yes No

13. Are these functions? Circle yes or no.

14. The function \(r(x)\) is defined by the following table of values.

   \[
   \begin{array}{c|cccc}
   x & 3 & 5 & 6 & 9 & 13 \\
   \hline
   r(x) & -9 & 3 & 2 & 2 & 1 \\
   \end{array}
   \]

   a. \(r(9) = \underline{\hspace{2cm}}\)       b. \(r(3) = \underline{\hspace{2cm}}\)       c. \(r(\underline{\hspace{2cm}}) = 1\)       d. \(r(\underline{\hspace{2cm}}) = 3\)

   e. The domain of \(r(x)\) is \{ \underline{\hspace{2cm}} \}  
   f. The range of \(r(x)\) is \{ \underline{\hspace{5cm}} \} 

15. Consider the function \(g = \{(2, 5), (0, 6), (5, 8), (-3, 7)\}\)

   a. \(g(0) = \underline{\hspace{2cm}}\)       b. \(g(5) = \underline{\hspace{2cm}}\)       c. \(g(\underline{\hspace{2cm}}) = 7\)       d. \(g(\underline{\hspace{2cm}}) = 5\)

   e. The domain of \(g\) is \{ \underline{\hspace{5cm}} \}  
   f. The range of \(g\) is \{ \underline{\hspace{5cm}} \} 

16. Given \(f(4) = 8, f(3) = 11, f(0) = 6\)

   a. The domain of \(f\) is \{ \underline{\hspace{5cm}} \}  
   b. The range of \(f\) is \{ \underline{\hspace{5cm}} \} 

   c. Write the function \(f\) as a set of ordered pairs.
17. The graph of \( g \) is given below.

a. Domain: _______________________

b. Range _______________________

c. \( g(-3) = \) _______

d. \( g(0) = \) _______

e. \( g(I=4 \text{ when } r = \) _______

f. \( g(I=0 \text{ when } r = \) _______

18. The graph of \( A(m) \) is given below.

a. Domain: _______________________

b. Range _______________________

c. \( A(-4) = \) _______

d. \( A(0) = \) _______

e. \( A(m) = -2 \text{ when } m = \) _______

f. \( A(m) = 0 \text{ when } m = \) _______

19. The graph of \( p(t) \) is given below.

a. Domain: _______________________

b. Range _______________________

c. \( p(-1) = \) _______

d. \( p(0) = \) _______

e. \( p(t) = -5 \text{ when } t = \) _______

f. \( p(t) = 3 \text{ when } t = \) ____________
20. The graph of \( f(n) \) is given below.

a. Domain: _______________________

b. Range: _______________________

c. \( f(-5) = \) ________

d. \( f(n) = 0 \) when \( n = \) ________

21. The graph of \( r(x) \) is given below.

a. Domain: _______________________

b. Range: _______________________

c. \( r(-10) = \) ________

d. \( r(x) = 300 \) when \( x = \) ________

22. Determine the domain and range of each of the graphs shown below. Use correct notation.
23. Let $W(p) = 4p^2 - 9p + 1$. Show all steps. Write each answer in function notation and as an ordered pair.
   a. Determine $W(5)$.
   b. Determine $W(0)$.
   c. Determine $W(-1)$.
   d. Determine $W(-10)$.

24. Let $k(m) = 8 - 3m$. Show all steps. Write each answer in function notation and as an ordered pair.
   a. Determine $k(5)$.
   b. Determine $k(-3)$.
   c. For what value of $m$ is $k(m) = 29$?
   d. For what value of $m$ is $k(m) = 0$?

25. Let $R(t) = 1500 + 40t$. Show all steps. Write each answer in function notation and as an ordered pair.
   a. Determine $R(18)$.
   b. For what value of $t$ is $R(t) = 3000$?

26. Let $h(x) = 4$. Show all steps. Write each answer in function notation and as an ordered pair.
   a. Determine $h(5)$.
   b. Determine $h(81)$.

27. Let $b(w) = \sqrt{w + 3}$. Show all steps. Write each answer in function notation and as an ordered pair. Round to the nearest hundredth as needed.
   a. Determine $b(1)$.
   b. Determine $b(8)$.
   c. Determine $b(-3)$.

28. Let $p(x) = \frac{45}{2x}$. Show all steps. Write each answer in function notation and as an ordered pair.
   a. Determine $p(5)$.
   b. Determine $p(-6)$.

29. For each of the following, determine $f(w)$, $f(-x)$, $f(x - 3)$, and $f(x + h)$. Simplify your answers.
   a. $f(x) = 3x + 7$
   b. $f(x) = 5x^2$
   c. $f(x) = x^2 - x - 11$
30. Exponential Function: \( P(t) = 5(1.2)^t \)

a) Use the TABLE feature of your graphing calculator to complete the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Use your graphing calculator to sketch the graph of \( P(t) = 5(1.2)^t \). Use the indicated viewing windows. Draw what you see on your calculator screen.

Standard Viewing Window
\[
\begin{align*}
\text{xmin} &= -10, \quad \text{xmax} = 10, \\
\text{ymin} &= -10, \quad \text{ymax} = 10
\end{align*}
\]

\[
\begin{align*}
\text{xmin} &= -10, \quad \text{xmax} = 20, \\
\text{ymin} &= -10, \quad \text{ymax} = 200
\end{align*}
\]

31. Absolute Value Function: \( H(t) = 2|x + 6| - 8 \)

a) Use the TABLE feature of your graphing calculator to complete the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>-12</th>
<th>-6</th>
<th>-2</th>
<th>0</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Use your graphing calculator to sketch the graph of \( P(t) = 5(1.2)^t \). Use the indicated viewing windows. Draw what you see on your calculator screen.

Standard Viewing Window
\[
\begin{align*}
\text{xmin} &= -10, \quad \text{xmax} = 10, \\
\text{ymin} &= -10, \quad \text{ymax} = 10
\end{align*}
\]

\[
\begin{align*}
\text{xmin} &= -12, \quad \text{xmax} = 0, \\
\text{ymin} &= -8, \quad \text{ymax} = 14
\end{align*}
\]
Unit 5: Graphs and Functions  
Practice Problems

32. Given each function \( f(x) \) shown below, evaluate: \( f(-1), f(0), f(2), \) and \( f(3). \)

(a) \[
\begin{align*}
&f(x) = \begin{cases} 
7x + 3 & \text{if } x < 0 \\
7x + 6 & \text{if } x \geq 0
\end{cases}
\end{align*}
\]

(b) \[
\begin{align*}
&f(x) = \begin{cases} 
4x - 9 & \text{if } x < 0 \\
4x - 18 & \text{if } x \geq 0
\end{cases}
\end{align*}
\]

(c) \[
\begin{align*}
&f(x) = \begin{cases} 
x^2 - 2 & \text{if } x < 2 \\
4 + |x - 5| & \text{if } x \geq 2
\end{cases}
\end{align*}
\]

(d) \[
\begin{align*}
&f(x) = \begin{cases} 
5x & \text{if } x < 0 \\
3 & \text{if } 0 \leq x \leq 3 \\
x^2 & \text{if } x > 3
\end{cases}
\end{align*}
\]

33. Given each function, evaluate: \( f(-1), f(0), f(2), \) and \( f(4). \)

(a) \[
\begin{align*}
&f(x) = \begin{cases} 
7x + 3 & \text{if } x < 0 \\
7x + 6 & \text{if } x \geq 0
\end{cases}
\end{align*}
\]

(b) \[
\begin{align*}
&f(x) = \begin{cases} 
4x - 9 & \text{if } x < 0 \\
4x - 18 & \text{if } x \geq 0
\end{cases}
\end{align*}
\]

(c) \[
\begin{align*}
&f(x) = \begin{cases} 
x^2 - 2 & \text{if } x < 2 \\
4 + |x - 5| & \text{if } x \geq 2
\end{cases}
\end{align*}
\]

(d) \[
\begin{align*}
&f(x) = \begin{cases} 
5x & \text{if } x < 0 \\
3 & \text{if } 0 \leq x \leq 3 \\
x^2 & \text{if } x > 3
\end{cases}
\end{align*}
\]

(e) \[
\begin{align*}
&f(x) = \begin{cases} 
x^3 + 1 & \text{if } x < 0 \\
4 & \text{if } 0 \leq x \leq 3 \\
3x + 1 & \text{if } x > 3
\end{cases}
\end{align*}
\]

(f) \[
\begin{align*}
&f(x) = \begin{cases} 
4 - x^3 & \text{if } x < 1 \\
\sqrt{x + 1} & \text{if } x \geq 1
\end{cases}
\end{align*}
\]
34. The graph below shows the population of a town over a 10-year time period.

![Graph showing population over time](image)

<table>
<thead>
<tr>
<th>Years Since 2004</th>
<th>Population (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

a. What is the input variable? ___________________________

b. What is the output variable? __________________________

d. The population of this town is (circle one) increasing decreasing

e. The population of this town in the year 2006 was approximately ________________.

f. The population of this town in the year 2011 was approximately ________________.

g. The population of this town in the year __________ was approximately 10,000 people.

h. Interpret the meaning of the ordered pair (9, 12).

i. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
35. Janey is selling homemade scented candles. The graph below shows her profit from selling the candles.

a. What is the input variable? __________________________

b. What is the output variable? __________________________

c. If Janey sells 90 candles, her profit will be ____________.

d. If Janey sells _________ candles, her profit will be $200.

e. If Janey sells 15 candles, her profit will be ____________.

f. Interpret the meaning of the ordered pair (60, 50).

g. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

h. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
36. Jordan is saving money for emergencies (or a trip to Europe). She has $420 under her mattress, and is adding $60 to it each week.
   a. Let $A$ represent the total amount of money under her mattress, and $w$ represent the number of weeks. Write an algebraic equation to represent this situation.
   b. Use the equation in part a. to complete the table below.

<table>
<thead>
<tr>
<th>$w$</th>
<th>0</th>
<th>8</th>
<th>37</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1800</td>
<td>2220</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Interpret the meaning of the ordered pair (18, 1500).
d. Identify the vertical intercept in this situation. Write it as an ordered pair and interpret its meaning in a complete sentence.
e. How much money will Jordan have saved after 3 weeks?
f. Calculate the horizontal intercept for the equation you found in part a. and write it as an ordered pair. Does this point make sense in the given situation? Why or why not?

37. Match the stories with the graphs below, and label the axes accordingly.
   Story A: Heidie is filling a pool with water. This graph shows the amount of water in the pool (in gallons) over time.
   Story B: John has $15,000 in his bank account for college. Every semester, he withdraws $3000 to pay for tuition and fees. This graph shows the remaining balance in his bank account over time.
   Story C: A caterer charges $12.50 per guest at a reception. This graph shows the cost for food at the reception, based on the number of guests attending.
   Story D: A car comes to a stop at a red light. This graph shows the speed of a car over time.
   Story E: Nik must complete all obstacles at each level of his video game before moving up to the next level. If he does not pass all obstacles, he has to restart at the beginning of the level. This graph shows Nik’s progress in the video game over time.

After matching all of the stories with their graphs, there should be one graph remaining. In the space below, write a story that corresponds to the remaining graph. Label the axes accordingly.
38. The function $D(t)$ shown below represents Sally’s distance from home over a 30-minute time period.

![Graph showing distance from home over time](image)

a. Identify the vertical intercept of $D(t)$. Write it as an ordered pair and explain its meaning in this situation.

b. Identify the horizontal intercepts of $D(t)$. Write them as an ordered pairs and explain their meaning in this situation.

c. Determine $D(15)$ and interpret its meaning in a complete sentence.

d. For what value of $t$ is $D(t) = 5$? Write a sentence explaining the meaning of your answer.

e. Determine the practical domain of $D(t)$.

f. Determine the practical range of $D(t)$.

39. The graph of the function $C(n)$ below shows the number of calories burned after riding a stationary bike for $n$ minutes.

![Graph showing calories burned over time](image)

a. Is this function increasing or decreasing?

b. Interpret the meaning of the statement $C(8) = 32$.

c. Determine $C(10)$ and interpret its meaning in a complete sentence.

d. For what value of $n$ is $C(n) = 80$? Write a sentence explaining the meaning of your answer.
40. A rock is dropped from the top of a building. The function \( h(t) = 144 - 16t^2 \) gives the height (measured in feet) of the rock after \( t \) seconds.

a. Complete the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Is this function increasing or decreasing? __________________________

c. Identify the vertical intercept and write a sentence explaining its meaning.

d. Identify the horizontal intercept and write a sentence explaining its meaning.

e. Determine \( h(2) \). Write a sentence explaining the meaning of your answer.

f. Determine \( h(0) \). Write a sentence explaining the meaning of your answer.

g. For what value of \( t \) is \( h(t) = 0 \)? Explain the meaning of your answer.

h. Determine the practical domain ________________________________

i. Determine the practical range ________________________________

j. Construct a good graph of \( h(t) \).

k. Use your graphing calculator to generate a graph of \( h(t) \). Use the practical domain and range to define your viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.
41. A local towing company charges $5.50 for each mile plus a reservation fee of $12. They tow a maximum of 30 miles.

a. Write a formula for the function \( C(x) \), representing the total cost to tow the car \( x \) miles.

b. Determine \( C(8) \). Show your work. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

c. Determine \( x \) when \( C(x) = 100 \). Show your work. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

d. Practical domain (include units): \( \text{________________________} \leq x \leq \text{________________________} \)

   Practical range (include units): \( \text{________________________} \leq C(x) \leq \text{________________________} \)

e. Construct a good graph of \( C(x) \).

f. Use your graphing calculator to generate a graph of \( C(x) \). Use the practical domain and range to define your viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

   \[ \text{Xmin=} \quad \text{Xmax=} \quad \text{Ymin=} \quad \text{Ymax=} \]
42. The height, \( h \) (in feet), of a golf ball is a function of the time, \( t \) (in seconds), it has been in flight. A golfer strikes the golf ball with an initial upward velocity of 96 feet per second. The maximum height of the ball is 144 feet. The height of the ball above the ground is given by the function \( h(t) = -16t^2 + 96t \).

a) Use the TABLE feature on your graphing calculator to complete the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Determine \( h(3) \). Write a sentence explaining the meaning of your answer.

c) For what values of \( t \) is \( h(t) = 0 \)? Explain the meaning of your answers.

d) Determine the practical domain. Use inequality notation and include units.

e) Determine the practical range. Use inequality notation and include units.

f) Construct a good graph of \( C(x) \).

[g] Use your graphing calculator to generate a graph of \( h(t) \). Use the practical domain and range to determine a “good” viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

\[
\begin{align*}
\text{Xmin} &= \underline{\quad} \\
\text{Xmax} &= \underline{\quad} \\
\text{Ymin} &= \underline{\quad} \\
\text{Ymax} &= \underline{\quad}
\end{align*}
\]
43. John is a door to door vacuum salesman. His weekly salary, in dollars, is given by the linear function $S(v) = 200 + 50v$, where $v$ is the number of vacuums sold.
   a. Determine $S(12)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.
   b. Determine $S(0)$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.
   c. Determine $v$ when $S(v) = 500$. Show your work. Write your answer as an ordered pair and interpret the meaning of this ordered pair in a complete sentence.

44. A rock is dropped from the top of a building. The function $H(t)$ gives the height (measured in meters) of the rock after $t$ seconds. In a complete sentence, explain the meaning of the statement $H(2) = 35$. Your answer must include correct units.

45. The function $P(n)$ represents a computer manufacturer’s profit, in dollars, when $n$ computers are sold. In a complete sentence, explain the meaning of the statement $P(40) = 1680$. Your answer must include correct units.

46. The function $E(t)$ gives the surface elevation (in feet above sea level) of Lake Powell $t$ years after 1999. In a complete sentence, explain the meaning of the statement $E(5) = 3675$. Your answer must include correct units.

47. The function $V(n)$ gives the value, in thousands of dollars, of an investment after $n$ months. In a complete sentence, explain the meaning of the statement $V(24) = 18$. Your answer must include correct units.

48. The function $P(t)$ can be used to approximate the population of a town, in thousands of people, $t$ years after 1980. In a complete sentence, explain the meaning of the statement $P(31) = 52$. Your answer must include correct units.

49. The function $E(t) = 3861 – 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell $t$ years after 1999. All answers must indicate the appropriate year.
   a. Determine $E(2)$ and write a sentence explaining the meaning of your answer.
   b. Identify the vertical intercept of this linear function. Write it as an ordered pair, then write a sentence explaining its meaning in this situation.
   c. (6 points) This function accurately models the surface elevation of Lake Powell from 1999 to 2004. Determine the practical range of this linear function. Use inequality notation and include units.
50. Sort the following terms into the two groups below.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Independent variable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Axis</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

51. For a part-time student, the cost of tuition at a local community college is $80 per credit hour. The function $C(n)$ gives the tuition cost for $n$ credit hours. As a part-time student, Gabe can take a maximum of 11 credit hours.

- a. Identify the input quantity in this situation: _____________________________
- b. Identify the output quantity in this situation: _____________________________
- c. Write a formula (symbolic rule) to represent this situation. Use the indicated variables and proper function notation:
- d. Use your formula from part c to determine $C(3)$. Write your answer as an ordered pair, then write a sentence explaining the meaning of your answer.
- e. Use your formula from part c to determine $n$ when $C(n)=800$. Write your answer as an ordered pair, then write a sentence explaining the meaning of your answer.
- f. Determine the practical domain of $C(n)$. Write your answer as an ordered list enclosed in curly brackets {}. Include units.
- g. Determine the practical range of $C(n)$. Write your answer as an ordered list enclosed in curly brackets {}. Include units.
- h. Complete the table below and construct a properly scaled and labeled graph of $C(n)$. Does it make sense to connect the points on the graph? Why or why not?

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
52. The graph below shows the *distance traveled* by a car. Draw a graph to represent the *speed* of the car during the same time period.

53. The graph below shows the *speed* of a car. Draw a graph to represent the *distance traveled* by the car during the same time period.

54. Draw a graph to represent each situation.
   a. The height above the ground of a child swinging on a swing.
   b. Bill is walking to school when he realizes that he forgot his math book. He runs home to get it, and then jogs to school.
   c. The speed of a car stuck morning traffic.
55. The graphs below shows Sara’s distance from home over time. Describe the story that each graph tells about the Sara’s journey.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Story</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Graph 2" /></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph 3" /></td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Graph 4" /></td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Graph 5" /></td>
<td></td>
</tr>
<tr>
<td><img src="image6.png" alt="Graph 6" /></td>
<td></td>
</tr>
<tr>
<td><img src="image7.png" alt="Graph 7" /></td>
<td></td>
</tr>
<tr>
<td><img src="image8.png" alt="Graph 8" /></td>
<td></td>
</tr>
<tr>
<td><img src="image9.png" alt="Graph 9" /></td>
<td></td>
</tr>
</tbody>
</table>

---

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Unit 5: Review

1. In the space below, draw a graph that represents an increasing function, a constant function, and a graph that does NOT represent a function.

   Increasing Function  
   Constant Function  
   Not a Function

2. The graph of \( f(x) \) is given below. Use interval notation for the domain and range.

   a. Domain: ____________________
   b. Range ____________________
   c. \( f(0) = \)_________
   d. \( f(x) = 0 \) when \( x = \)_________

3. Let \( f(x) = 5x - 8 \).
   a. Determine \( f(-2) \).
   b. Determine \( x \) when \( f(x) = 42 \)

   b. Determine \( f(x - 6) \)
   c. Determine \( f(x + h) \)
4. A rock is dropped from the top of a building. The function \( h(t) = 100 - 16t^2 \) gives the height (measured in feet) of the rock after \( t \) seconds.

a. Use the Table feature of your Graphing Calculator to complete the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Is this function increasing or decreasing? __________________________

c. Determine \( h(1) \). Write a sentence explaining the meaning of your answer.

d. For what value of \( t \) is \( h(t) = 0 \)? Explain the meaning of your answer.

e. Determine the practical domain ______________________________________

f. Determine the practical range _________________________________________

g. Construct a good graph of \( h(t) \).

h. Use your graphing calculator to generate a graph of \( h(t) \). Use the practical domain and range to define your viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

\[ X_{\text{min}} = \text{__________} \quad X_{\text{max}} = \text{__________} \]
\[ Y_{\text{min}} = \text{__________} \quad Y_{\text{max}} = \text{__________} \]
Unit 6: Linear Functions

Section 6.1: Linear Functions
Section 6.2: Graphing Linear Functions
Section 6.3: Interpreting the Slope of a Linear Function
Section 6.4: Using Rates of Change to Build Tables and Graphs
Section 6.5: The Equation of a Linear Function
Section 6.6: Writing Linear Equations in Slope-Intercept Form
Section 6.7: Parallel and Perpendicular Lines
Section 6.8: Applications – Slope-Intercept Form
Section 6.9: Interpreting a Linear Function in Slope-Intercept Form

UNIT 6 LEARNING OBJECTIVES

- Calculate the slope between two ordered pairs and interpret it as rate of change
- Build tables and graphs using slope (rate of change) and a data point
- Describe how the rate of change of a linear function relates to the behavior of the graph.
- Compare the slopes of increasing lines, decreasing lines, horizontal lines, and vertical lines
- Describe the relationship between vertical and horizontal lines and the concept of slope.
- Determine and interpret slope (rate of change) in an application problem
- Determine if a data set is linear by determining if the slope is the same between all data points
- Given a linear equation in slope-intercept form, identify the slope and intercepts
- Graph a linear equation given in slope-intercept form
- Model and solve contextual problems using linear equations in slope-intercept form.
- Interpret the rate of change (slope) and the vertical intercept of a linear model contextually
- Model data that exhibit a constant rate of change with linear functions, equations and graphs
- Construct an equation of a line when given the slope and vertical intercept or given the slope and a point or given two points, and express the equation in the various forms

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Functions</td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td></td>
</tr>
<tr>
<td>Using Slope to Graph a Linear Function</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Units of Slope</td>
<td></td>
</tr>
<tr>
<td>Rate of Change</td>
<td></td>
</tr>
<tr>
<td>Constant Rate of Change</td>
<td></td>
</tr>
<tr>
<td>Interpreting the Slope of a Linear Function</td>
<td></td>
</tr>
<tr>
<td>Slope-Intercept Form</td>
<td></td>
</tr>
<tr>
<td>Parallel Lines</td>
<td></td>
</tr>
<tr>
<td>Perpendicular Lines</td>
<td></td>
</tr>
</tbody>
</table>
A linear function is a function that fits the form:

A linear function can be graphically represented by a ______________________________
Unit 6: Linear Functions

Increasing Linear Function
Slope > 0

Decreasing Linear Function
Slope < 0

Constant Function
Slope = 0

Not a Function
Slope is Undefined (No Slope)

\[ m = \text{slope} = \frac{\text{change in output}}{\text{change in input}} = \frac{\Delta \text{output}}{\Delta \text{input}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]
**Example 1:** Determine the slope for each of the following:

a. \((-2, 3)\) and \((4, -1)\)

b. \((-3, -1)\) and \((4, 2)\)

c. \((3, 2)\) and \((-1, 2)\)
d. (2, –3) and (2, 1)

Section 6.1 – You Try

Plot the points (–4, –1) and (5, –6) and draw a line connecting them. Determine the slope of this line. Show all steps, as in the media examples.
Section 6.2: Graphing Linear Functions

**USING THE SLOPE TO GRAPH A LINEAR FUNCTION**

\[ m = \text{Slope} = \frac{\text{Change in OUTPUT}}{\text{Change in INPUT}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} \rightarrow \uparrow \leftrightarrow \]

\[ m = \frac{2}{5} \rightarrow \text{up 2 right 5} \]

\[ m = \frac{2}{5} = -\frac{2}{5} \rightarrow \text{down 2 left 5} \]

**Example 1:** Draw an accurate graph for each of the following

a. \((-2, -3)\) slope \(\frac{1}{2}\)

b. \((0, -1)\) slope \(-\frac{2}{3}\)
c. (2, 1) slope 3

d. (1, –4) slope 0

e. (5, 2) undefined slope

Section 6.2 – You Try

Sketch the graph of a linear function that passes through the point (1, –2) with slope $= -\frac{3}{5}$

Your line must extend accurately from edge to edge of the graph shown.

Give the coordinates of at least two additional points on the line.

_________________________________
Section 6.3: Interpreting the Slope of a Linear Function

\[
\text{Slope} = \frac{\text{Change in Output}}{\text{Change in Input}} \quad \text{Units of Slope} = \frac{\text{Output Units}}{\text{Input Units}} \rightarrow \text{Rate of Change}
\]

Example: \( \text{Output} = \text{Height in Feet} \) \quad \text{Input} = \text{Time in Seconds} \n
\[
\text{Slope} = \frac{\text{Change in Height}}{\text{Change in Time}} \quad \text{Units of Slope} = \frac{\text{feet}}{\text{second}} = \text{feet/second}
\]

What is the meaning of a slope of \(-5\)?

What is the meaning of a slope of 8?

\text{Example 1:} \text{ Consider the graph shown below.}

\[\text{Amount of Water (gallons)}\]
\[\text{Time (minutes)}\]

a. Identify the vertical intercept and interpret its meaning.

b. Identify the horizontal intercept and interpret its meaning.

c. Determine the slope, and interpret its meaning.
**Example 2:** In the year 2003, an investment was worth $31,400. In the year 2016, this investment was worth $47,650. Assume a constant rate of change. Complete the sentence below:

The value of this investment is ___________ at a rate of ________________________.

**Section 6.3 – You Try**

The graph below shows Sally’s distance from home over a 30 minute time period.

a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning. Show your work and write your answer in a complete sentence.
Section 6.4: Using Rates of Change to Build Tables and Graphs

For each of the examples below, circle the rate of change in each situation and underline the starting value. Then use the given information to complete the table. Graph the results, and decide if it would make sense to connect the data points on the graph.

**Example 1:** A local carpet cleaning company charges $15 for each room plus a nonrefundable reservation fee of $25.

<table>
<thead>
<tr>
<th>Number of Rooms</th>
<th>Total Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

**Example 2:** Water is leaking out of a tank at a constant rate of 2 gallons per minute. The tank initially held 12 gallons of water.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Amount of Water in Tank (gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Sara is selling snow cones at the local carnival for $3 each.

Identify the rate of change in this situation. Be sure to include units in your answer.

Complete the table to show Sara’s revenue from selling the snow cones. Graph the results, and decide if it would make sense to connect the data points on the graph.

<table>
<thead>
<tr>
<th>Number of Snow Cones</th>
<th>Revenue (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Section 6.5: The Equation of a Linear Function

SLOPE-INTERCEPT FORM:
\[ y = mx + b \]
\[ y = b + mx \]
\[ f(x) = mx + b \]

<table>
<thead>
<tr>
<th>Slope</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m &gt; 0 )</td>
<td>Increasing</td>
</tr>
<tr>
<td>( m &lt; 0 )</td>
<td>Decreasing</td>
</tr>
<tr>
<td>( m = 0 )</td>
<td>Horizontal</td>
</tr>
<tr>
<td>( m ) is undefined</td>
<td>Vertical</td>
</tr>
</tbody>
</table>

**Example 1:** Fill in the table below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Behavior I, D, H, V</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 3x + 5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 8 - x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 2x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = -8 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 2:** Determine the horizontal intercepts of each of the following.

\[ y = 3x + 5 \] \quad \[ y = 8 - x \] \quad \[ y = 2x \] \quad \[ y = -8 \]

*To find a horizontal intercept:* ____________________________
Example 3: The equation of a vertical line

Example 4: Draw an accurate graph of the function \( f(x) = 4 - 3x \).

Slope: ____________

Vertical Intercept: ____________

Horizontal Intercept: ____________

To find the Horizontal Intercept:

_________________________

_________________________

Two additional points on the line:

__________    ____________
Section 6.5 – You Try

Complete the problems below.

a. Fill in the table below. Write intercepts as ordered pairs. Write “DNE” if the answer does not exist.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Behavior</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x - 11$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G(x) = -2x$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = 5$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw an accurate graph of the function $y = \frac{3}{4}x - 5$. Identify the slope, intercepts, and two additional points on the line.

Slope: ____________

Vertical Intercept: ____________

Horizontal Intercept: ____________

Two additional points on the line:

___________    ___________
Section 6.6: Writing the Equation of a Line in Slope-Intercept Form

Slope-Intercept Form \( y = mx + b \)

**Example 1**: Give the equation of the line in slope-intercept form

a. With vertical intercept \((0, 2)\) and slope \(-9\)

b. Passing through \((2, 3)\) with slope \(-5\)

c. Passing through \((2, 6)\) and \((4, 16)\)
Example 2: Give the equation of the linear function that would generate the following table of values. Use your calculator to check.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−5</td>
<td>238</td>
</tr>
<tr>
<td>−3</td>
<td>174</td>
</tr>
<tr>
<td>−1</td>
<td>110</td>
</tr>
<tr>
<td>1</td>
<td>46</td>
</tr>
<tr>
<td>7</td>
<td>−146</td>
</tr>
<tr>
<td>12</td>
<td>−306</td>
</tr>
</tbody>
</table>

Example 3: Give the equation of the linear function shown below.

Example 4: Give the equation of the horizontal line passing through the point (1, 3).

Example 5: Give the equation of the vertical line passing through the point (1, 3).
Section 6.6 – You Try

Complete the problems below. Show as much work as possible, as demonstrated in the Media Examples.

a. Give the equation of the line passing through the points (1, 7) and (3, –9).

b. Give the equation of the horizontal line passing through the point (5, 11).

c. Give the equation of the linear function shown below.
Section 6.7: Parallel and Perpendicular Lines

Parallel Lines

The slopes of Parallel Lines are ______________________________________________________________________

Slope-Intercept Form

\[ y = mx + b \]
\[ f(x) = mx + b \]
\[ m = \text{slope} \]
\[ b = \text{vertical intercept (0,} b) \]

Example 1: Give the equation of the line passing through the point \((8, 3)\) that is parallel to the line \(y = -2x + 3\).

Perpendicular Lines

The slopes of perpendicular lines are ______________________________________________________________________

175
If Line 1 and Line 2 are perpendicular to each other, then

<table>
<thead>
<tr>
<th>Slope of Line 1</th>
<th>Slope of Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$-8$</td>
<td></td>
</tr>
<tr>
<td>$-\frac{4}{5}$</td>
<td></td>
</tr>
</tbody>
</table>

**Example 2:** Give the equation of the line passing through the point $\left(8, 3\right)$ that is **perpendicular** to the line $y = -2x + 3$.

---

**Section 6.7 – You Try**

Give the equation of the line passing through the point $\left(-6, 1\right)$ that is:

a. **Parallel** to the line $y = 3x - 5$.

b. **Perpendicular** to the line $y = 3x - 5$. 
Section 6.8: Applications – Slope-Intercept Form

<table>
<thead>
<tr>
<th>Slope-Intercept Form</th>
<th>If we are not given the slope and vertical intercept, we need:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = mx + b )</td>
<td>• One point and the slope</td>
</tr>
<tr>
<td>( f(x) = mx + b )</td>
<td>• Two points</td>
</tr>
<tr>
<td>( m = \text{slope} = \text{rate of change} )</td>
<td></td>
</tr>
<tr>
<td>( b = \text{vertical intercept (initial value)} )</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1:** You have just bought a new Sony 55” 3D television set for $2300. The TV’s value decreases at a rate of $250 per year. Construct a linear function to represent this situation.

**Example 2:** In 1998, the cost of tuition at a large Midwestern university was $144 per credit hour. In 2008, tuition had risen to $238 per credit hour. Determine a linear equation to represent the cost, \( C \), of tuition as a function of \( x \), the number of years since 1990.

**Example 3:** When a new charter school opened in 2005, there were 520 students enrolled. Write an equation for \( N \) representing the number of students attending the school \( t \) years after 2005, assuming that the student population:

Increased by 32 students per year: ______________________________________

Decreased by 48 students per year: _______________________________________
Increased by 20 students every 2 years: ________________________________

Decreased by 28 students every 4 years: ________________________________

Remained constant (did not change): ________________________________

Increased by 10 students every semester: ________________________________

---

**Section 6.8 – You Try**

For each of the following, determine a linear equation to represent the given situation. Use the indicated variables and proper function notation. Show all possible steps.

a. A tree is 3 feet tall when it is planted, and it grows by approximately 1 foot every 2 years. Let \( H(t) \) represent the height of the tree (in feet) after \( t \) years.

b. The enrollment at a local charter has been decreasing linearly. In 2008, there were 857 students enrolled. By 2017, there were only 785 students enrolled. Let \( E(n) \) represent the number of students enrolled in this school \( n \) years after the year 2008.
Section 6.9: Interpreting a Linear Function in Slope-Intercept Form

Example 1: The function \( A(m) = 200 - 1.25m \) represents the balance in a bank account (in thousands of dollars) after \( m \) months.

a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

   Ordered Pair: _________________

c. Determine the horizontal intercept of this linear function. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

   Ordered Pair: _________________

d. Determine \( A(12) \). Write your answer as an ordered pair and interpret its practical meaning in a complete sentence.

   Ordered Pair: _________________

e. How long will it take for the balance in this account to reach $80,000? Write the corresponding ordered pair.

   Ordered Pair: _________________
The function \( E(t) = 3860 - 77.2t \) gives the surface elevation (in feet above sea level) of Lake Powell \( t \) years after 1999. Your answers must include all appropriate units.

a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

Ordered Pair:_________________

Ordered Pair:_________________

c. Determine \( E(5) \). Write your answer as an ordered pair and interpret its practical meaning in a complete sentence. Show your work.

Ordered Pair:_________________
Unit 6: Answers to You-Try Problems

Section 6.1:  Slope: $-\frac{5}{9}$

Section 6.2:  Additional Points: $(-4, 1), (6, -5)$

Section 6.3:  

a. Vertical Intercept: $(0,15)$  Sally was 15 miles from home after 0 minutes.
b. Horizontal Intercept: $(30,0)$  Sally arrives at home after 30 minutes
c. Slope: $-\frac{1}{2}$ mile per minute  
   Sally’s distance from home is decreasing at a rate of $\frac{1}{2}$ mile per minute.

Section 6.4:  Rate of change: $3$ per snow cone.

<table>
<thead>
<tr>
<th>Number of Snow Cones</th>
<th>Revenue (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Graphs will vary. Do not connect the data points.

Section 6.5:  

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Behavior</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x - 11$</td>
<td>1</td>
<td>Increasing</td>
<td>$(0, -11)$</td>
</tr>
<tr>
<td>$G(x) = -2x$</td>
<td>$-2$</td>
<td>Decreasing</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$x = 5$</td>
<td>DNE</td>
<td>Vertical</td>
<td>DNE</td>
</tr>
</tbody>
</table>
Section 6.6:  
- a. \( y = -8x + 15 \)  
- b. \( y = 11 \)  
- c. \( y = 2000 - 100x \) or \( y = -100x + 2000 \)

Section 6.7:  
- a. \( y = 3x + 19 \)  
- b. \( y = -\frac{1}{3}x - 1 \)

Section 6.8:  
- a. \( H(t) = \frac{1}{2}t + 3 \) or \( H(t) = 3 + \frac{1}{2}t \)  
- b. \( E(n) = -8n + 857 \) or \( E(n) = 857 - 8n \)

Section 6.9:  
- a. The surface elevation is decreasing at a rate of 77.2 feet per year.  
- b. (0, 3860) In 1999, the surface elevation of Lake Powell was 3860 feet above sea level.  
- c. (5, 3474) In 2004, the surface elevation of Lake Powell was 3474 feet above sea level.
Unit 6: Practice Problems

Skills Practice

1. Determine the slope of the line between each of the following pairs of points. Show all steps, and reduce your answer to lowest terms.

a. (4, –5) and (–2, 3)  
   b. (–3, 2) and (1, 8)

c. (5, –9) and (5, 2)  
   d. (2, –1) and (–2, 3)

e. (4, 3) and (12, –3)  
   f. (2, –4) and (7, –4)

2. Determine the slope of each of the lines shown below.

   a.  
   b.  

   Slope = ___________

   c.  
   d.  

   Slope = ___________

   e.  
   f.  

   Slope = ___________
3. Draw an accurate graph for each of the following by
   - Plotting the point
   - Using the slope to find at least two additional points

   a. (1, −2) with slope = \( \frac{1}{3} \)
   b. (−1,3) with slope = \( -\frac{3}{2} \)
   
   c. (3, 0) with slope = 5
   d. (0, −1) with slope = −3
   
   e. (2, −3) with undefined slope
   f. (−3, 1) with slope = 0
4. Determine the slope, behavior (increasing, decreasing, constant, or vertical), and vertical intercept (as an ordered pair) of each of the following. Write “DNE” if an answer does not exist.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Behavior</th>
<th>Vertical Intercept</th>
<th>Horizontal Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x - 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(a) = 6 - 4a )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P(n) = 3n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 7 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{3}{5}x - 4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B(x) = 8 - x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V(t) = -70 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Draw an accurate graph of the function \( f(x) = 4x + 5 \).

Slope: __________

Vertical Intercept: __________

Horizontal Intercept: __________
6. Draw an accurate graph of the function  \( y = \frac{2}{5}x - 3 \)

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

7. Draw an accurate graph of the function  \( g(x) = 3 - x \).

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

8. Draw an accurate graph of the function  \( y = -2x \).

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________
9. Draw an accurate graph of the function $r(a) = 5$.

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

10. Draw an accurate graph of the function $C(x) = \frac{x}{5}$.

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________

11. Draw an accurate graph of the function $y = x$.

Slope: ___________

Vertical Intercept: ____________

Horizontal Intercept: ____________
12. Determine the equation of the line between each of the following pairs of points.

a. (4, –5) and (2, 3)  
b. (–3, 2) and (1, 8)

c. (5, –9) and (5, 2)  
d. (2, –1) and (–2, 3)

e. (4, 3) and (12, –3)  
f. (2, –4) and (7, –4)

13. Give the equation of the linear function that generates the following table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>–5</td>
<td>91</td>
</tr>
<tr>
<td>–2</td>
<td>67</td>
</tr>
<tr>
<td>1</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>–21</td>
</tr>
</tbody>
</table>

14. Give the equation of the linear function that generates the following table of values.

<table>
<thead>
<tr>
<th>t</th>
<th>C(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>–1250</td>
</tr>
<tr>
<td>15</td>
<td>–900</td>
</tr>
<tr>
<td>20</td>
<td>–725</td>
</tr>
<tr>
<td>35</td>
<td>–200</td>
</tr>
<tr>
<td>45</td>
<td>150</td>
</tr>
</tbody>
</table>

15. Give the equation of the linear functions shown below

a.  

b.  

188
16. Give the equation of the horizontal line passing through the point \((-6, 11)\). ______________

17. Give the equation of the vertical line passing through the point \((4, 7)\). _______________

18. Give the equation of the \(x\)-axis. ______________

19. Give the equation of the \(y\)-axis. ______________

20. Graph the following piecewise-defined functions.

\[ f(x) = \begin{cases} 
3x + 4 & \text{if } x < 1 \\
-x + 6 & \text{if } x \geq 1
\end{cases} \]

21. Graph the lines A, B, C, and D on the grid below.

A: Passes through the point \((0, -5)\) with slope -2
B: Passes through the point \((0, -1)\) with slope -2
C: Passes through the point \((0, 3)\) with slope -2
D: Passes through the point \((0, 7)\) with slope -2

How are these lines geometrically related?
22. Give the equation of the line passing through the point \((1, -5)\) that is parallel to \(y = 12 - 8x\).

23. Give the equation of the line passing through the point \((4, 0)\) that is parallel to \(y = 9 - \frac{3}{2}x\).

24. Give the equation of the line passing through the point \((10, 3)\) that is perpendicular to \(y = \frac{2}{5}x + 1\).

25. Give the equation of the line passing through the point \((-12, -1)\) that is perpendicular to \(y = 3 - 4x\).

### Applications

26. The enrollment at a local charter has been decreasing linearly. In 2006, there were 857 students enrolled. By 2015, there were only 785 students enrolled. Determine the rate of change of this school’s enrollment during this time period.

27. A tree grows 2 feet taller every 3 years. Determine the rate of change in this situation.

28. Oil is leaking from a tanker at a rate of 18 gallons every 30 minutes. Determine the rate of change in this situation.

29. In the year 1987, an investment was worth $30,200. In the year 1996, this investment was worth $43,700. Determine the rate of change in this situation, and write a sentence explaining its meaning.

30. In the year 1998, the surface elevation of Lake Powell was 3,843 feet above sea level. In the year 2001, the surface elevation of Lake Powell was 3,609 feet above sea level. Determine the rate of change in this situation, and write a sentence explaining its meaning.
31. The function \( D(t) \) below shows Sally’s distance from home over a 30 minute time period.

![Graph showing distance from home over time]

a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope of \( D(t) \), and interpret its meaning.

d. Determine the practical domain of this linear function. Use inequality notation and include units.

e. Determine the practical range of this linear function. Use inequality notation and include units.

32. Janey is selling homemade scented candles. The graph below shows her profit from selling the candles.

![Graph showing profit from selling candles]

a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.

b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.

c. Determine the slope, and interpret its meaning.

33. With good credit, and a $5000 down payment, you can finance a new 2012 Chevrolet Camaro convertible for 60 months for $615.17 per month.

a. Determine the equation of the linear function, \( T(n) \), that represents the total amount paid for this car after \( n \) months.

b. Use the equation from part a to determine the total payment over the 60-month time period.

c. A new 2012 Chevrolet Camaro convertible has a base MSRP of $35,080. Why is this value lower than your answer in part b?
34. The function $P(n) = 455n - 1820$ represents a computer manufacturer’s profit when $n$ computers are sold.
   a. Identify the slope, and interpret its meaning in a complete sentence.
   b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
   c. Determine the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

35. John is a door to door vacuum salesman. His weekly salary is given by the linear function $S(v) = 200 + 50v$, where $v$ is the number of vacuums sold.
   a. Identify the slope, and interpret its meaning in a complete sentence.
   b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

36. The function $V(n) = 221.4 + 4.25n$ gives the value, in thousands of dollars, of an investment after $n$ years.
   a. Identify the slope, and interpret its meaning in a complete sentence.
   b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

37. The function $V(t) = 86.4 - 1.2t$ gives the value, in thousands of dollars, of an investment after $t$ years.
   a. Identify the slope, and interpret its meaning in a complete sentence.
   b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
   c. Determine the horizontal intercept. Write it as an ordered pair and discuss its meaning.

38. In the year 2000, the median cost for in-state tuition and fees at a public 4-year college was $3412. In the year 2010, the median cost for tuition had risen to $7231.
   a. Determine a linear function, $C(t)$ to represent the cost for tuition and fees $t$ years since 2000. Show all of your work. Write your answer in function notation, $C(t) = mt + b$.
   b. Determine $C(13)$. Show all of your work. Write your answer in a complete sentence.
   c. Identify the slope of this linear function and write a sentence explaining its meaning in this situation.
39. When a new charter school opened in 2005, there were 300 students enrolled. Write a formula for the function \( N(t) \) representing the number of students attending this charter school \( t \) years after 2005, assuming that the student population

\[ a. \] Increases by 20 students per year.
\[ b. \] Decreases by 40 students per year.
\[ c. \] Increases by 100 students every 4 years.
\[ d. \] Decreases by 60 students every two years.
\[ e. \] Remains constant (does not change).

**Extension**

40. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function \( R(x) \) gives the revenue earned when \( x \) candles are sold. The function \( C(x) \) gives the total cost to produce \( x \) candles.

\[ a. \] Identify the vertical intercept of \( C(x) \). Write it as an ordered pair, and interpret its meaning.
\[ b. \] Determine the slope of \( C(x) \). Interpret its meaning.
\[ c. \] Identify the vertical intercept of \( R(x) \). Write it as an ordered pair, and interpret its meaning.
\[ d. \] Determine the slope of \( R(x) \). Interpret its meaning.
\[ e. \] Discuss the significance of the point (40, 100) in terms of the cost, revenue, and profit for this company.
41. Write the formula for each of the piecewise functions below

(a) [Graph of a line with points at (-2, 1), (1, 2), (3, 4)]

(b) [Graph of a line with points at (-3, -1), (0, 0), (3, 2)]

(c) [Graph of a line with points at (-4, -3), (-2, -2), (2, 1)]

(d) [Graph of a line with points at (-3, -2), (0, 0), (3, 3), (5, 5)]
1. Draw an accurate graph of the linear function \( y = 5 - \frac{1}{3}x \).

Slope: ___________
Vertical Intercept: ___________
Horizontal Intercept: ___________

2. The function \( V(n) = 221 - 4.25n \) gives the value, in thousands of dollars, of an investment after \( n \) years.

a. Identify the slope of this linear function and interpret its meaning in a complete sentence.

b. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

   Ordered Pair: _________________

   Ordered Pair: _________________

c. Determine the horizontal intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

   Ordered Pair: _________________
3. When it first opened, there were 800 students enrolled in a new charter school. Using correct notation, write a formula for the number, \( N \), of students enrolled in this charter school after \( x \) years, assuming that the enrollment:
   a) Increases by 32 students per year.            b) Decreases by 20 students per year.
   c) Increases by 100 students every 4 years.       d) Remains constant.

4. Give the equation of the vertical line passing through the point (7, 3): ________________

5. Give the equation of the horizontal line passing through the point (7, 3): ________________

6. In 2010, the cost of tuition at a large Midwestern university was $88 per credit hour. In 2016, tuition had risen to $118 per credit hour. Determine a linear function \( C(x) \) to represent the cost of tuition as a function of \( x \), the number of years since 2010.

7. Determine the equation of the line between the points (4, 3) and (12, –3). Your answer must be written in slope-intercept form, \( y = mx+b \).

8. Kayla is selling homemade scented candles. The function \( P(c) \) shown below represents Kayla’s profit earned from selling \( c \) candles. Write the formula for \( P(c) \). Use the indicated variables and proper function notation.
Unit 7: Systems of Linear Equations

Section 7.1: General Form: \( ax + by = c \)
Section 7.2: Applications – General Form
Section 7.3: Systems of Linear Equations
Section 7.4: The Substitution Method
Section 7.5: The Addition (Elimination) Method
Section 7.6: Applications

UNIT 7 LEARNING OBJECTIVES

- Given a linear equation in general form, identify the slope and intercepts
- Graph a linear equation given in general form
- Convert a linear equation given in general form to slope-intercept form
- Model and solve contextual problems using linear equations in general form
- Verify that a given point is a solution to a system of linear equations
- Solve a system of linear equations by graphing by hand and using the intersection method on the graphing calculator
- Use the Substitution Method to solve a system of linear equations
- Use the Addition (Elimination) method to solve a system of linear equations
- Model contextual problems with systems of two linear equations, solve using graphing and algebraic techniques and interpret the solutions

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

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<th>Key Term / Concept</th>
<th>Definition and examples</th>
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Unit 7: Media Lesson

Section 7.1: General Form: \( ax + by = c \)

<table>
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<tr>
<th>Slope-Intercept Form of a Linear Equation</th>
<th>General (Standard) Form of a Linear Equation</th>
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<tr>
<td>( y = mx + b )</td>
<td>( ax + by = c )</td>
</tr>
<tr>
<td>( x = \text{input}, \ y = \text{output} )</td>
<td>( x = \text{input}, \ y = \text{output} )</td>
</tr>
<tr>
<td>( m = \text{slope} )</td>
<td>( a, b, \text{ and } c ) \text{ are constants}</td>
</tr>
<tr>
<td>( b = \text{vertical intercept} (0, b) )</td>
<td></td>
</tr>
</tbody>
</table>

Example 1: Consider the linear equation \( 3x - 5y = 30 \)

a. Write this equation in slope-intercept form.

b. Identify the slope.

d. Determine the horizontal intercept.

c. Determine the vertical intercept.
Example 2: Draw an accurate graph of the linear equation $3x + 2y = 16$.

- **Slope-Intercept Form:**
  - _______________________

- **Slope:** ________
- **Vertical Intercept:** __________
- **Horizontal Intercept:** ________
- **Additional points on the line:**
  - _______________________

Section 7.1 – YOU TRY

Draw an accurate graph of the linear equation $4x - y = 7$

- **Slope-Intercept Form:**
  - _______________________

- **Slope:** ________
- **Vertical Intercept:** __________
- **Horizontal Intercept:** ________
- **Additional points on the line:**
  - _______________________

200
Section 7.2: Applications – General Form

**Example 1:** Movie tickets cost $7 for adults (matinee), $5.50 for children. A total of $668 was collected in ticket sales for the Saturday matinee.

a. Write an equation representing the total amount of money collected.

b. If 42 adult tickets were purchased for this matinee, how many children were there?

**Example 2:** Juan has a pocket full of dimes and quarters. The total value of his change is $6.25.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Juan has 7 quarters in his pocket, how many dimes are there?

**Example 3:** Ivan invested money into two mutual funds. Fund A earned 6% interest during the first year, while Fund B earned 8% interest. At the end of the year, he receives a total of $774 in interest.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Ivan invested $8500 in Fund A, how much did he invest in Fund B?
Example 4: Kim invested money into two mutual funds. Fund A earned 6% profit during the first year, while Fund B suffered a 3.5% loss. At the end of the year, she receives a total of $177 in profit.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. If Kim invested $3650 in Fund A, how much did she invest in Fund B?

Section 7.2 – YOU TRY

The Science Museum charges $18 for adult admission and $10 for each child. The museum bill for a school field trip was $1150.

a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

b. Fifteen adults attended the field trip. How many children were there?
Section 7.3: Systems of Linear Equations

**Definitions**

Two linear equations that relate the same two variables are called a **system of linear equations**. A **solution** to a system of linear equations is an **ordered pair** that satisfies both equations.

---

**Example 1**: Verify that the point $(5, 4)$ is a solution to the system of equations

\[
\begin{align*}
y &= 2x - 6 \\
y &= x - 1
\end{align*}
\]

---

**Types of Solutions to a Linear System of Equations**

Graphically, the solution to a system of linear equations is a point at which the graphs intersect.

**Types of Solutions to a Linear System of Equations**:

- **One unique solution**: The lines intersect at exactly one point
- **No solution**: The two lines are parallel and will never intersect
- **Infinitely many solutions**: This occurs when both lines graph as the same line

---

![Graphs of One Unique Solution, No Solution, and Infinitely Many Solutions](attachment:image.png)
Example 2: Solve the system of equations by graphing. Check your answer.

\[
\begin{align*}
y &= 6 - \frac{2}{3}x \\
y &= x + 1
\end{align*}
\]

Example 3: Solve the system of equations by graphing. Check your answer.

\[
\begin{align*}
4x - 3y &= -18 \\
2x + y &= -4
\end{align*}
\]
**Example 4:** Solve the system of equations by graphing. Check your answer.

\[ x - 3y = 3 \]
\[ 3x - 9y = -18 \]

**Example 5:** Solve the system of equations by graphing. Check your answer.

\[ 2x + y = 3 \]
\[ 6x + 3y = 9 \]

**Example 6:** Use your graphing calculator to solve the system of equations.

\[ 3x + 2y = 6 \]
\[ x - 2y = -6 \]
Section 7.3 – You Try

Solve the system of equations by graphing. Write your answer as an ordered pair.

\[ x - y = 2 \]
\[ x + y = 6 \]

Use the intersection method on your Graphing Calculator to verify that your solution is correct. Use the standard viewing window and sketch what you see on your calculator screen in the space below:

Verify algebraically that your solution is correct:
Section 7.4: The Substitution Method

Consider the following equations: 
\[ y = 2x \]
\[ x + y = 3 \]

<table>
<thead>
<tr>
<th>Using Substitution to Solve a Linear System of Equations</th>
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<tbody>
<tr>
<td>Step 1: Solve one of the equations of the system for one of the variables.</td>
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<tr>
<td>Step 2: Substitute the expression for the variable obtained in step 1 into the other equation.</td>
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<tr>
<td>Step 3: Solve the equation.</td>
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<tr>
<td>Step 4: Substitute the result back into one of the original equations to find the ordered pair solution.</td>
</tr>
<tr>
<td>Step 5: Check your result by substituting your result into either one of the original equations.</td>
</tr>
</tbody>
</table>

**Example 1:** Solve the system of equations using the Substitution Method.
\[ 3x - 2y = 16 \]
\[ 2x + y = 20 \]

**Example 2:** Solve the system of equations using the Substitution Method.
\[ 5x - 4y = 9 \]
\[ x - 2y = -3 \]
Example 3: Solve the system of equations using the Substitution Method.

\[ 3x + y = 5 \]
\[ 6x + 2y = 11 \]

Example 4: Solve the system of equations using the Substitution Method.

\[ x - y = -1 \]
\[ y = x + 1 \]

Section 7.4 – You Try

Solve the system of equations using the Substitution Method. Show all steps. Check your answer.

\[ x - 2y = -11 \]
\[ 5x + 2y = 5 \]
Section 7.5: The Addition (Elimination) Method

Consider the following systems of equations:

\[ x - 2y = -11 \]
\[ 5x + 2y = 5 \]

---

**Using the Addition (Elimination) Method to Solve a Linear System of Equations**

Step 1: “Line up” the variables.

Step 2: Determine which variable you want to eliminate. Make those coefficients opposites.

Step 3: Add straight down (one variable should “drop out”)

Step 4: Solve resulting equation

Step 5: Substitute this result into either of the ORIGINAL equations

Step 6: Solve for the variable

Step 7: CHECK!!!!!!! Plug solution into BOTH equations!

---

**Example 1:** Solve the system of equations using the Addition (Elimination) Method.

\[ 4x - 3y = -15 \]
\[ x + 5y = 2 \]
Example 2: Solve the system of equations using the Addition (Elimination) Method.

\[ 3x - 2y = -12 \]
\[ 5x - 8y = 8 \]

Example 3: Solve the system of equations using the Addition (Elimination) Method.

\[ 7x - 2y = 41 \]
\[ 3x - 5y = 1 \]

Section 7.5 – You Try

✍️ Solve the system of equations using the Addition (Elimination) Method. Show all steps. Check your answer.

\[ 2x + 3y = 18 \]
\[ x - y = 4 \]
**Section 7.6: Applications**

**Example 1:** In supply and demand problems, $y$ is the number of items the supplier will produce (or consumers will buy) if the price of the item is $x$ dollars.

Supply equation: $y = 6x + 530$

Demand equation: $y = -8x + 740$

What is the intersection point of the two lines?

What is the selling price when supply and demand are in equilibrium?

What is the number of items in the market when the supply and demand are in equilibrium?

**Example 2:** You are offered two different sales jobs. Company A offers a straight commission of 5% of the sales. Company B offers a salary of $510 per week plus 1% of the sales. How much would you need to sell in a week in order for Company A’s offer to be at least as good as Company B’s offer?
Example 3: A woman has 21 coins in her pocket, all of which are dimes and quarters. If the total value of her change is $3.90, how many dimes and how many quarters does she have?

Example 4: Jun had $26,100 and chose to split the money into two different mutual funds. During the first year, Fund A earned 7% interest and Fund B earned 2% interest. If Jun received a total of $1,107 in interest, how much had he invested into each account?

Example 5: The Nut Shack sells hazelnuts for $6.80 per pound and peanuts for $5.30 per pound. How much of each type should be used to make a 37 pound mixture that sells for $5.91 per pound? Round your answers to the nearest pound.

________pounds of hazelnuts

________pounds of peanuts
Example 6: A chemist needs to make 2 liters of a 15% acid solution from a 10% acid solution and a 35% acid solution. How many liters of each solution should she mix to get the desired solution?

Section 7.6 – You Try

Tickets to a 3D movie cost $12.50 for adults and $8.50 for children. The theater can seat up to 180 people. A total of $1,826 was collected in ticket sales for the sold-out 7:15PM show. Determine the number of adult tickets and the number of children’s tickets that were sold.

a. Write an equation representing the total number of tickets sold. Clearly indicate what each variable represents.

b. Write an equation representing the total amount of money collected from the sale of all tickets.

c. Solve this system of linear equations. Show all steps.

Number of adult tickets sold: ____________

Number of children’s tickets sold: ____________
Unit 7: Answers to You-Try Problems

Section 7.1: Slope-Intercept Form: \( y = 4x - 7 \)

- Slope: 4
- Vertical Intercept: (0, –7)
- Horizontal Intercept: \( \left( \frac{7}{4}, 0 \right) \)
- Additional points line: (1, –3) (2,1) (3,5) (4,9)

Section 7.2:

a. \( 10C + 18A = 1150 \)   \( C \) = the number of children; \( A \) = the number of adults

b. There were 88 children attending the field trip

Section 7.3:  \( (4, 2) \)

Section 7.4:  \( (-1, 5) \)

Section 7.5:  \( (6, 2) \)

Section 7.6:

a. \( A \) = number of adult tickets, \( C \) = number of child tickets   \( A + C = 180 \)

b. \( 12.5A + 8.5C = 1826 \)

c. 74 adult tickets sold, 106 children’s tickets sold.
1. Write the equation \( x - y = 5 \) in Slope-Intercept Form.

2. Write the equation \( 2x + 3y = 6 \) in Slope-Intercept Form.

3. Draw an accurate graph of the linear equation \( 2x + 4y = 12 \).

4. Draw an accurate graph of the linear equation \( 3x - 2y = 10 \).

5. Is the point \((6, 1)\) a solution to the system of equations below?

\[
\begin{align*}
  y &= x - 5 \\
  y &= 2x + 4
\end{align*}
\]

6. Is the point \((-2, 5)\) a solution to the system of equations below?

\[
\begin{align*}
  2x + y &= 1 \\
  3x - 2y &= -16
\end{align*}
\]
7. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

\[ y = 7 - x \]
\[ y = 3x - 5 \]

Solution: _____________

8. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

\[ x - y = -2 \]
\[ x + y = 4 \]

Solution: _____________
9. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

\[
\begin{align*}
    x - 2y &= 10 \\
    5x - y &= -4
\end{align*}
\]

Solution: _____________

10. Solve the system of equations by **graphing**. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.

\[
\begin{align*}
    3x - y &= 8 \\
    -3x + y &= 1
\end{align*}
\]

Solution: _____________
11. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.
\[
\begin{align*}
x + 2y &= -4 \\
2x + 4y &= -8
\end{align*}
\]
Solution: ______________

12. Solve the system of equations using the substitution method. Show all steps.
\[
\begin{align*}
5x + y &= 2 \\
3x - 4y &= 15
\end{align*}
\]
Solution: ______________

13. Solve the system of equations using the substitution method. Show all steps.
\[
\begin{align*}
2x + y &= 8 \\
6x + 3y &= 24
\end{align*}
\]
Solution: ______________

14. Solve the system of equations using the substitution method. Show all steps.
\[
\begin{align*}
x - y &= 9 \\
5x + 3y &= 21
\end{align*}
\]
Solution: ______________

15. Solve the system of equations using the addition (elimination) method. Show all steps.
\[
\begin{align*}
-3x + 2y &= 12 \\
x + y &= 16
\end{align*}
\]
Solution: ______________

16. Solve the system of equations using the addition (elimination) method. Show all steps.
\[
\begin{align*}
3x - 2y &= -12 \\
12x - 8y &= 22
\end{align*}
\]
Solution: ______________
17. Solve the system of equations using the addition (elimination) method. Show all steps.
   \[ 3x + 2y = -18 \]
   \[ 4x - 3y = -24 \]
   Solution: ______________

18. Solve the system of equations using the addition (elimination) method. Show all steps.
   \[ 5x + 2y = -10 \]
   \[ 3x + 4y = 8 \]
   Solution: ______________

19. The functions \( f(x) \) and \( g(x) \) are defined by the following tables. At what point is \( f(x) = g(x) \)?

   \[
   \begin{array}{c|cccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   \hline
   f(x) & 11 & 8 & 5 & 2 & -1 & -4 & -7 \\
   \end{array}
   \]

   \[
   \begin{array}{c|cccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   \hline
   g(x) & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]

   Solution (write the ordered pair): ________________

Applications

20. At a concession stand, three hot dogs and five sodas cost $18.50.
   a. Let \( h \) represent the price of each hot dog, and \( s \) represent the price of each soda. Write a linear equation in general form to represent this situation.
   b. If hot dogs cost $3.25 each, how much is each soda?

21. The Science Museum charges $14 for adult admission and $11 for each child. The museum bill for a school field trip was $896.
   b. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.
   c. Nine adults attended the field trip. How many children were there?

22. Jamaal invested money into two mutual funds. Fund A earned 6% interest during the first year, while Fund B earned 2.5% interest. At the end of the year, he receives a total of $390 in interest. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

23. Marisol invested money into two mutual funds. Fund A earned 4% profit during the first year, while Fund B suffered a 2% loss. At the end of the year, she receives a total of $710 in profit. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.
24. Jake has a pocket full of dimes and quarters. The total value of his change is $4.00. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.

25. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $80 consultation fee plus $14 per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $30 per hour for the actual work.
   a. Write an equation that describes the cost, C, if you hire The Greenhouse for \( h \) hours of work.
   b. Write a second equation that describes Garden Pros’ charge, C, for \( h \) hours of work.
   c. Solve this system of linear equations. Write your answer as an ordered pair.
   d. Interpret the solution in a complete sentence.
   e. Your yard needs a lot of work, and you anticipate that the job will take at least 6 hours. Which service do you choose? Why?

26. The graph below shows the cost and revenue for a company that produces and sells scented candles. The function \( R(x) \) gives the revenue earned when \( x \) candles are sold. The function \( C(x) \) gives the total cost to produce \( x \) candles.

![Graph of cost and revenue](image)

   a. Discuss the significance of the point (40, 100) in terms of the cost, revenue, and profit for this company.
   b. What happens if fewer than 40 candles are sold?
   c. What happens if more than 40 candles are sold?

27. At a concession stand, five hot dogs and five sodas cost $30. Two hot dogs and four sodas cost $15. Determine the price of each hot dog and each soda.

28. The Science Museum charges $14 for adult admission and $11 for each child. The total bill for 68 people from a school field trip was $784. How many adults and how many children went to the museum?
29. Tickets to a 3D movie cost $12.50 for adults and $8.50 for children. The theater can seat up to 260 people. A total of $1,734 was collected in ticket sales for the 7:15PM show, in which only 60% of the tickets were sold. How many adults and how many children were in the theater?

30. Jake has 20 coins in his pocket, all of which are dimes and quarters. If the total value of his change is $4.10, how many dimes and how many quarters does he have?

31. Juan had $17400 and chose to split the money into two different mutual funds. During the first year, Fund A earned 3% interest and Fund B earned 6% interest. If he received a total of $774 in interest, how much had he invested into each account?

32. Emery invested $10,000 in two mutual funds. Fund A earned 4% profit during the first year, while Fund B suffered a 2% loss. If she received a total of $130 profit, how much had she invested in each mutual fund?

33. The Nutty Professor sells cashews for $7.80 per pound and Brazil nuts for $4.20 per pound. How much of each type should be used to make a 50 pound mixture that sells for $6.00 per pound?

34. A candy distributor needs to mix a 40% fat-content chocolate with a 60% fat-content chocolate to create 200 kilograms of a 49% fat-content chocolate. How many kilograms of each kind of chocolate must they use?

35. Katy needs to mix a 20% alcohol solution with a 40% alcohol solution to create 200 milliliters of a 24% solution. How many milliliters of each solution must Katy use?

36. James needs to mix a 20% fungicide solution with a 50% fungicide solution to create 100 milliliters of a 29% solution. How many milliliters of each solution must James use?

37. A coffee distributor needs to mix a(n) Costa Rican coffee blend that normally sells for $10.50 per pound with a House coffee blend that normally sells for $14.60 per pound to create 70 pounds of a coffee that can sell for $11.61 per pound. How many pounds of each kind of coffee should they mix?

38. Bill begins a 100 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. The whole trip takes 6 hours. If Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 20 miles per hour, find the amount of time he spent walking. Write your answer in a complete sentence. (Hint: Distance = rate \times time)
39. *Refer to your course syllabus*

   a. The Final Exam for this class is worth ____________ % of your course grade.

   b. Let \( x \) represent the score you make on the Final Exam (as a percent), and \( y \) represent your grade in the class (as a percent) just prior to taking the Final Exam. Write a linear inequality in general form to represent this situation, assuming that you want your final course grade to be:

   A: At least 90%  
   B: At least 80%  
   C: At least 70%  

   Hint: If your Final Exam is worth 30% of your course grade, then everything else would be worth 100% – 30% = 70% of your course grade.

   c. Suppose you have a 77% in the class just before taking the final exam. What score do you need to make on the Final Exam to earn an A, B, or C in the class? Assume that your instructor does not round up!

40. Construct a system of linear equations (in slope-intercept form) that has the ordered pair \((3,5)\) as a solution.

41. Construct a system of linear equations (in general form) that has the ordered pair \((2,4)\) as a solution.

42. The functions \( f(x) \) and \( g(x) \) are defined by the following tables.

At what point(s) is \( f(x) = g(x) \)?

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

Solutions (write the ordered pairs): ________________
Unit 7: Review

1. Draw an accurate graph of the linear equation $2x + 3y = 6$. Determine the slope and intercepts of this linear equation and rewrite this equation in Slope-Intercept Form.

   Slope-Intercept Form:

   Slope: ___________

   Vertical Intercept: ____________

   Horizontal Intercept: ____________

2. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Use the Intersection Method on your graphing calculator to verify that your solution is correct.

   $4x - 3y = -18$
   $3x + y = -7$

   Solution: _______________
3. Solve the system of equations using the **substitution** method. Show all steps. Verify that your solution is correct.

\[ 2x - 3y = -19 \]
\[ x + 2y = 8 \]

Solution: ______________

4. Solve the system of equations using the **elimination** method. Show all steps. Verify that your solution is correct.

\[ 2x - 3y = -19 \]
\[ x + 2y = 8 \]

Solution: ______________
Unit 8: Exponential Functions

Section 8.1: Introduction to Exponential Functions
Section 8.2: Characteristics of Exponential Functions
Section 8.3: Growth and Decay Rates
Section 8.4: Linear and Exponential Models
Section 8.5: Exponential Equations by Graphing
Section 8.6: Applications

UNIT 8 LEARNING OBJECTIVES

- Construct and compare linear and exponential models
- Identify the characteristics of an exponential function
- Solve exponential equations by graphing

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential Function</td>
<td></td>
</tr>
<tr>
<td>Asymptote</td>
<td></td>
</tr>
<tr>
<td>Growth / Decay Factor</td>
<td></td>
</tr>
<tr>
<td>Doubling Time</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>---</td>
</tr>
</tbody>
</table>

| Halving Time |   |
Section 8.1: Introduction to Exponential Functions

**Example 1:** Complete the input/output table for each of the functions below then graph each function on the grid provided.

<table>
<thead>
<tr>
<th>x</th>
<th>$f(x) = 2^x$</th>
<th>$g(x) = 2^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What do you notice about the outputs in the table above and about the graphs for each function?
Example 2: After graduating from college, Tristan receives two different job offers. Both pay a starting salary of $58,000 per year, but one job promises a $2,000 raise per year, while the other guarantees a 5% raise each year.

Complete the tables below to determine what Tristan’s salary will be after $t$ years. *Round your answers to the nearest dollar.*

<table>
<thead>
<tr>
<th>Year, $t$</th>
<th>Salary with $2000 raise each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year, $t$</th>
<th>Salary with 5% raise each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
After graduating from college, Erik receives two different job offers. Both pay a starting salary of $47,000 per year, but one job promises a $1,500 raise per year, while the other guarantees a 3% raise each year. Use this information to complete the tables below. Round your answers to the nearest dollar as needed.

<table>
<thead>
<tr>
<th>Year, t</th>
<th>Salary with $1500 raise each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year, t</th>
<th>Salary with 3% raise each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Section 8.2: Characteristics of Exponential Functions  \( f(x) = ab^x \)

**Example 1:** Use the equation \( y = 2^x \) to complete the table below. Graph your results.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exponential Functions** are of the form \( f(x) = ab^x \) where:

- The input, \( x \), is located in the **exponent**.
- \( a \) is the **initial value**. This can be found by computing \( f(0) = ab^0 = a(1) = a \).
- \( b \) is the **base** \((b > 0 \text{ and } b \neq 1)\); also called the **growth factor** or **decay factor**.
  - If \( b > 1 \), the function is an **exponential growth function**, and the graph increases from left to right.
  - If \( 0 < b < 1 \), the function is an **exponential decay function**, and the graph decreases from left to right.
- Vertical intercept: \((0, a)\)
- Horizontal Intercept: DNE
- Domain: All Real Numbers or \((-\infty, \infty)\)
- Range: \( f(x) > 0 \) or \((0, \infty)\)
- Horizontal Asymptote: \( y = 0 \)
**Example 2:** Complete the table below for each of the functions. Draw your graph by hand first, then confirm using your calculator and the window: $X[-10..10]$, $Y[0..500]$.

<table>
<thead>
<tr>
<th></th>
<th>$f(x) = 150(1.25)^x$</th>
<th>$g(x) = 150(0.75)^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Graph</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial Value</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Domain</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Horizontal Intercept</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertical Intercept</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Horizontal Asymptote</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Increasing or Decreasing</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 8.2 – YOU TRY

Complete the table below for each of the functions. Draw your graph by hand first, then confirm using your calculator and the window: X[-10..10], Y[0..500].

<table>
<thead>
<tr>
<th></th>
<th>( f(x) = 235(1.14)^x )</th>
<th>( g(x) = 235(0.95)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X[-10..10] Y[0..500]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth or Decay?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Intercept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 8.3: Growth and Decay Rates

An exponential function $f(x) = ab^x$ grows or decays at a constant percent rate, $r$.

$r = \text{growth/decay rate in decimal form}$

Growth Factor: $b = 1 + r$  \quad \text{Growth Rate: } r = b - 1

Decay Factor: $b = 1 - r$  \quad \text{Decay Rate: } r = 1 - b

Example 1: Complete the following table.

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Growth or Decay?</th>
<th>Initial Value</th>
<th>Growth/Decay Factor</th>
<th>Growth/Decay Rate, $r$ (as a decimal)</th>
<th>Growth/Decay Rate, $r$ (as a %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 812(0.71)^x$</td>
<td>Growth</td>
<td></td>
<td></td>
<td></td>
<td>0.5%</td>
</tr>
<tr>
<td>$g(t) = 64.5(1.32)^t$</td>
<td>Decay</td>
<td>150</td>
<td></td>
<td></td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>Growth</td>
<td>8.24</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 2: The equation \( V = 355000(1.06)^t \) represents the value (in dollars) of a house \( t \) years after its purchase. Use this equation to complete the statements below.

The value of this house is ______________ at a rate of _____________________.

The purchase price of the house was _____________________________.

The value of the house after 5 years is ______________.

Round to the nearest dollar.

Example 3: The equation \( V = 37500(0.84)^t \) represents the value (in dollars) of a car \( t \) years after its purchase. Use this equation to complete the statements below.

The value of this car is ______________ at a rate of _____________________.

The purchase price of the car was _____________________________.

The value of the car after 3 years is ______________.

Round to the nearest dollar.

Section 8.3 – YOU TRY

Complete the following table.

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Growth or Decay?</th>
<th>Initial Value</th>
<th>Growth/Decay Rate (as a %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 300(0.88)^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 213(1.2)^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Growth</td>
<td>177</td>
<td>9.8%</td>
</tr>
<tr>
<td></td>
<td>Decay</td>
<td>5.41</td>
<td>7%</td>
</tr>
</tbody>
</table>
Section 8.4: Linear and Exponential Models

Example 1: When it first opened, there were 700 students enrolled in a new charter school. Using function notation, write a formula for the function $N(t)$ which represents the number of students enrolled in this charter school after $t$ years, assuming that the enrollment

a) Increases by 20 students per year.  

b) Increases by 20% per year.

c) Decreases by 20 students each year.  

d) Decreases by 20% per year.

Section 8.4 – YOU TRY

You purchase a house for $210,000. Using function notation, $V(x)$, write a formula for the value of the house, in dollars, $x$ years after purchase, assuming that its value:

a) Increases by $1500 per year.  

b) Increases by 30% per year.

c) Decreases by 15% per year.  

d) Remains constant (does not change).
Section 8.5: Solving Exponential Equations by Graphing

Example 1: Solve the following equations by graphing. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) Solve $50 = 25(1.15)^x$  
Solution: $x = \text{________________} $

\[ \begin{array}{c}
\text{Xmin} = \text{__________} \\
\text{Xmax} = \text{__________} \\
\text{Ymin} = \text{__________} \\
\text{Ymax} = \text{__________}
\end{array} \]

b) Solve $250 = 25(1.15)^x$  
Solution: $x = \text{________________} $

\[ \begin{array}{c}
\text{Xmin} = \text{__________} \\
\text{Xmax} = \text{__________} \\
\text{Ymin} = \text{__________} \\
\text{Ymax} = \text{__________}
\end{array} \]

c) Solve $5 = 10(0.86)^t$  
Solution: $t = \text{________________} $

\[ \begin{array}{c}
\text{Xmin} = \text{__________} \\
\text{Xmax} = \text{__________} \\
\text{Ymin} = \text{__________} \\
\text{Ymax} = \text{__________}
\end{array} \]
d) Solve $40 = 10(0.86)^t$
Solution: $t = \underline{\hspace{2cm}}$

- $X_{\text{min}} = \underline{\hspace{2cm}}$
- $X_{\text{max}} = \underline{\hspace{2cm}}$
- $Y_{\text{min}} = \underline{\hspace{2cm}}$
- $Y_{\text{max}} = \underline{\hspace{2cm}}$

e) Solve $1500 = 1000(2.32)^x$
Solution: $x = \underline{\hspace{2cm}}$

- $X_{\text{min}} = \underline{\hspace{2cm}}$
- $X_{\text{max}} = \underline{\hspace{2cm}}$
- $Y_{\text{min}} = \underline{\hspace{2cm}}$
- $Y_{\text{max}} = \underline{\hspace{2cm}}$

e) Solve $400 = 1000(2.32)^x$
Solution: $x = \underline{\hspace{2cm}}$

- $X_{\text{min}} = \underline{\hspace{2cm}}$
- $X_{\text{max}} = \underline{\hspace{2cm}}$
- $Y_{\text{min}} = \underline{\hspace{2cm}}$
- $Y_{\text{max}} = \underline{\hspace{2cm}}$

Section 8.5 – You Try

- Solve $1400 = 215(0.958)^x$
Solution: $x = \underline{\hspace{2cm}}$

- $X_{\text{min}} = \underline{\hspace{2cm}}$
- $X_{\text{max}} = \underline{\hspace{2cm}}$
- $Y_{\text{min}} = \underline{\hspace{2cm}}$
- $Y_{\text{max}} = \underline{\hspace{2cm}}$
Section 8.6: Applications of Exponential Growth / Decay

Example 1: Jolene invested $13,200 in a savings account that paid 7.5% interest compounded annually. The function used to represent this investment is $A(t) = 13200(1.075)^t$ where $A(t)$ is the accrued value of the savings account and $t$ is the length of the investment in years. Use the function to answer the following questions.

Determine how much money Jolene will have after 12 years.

Determine how long it will take for Jolene to have $69,657.62 accrued.

Example 2: Since 1990, the number of fish in Lake Beckett has been decreasing at a rate of 1.7% per year. In 1990, the population of fish was estimated to be 121 million. Use this information to answer the following:

a. Write the exponential function $P(t)$ for this scenario, where $P(t)$ is the fish population in millions $t$ years after 1990.

b. Determine the number of fish in Lake Beckett in 1993. Round your answer to two decimal places.

c. Determine in what year the population will be half what it was in 1990. Round your answer to the nearest year.
Example 3: In 2001, the population of a particular city was 22,395 with an identified growth rate of 6.2% per year. Assume that this growth rate is fairly consistent from year to year.

a) Write the exponential growth model for this situation.

   Initial Population: ____________

   Given growth rate as a decimal: ____________

   Growth factor: ________________

   Write the model: \[ P(t) = a(b)^t \]

b) What is the approximate population of the city in 2006? Be sure to round to the nearest whole person.

c) Estimate the number of years (to the nearest whole year) that it will take for the population to double. In what actual year will this take place? Be sure to set up and clearly identify the Doubling Equation. Then, draw a sketch of the graph you obtain when using the Graphing/Intersection Method to solve. Round to the nearest whole year.

   Doubling Equation: ________________________________
Example 4: The 2000 U.S. Census reported the population of Tulsa, Oklahoma to be 382,872. Since the 2000 Census, Tulsa’s population has been decreasing at approximately 2.6% per year.

a) Write the exponential growth model, \( P(t) \), that predicts the population of Tulsa, OK at time \( t \) in years after 2000.

   Initial Population: __________

   Given growth rate as a decimal: __________

   Growth factor: _______________

   Write the model: \( P(t) = a(b)^t \) _______________

b) Use the function you wrote for \( P(t) \) to predict the population of Tulsa, OK in 2013.

c) In how many years will the population of Tulsa decrease to half of the initial (2000) population? Round to the nearest whole year.

   Equation: ____________________________
Section 8.6 – You Try

After graduating from college in 2010, Sara accepts a job that pays $52,000 per year. At the end of each year, she expects to receive a 3% raise.

a) Let $t$ represent the number of years Sara works at her new job. Write the exponential function, $S(t) = a(b)^t$, that models Sara’s annual salary given the information above.

b) If Sara’s salary continues to increase at the rate of 3% each year, determine how much she will make in 2015. Show your work clearly here. Round your answer to the nearest dollar.

c) How many years will she have to work before her salary will be double what it was in 2010 (assuming the same growth rate)? Be sure to set up and clearly identify the Doubling Equation. Then, draw a sketch of the graph you obtain when using the Graphing/Intersection Method to solve. Round your answer to the nearest whole year.
Unit 8 Answers to You-Try Problems

Section 8.1:

<table>
<thead>
<tr>
<th>Year, t</th>
<th>$1500 raise</th>
<th>Year, t</th>
<th>3% raise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48500</td>
<td>1</td>
<td>48410</td>
</tr>
<tr>
<td>2</td>
<td>50000</td>
<td>2</td>
<td>49862</td>
</tr>
<tr>
<td>3</td>
<td>51500</td>
<td>3</td>
<td>51358</td>
</tr>
<tr>
<td>5</td>
<td>54500</td>
<td>5</td>
<td>54486</td>
</tr>
<tr>
<td>10</td>
<td>62000</td>
<td>10</td>
<td>63164</td>
</tr>
</tbody>
</table>

Section 8.2:

<table>
<thead>
<tr>
<th></th>
<th>f(x) = 235(1.14)^x</th>
<th>g(x) = 235(0.95)^x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X[-10..10],</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y[0..500]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Value</td>
<td>235</td>
<td>235</td>
</tr>
<tr>
<td>Growth or Decay?</td>
<td>Growth</td>
<td>Decay</td>
</tr>
<tr>
<td>Domain</td>
<td>(-∞,∞)</td>
<td>(-∞,∞)</td>
</tr>
<tr>
<td>Range</td>
<td>(0,∞)</td>
<td>(0,∞)</td>
</tr>
<tr>
<td>Horizontal Intercept</td>
<td>DNE</td>
<td>DNE</td>
</tr>
<tr>
<td>Vertical Intercept</td>
<td>(0,235)</td>
<td>(0,235)</td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td>y = 0</td>
<td>y = 0</td>
</tr>
</tbody>
</table>

Section 8.3:  
y = 300(0.88)^t, Decay, 300, 12%  
y = 213(1.2)^t, Growth, 213, 20%  
y = 177(1.098)^t, Growth, 177, 9.8%  
y = 5.41(0.93)^t, Decay, 5.41, 7%  

Section 8.4:  
a) V(x) = 210000 + 1500x  
b) V(x) = 210000(1.3)^x  
c) V(x) = 210000(0.85)^x  
d) V(x) = 210000

Section 8.5  
x = -43.67

Section 8.6:  
a) 52000(1.03)^t  
b) $60282.25  
c) 104000 = 52000(1.03)^t, 23 years  
In the year 2033, Sara will make twice her initial salary from 2010.
Unit 8: Practice Problems

Skills Practice

1. Complete the table below for each exponential function.

<table>
<thead>
<tr>
<th>Growth or Decay</th>
<th>Domain</th>
<th>Range</th>
<th>Horizontal Intercept</th>
<th>Vertical Intercept</th>
<th>Horizontal Asymptote</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 38(1.12)^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(x) = 4.6(0.983)^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 1.8^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write the exponential function for each of the following.

<table>
<thead>
<tr>
<th>Initial Value</th>
<th>Rate</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1500</td>
<td>Growth Rate = 15%</td>
<td></td>
</tr>
<tr>
<td>b) 75</td>
<td>Decay Rate = 15%</td>
<td></td>
</tr>
<tr>
<td>c) 1250</td>
<td>Growth Rate = 7.5%</td>
<td></td>
</tr>
<tr>
<td>d) 12</td>
<td>Growth Rate = 112%</td>
<td></td>
</tr>
<tr>
<td>e) 1000</td>
<td>Decay Rate = 12%</td>
<td></td>
</tr>
<tr>
<td>f) 56</td>
<td>Decay Rate = 5%</td>
<td></td>
</tr>
<tr>
<td>g) 100</td>
<td>Decay Rate = 0.5%</td>
<td></td>
</tr>
<tr>
<td>h) 57</td>
<td>Decay Rate = 6.2%</td>
<td></td>
</tr>
</tbody>
</table>
3. For each exponential function, identify the Initial Value and the Growth/Decay Rate.

   a) $f(x) = 3200(0.82)^x$
      
      Initial Value = ____________
      
      Decay Rate = ____________

   b) $g(x) = 1000(1.32)^x$
      
      Initial Value = ____________
      
      Growth Rate = ____________

   c) $p(t) = 50(2.5)^t$
      
      Initial Value = ____________
      
      Decay Rate = ____________

   d) $f(x) = 5(3)^x$
      
      Initial Value = ____________
      
      Growth Rate = ____________

4. In each situation below, you will need to graph to find the solution to the equation using the Graphing/Intersection Method described in this lesson. Round answers to two decimal places.

   a) Solve $25(1.3)^x = 400$
   b) Solve $25(1.3)^x = 4$
   c) Solve $300(0.77)^x = 90$
   d) Solve $300(0.77)^x = 550$

5. Given $f(x) = 44(1.09)^x$, determine each of the following and show complete work.

   a) $f(12)$
   b) $f(-30)$
   c) Find $x$ when $f(x) = 88$
   d) Find $x$ when $f(x) = 22$

6. Given $f(x) = 320(0.9)^x$, determine each of the following and show complete work.

   a) $f(5)$
   b) $f(-10)$
   c) Find $x$ when $f(x) = 160$
   d) Find $x$ when $f(x) = 600$

7. When a new charter school opened in 2005, there were 300 students enrolled. Using correct notation, write a function model for the number of students, $N(t)$, enrolled in this charter school $t$ years after 2005, assuming that the enrollment:

   a) Decreases by 20 students per year
   b) Decreases by 2% per year
   c) Increases by 30 students per year
   d) Increases by 6% per year
   e) Decreases by 32 students per year
   f) Increases by 30% per year
   g) Remains constant (does not change)
   h) Increases by 100% per year
8. Fred and Wilma purchase a home for $180,000. Using function notation, write a formula for the value, \( V(t) \), of the house \( t \) years after its purchase, assuming that the value:
   a) Decreases by $1,500 per year  
   b) Decreases by 2% per year  
   c) Increases by $3,300 per year  
   d) Increases by 6% per year

9. One 12-oz can of Dr. Pepper contains about 39.4 mg of caffeine. The function \( A(x) = 39.4(0.8341)^x \) gives the amount of caffeine remaining in the body \( x \) hours after drinking a can of Dr. Pepper.
   a) How much caffeine is in the body eight hours after drinking one can of Dr. Pepper? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.
   b) How long after drinking one can of Dr. Pepper will only 1 mg of caffeine remain in the body? Show all of your work. Write your answer in a complete sentence. Round your answer to two decimal places as needed.

10. The fox population in a certain region has a relative growth rate of 7 percent per year. It is estimated that the fox population in the year 2000 was 12,800 foxes. If this growth rate continues, determine the fox population in the year 2008. Round your answer to the nearest fox. Write your answer in a complete sentence.

11. Bismuth-210 is an isotope that radioactively decays by about 13% each day, meaning 13% of the remaining Bismuth-210 transforms into another atom (polonium-210 in this case) each day. If you begin with 247 mg of Bismuth-210, how much remains after 6 days? Round your answer to the nearest tenth. Write your answer in a complete sentence.

12. After graduating from college, Carlos receives two different job offers. Both pay a starting salary of $50,000 per year, but one job promises a $2,000 raise per year, while the other guarantees a 4% raise each year. Use this information to complete the tables below. Round your answers to the nearest dollar.

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>1</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary with $2,000 raise per year</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>1</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary with 4% raise per year</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
13. When a new charter school opened in 2005, there were 300 students enrolled. Using correct notation, write a function model for the number of students, \( N(t) \), enrolled in this charter school \( t \) years after 2005, assuming that the enrollment:

a) Decreases by 20 students every 2 years  
b) Decreases by 10% every 3 years  

c) Increases by 30 students every 3 years  
d) Increases by 6% every 4 years  

e) Increases by 8 students every 6 months  
f) Increases by 1% every month  

g) Remains constant (does not change)  
h) Doubles every 5 years.
Unit 8: Review

1. Consider the function \( g(x) = 132(1.18)^x \). Answer the questions below. Use interval notation for the domain and range. Write intercepts as ordered pairs. Write “DNE” if the answer does not exist.
   a) What is the domain of \( g(x) \)? ________________________________
   b) What is the range of \( g(x) \)? ________________________________
   c) What is the vertical intercept? __________________________
   d) What is the horizontal intercept? __________________________
   e) Give the equation of the horizontal asymptote for \( g(x) \). _______________________

2. Complete the following table.

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th>Growth or Decay?</th>
<th>Vertical Intercept (as an ordered pair)</th>
<th>Growth/Decay Rate (as a %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 500(0.93)^x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = 87(1.2)^x )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. When it first opened, there were 700 students enrolled in a new charter school. Using function notation, write a formula the function \( N(x) \) which represents the number of students enrolled in this charter school after \( x \) years, assuming that the enrollment
   a) Increases by 15% per year.
   b) Increases by 20 students per year.
   c) Decreases by 35 students each year.
   d) Decreases by 3% per year.

4. A car’s value depreciates by about 8% per year. You purchase a new car for $24,000. Determine the value of the car after 5 years (when the loan is paid off). Show all of your work. Round to the nearest dollar. Write your answer in a complete sentence.
5. Solve the following equation by graphing. In the space provided, draw a *detailed sketch* of the graph for this equation. Clearly label the intercepts and intersection point on your sketch. Round your solution to the nearest hundredth as needed.

\[ 83(0.95)^x = 515 \]

Solution: \( x = \) ________________

6. The fox population in a certain region has a relative growth rate of 7 percent per year. It is estimated that the fox population in the year 2000 was 12,800. If this growth rate continues, determine the fox population in the year 2011. Show all of your work. Write your answer in a complete sentence. Round your answer to the nearest fox.

7. A house purchased for $210,000 gains 5% of its value every year. If this growth rate continues, how long will it take for the house to be worth twice its original value? Show all of your work. Round your answer to the nearest tenth as needed. Write your answer in a complete sentence.
Unit 9: Exponents and Roots

Section 9.1: Properties of Exponents

Section 9.2: Division Properties of Exponents

Section 9.3: Negative Exponents

Section 9.4: Rational Exponents

Section 9.5: Simplifying Square Roots

Section 9.6: Complex Numbers

UNIT 9 LEARNING OBJECTIVES

- Apply properties of exponents to simplify expressions.
- Work with negative exponents.
- Work with rational exponents.
- Simplify square roots
- Write complex numbers in exact and approximate form

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication Property of Exponents</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Raising a Power to a Power</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Raising a Product to a Power</td>
<td></td>
</tr>
<tr>
<td>Division Property of Exponents</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>---</td>
</tr>
<tr>
<td>Raising a Quotient to a Power</td>
<td></td>
</tr>
<tr>
<td>Negative Exponent</td>
<td></td>
</tr>
<tr>
<td>Rational Exponent</td>
<td></td>
</tr>
<tr>
<td>Complex Numbers</td>
<td></td>
</tr>
</tbody>
</table>
**Unit 9: Media Lesson**

**Section 9.1: Properties of Exponents**

Given any real numbers \( a, b, c, m, \) and \( n \)

\[
\begin{align*}
n^1 &= \underline{\quad} & 1^n &= \underline{\quad} & n^0 &= \underline{\quad} & 0^0 &= \underline{\quad} \\
& & & & & & n\neq0 & n\neq0
\end{align*}
\]

\[
\begin{align*}
3^4 &= \underline{\quad} \\
3^3 &= \underline{\quad} \\
3^2 &= \underline{\quad} \\
3^1 &= \underline{\quad} \\
3^0 &= \underline{\quad} \\
3^{-1} &= \underline{\quad} \\
3^{-2} &= \underline{\quad} \\
3^{-3} &= \underline{\quad} \\
3^{-4} &= \underline{\quad}
\end{align*}
\]

**Multiplication Properties of Exponents**

\[
\begin{align*}
\alpha^m \cdot \alpha^n &= \alpha^{m+n} \\
(a^m)^n &= a^{mn}
\end{align*}
\]

**Example 1:** Evaluate and simplify the following expressions.
Assume \( x \neq 0, \ x \neq -1/2, \ a \neq 0, \ b \neq 0, \) and \( c \neq 0.\)

\[
\begin{align*}
5x^0 &= \underline{\quad} \\
(2x + 1)^0 &= \underline{\quad} \\
\alpha^0 + b^0 + c^0 &= \underline{\quad}
\end{align*}
\]

The Multiplication Property: \( \alpha^m \cdot \alpha^n = \alpha^{m+n} \)

**Example 2:** Simplify the following expressions

\[
\begin{align*}
n^3n^9 &= \underline{\quad} \\
b^5 \cdot b^4 \cdot b &= \underline{\quad} \\
5x^2y^5(7xy^3) &= \underline{\quad}
\end{align*}
\]
Raising a Power to a Power: \((a^m)^n = a^{mn}\)

**Example 3:** Simplify the following expressions

\[(x^3)^9 \quad 5b^2(b^5)^8\]

Raising a Product to a Power: \((ab)^n = a^nb^n\)

**Example 4:** Simplify the following expressions

\[(5x)^2 \quad (x^3y^2)^9 \quad (-8ab^5)^2\]

\[5(-2w^7)^3 \quad 5n^4(-3n^3)^2\]

**Section 9.1 – You Try**

Simplify the following expressions. Show all steps as in the media examples.

a. \((2x^4)^2\)  
b. \(2(x^3)^3\)

c. \(8g^3 \cdot 5g^4\)  
d. \(2n^0\)
Section 9.2: Division Properties of Exponents

The Division Property: \( \frac{a^m}{a^n} = a^{m-n} \) for \( a \neq 0 \)

Example 1: Simplify the following expressions. Variables represent nonzero quantities.

\[
\frac{x^{50}}{x^4} = \\
\frac{4a^{10}b^5}{6ab^2} =
\]

Raising a Quotient to a Power: \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \) for \( b \neq 0 \)

Example 2: Simplify the following expressions. Variables represent nonzero quantities.

\[
\left( \frac{5}{7} \right)^2 = \\
\left( \frac{x^5}{y^3} \right)^4 = \\
\left( \frac{-4t^{10}}{u^6} \right)^2 =
\]

Section 9.2 – You Try

Simplify the following expressions. Variables represent nonzero quantities.

a. \( \left( \frac{3a^{10}}{7} \right)^2 = \)

b. \( \frac{6x^3y^8}{9xy^5} = \)
Section 9.3: Negative Exponents

For any real numbers \( a \neq 0, b \neq 0, \) and \( m \):

\[
\left( \frac{a}{b} \right)^{-m} = \left( \frac{b}{a} \right)^{m} \quad a^{-m} = \frac{1}{a^{m}} \quad \frac{1}{a^{-m}} = a^{m}
\]

**Example 1:** Rewrite each of the following with only positive exponents.

Variables represent nonzero quantities

a. \( x^{-3} = \)

b. \( \frac{1}{x^{-3}} = \)

c. \( 2^{-3} = \)

d. \( \left( \frac{4}{5} \right)^{-2} = \)

e. \( 3x^{-4} = \)

f. \( (3x)^{-4} = \)

**Example 2:** Simplify the following expressions. Variables represent nonzero quantities. Write your answer with only positive exponents.

a. \( p^{4} \cdot p^{2} \cdot p = \)

b. \( \frac{2}{3} a^{-5} b^{-3} c^{2} = \)

c. \( \frac{d^{-2}}{d^{-7}} = \)

d. \( \frac{4t^{-10}u}{6t^{-3}u^{-1}} = \)
Simplify the following expressions. Write your answers with only positive exponents. Variables represent nonzero quantities.

a. \( 7a^2 = \)

b. \( \frac{7}{a^{-2}} = \)

c. \( (7a)^2 = \)

d. \( \left( \frac{7}{a} \right)^{-2} = \)
Section 9.4: Rational Exponents

Basic Properties of Exponents

1. \( a^m \cdot a^n = a^{m+n} \)

2. \( \frac{a^m}{a^n} = a^{m-n} \)

3. \( (a^m)^n = a^{m \cdot n} \)

4. \( a^{-m} = \frac{1}{a^m} \)

<table>
<thead>
<tr>
<th>Basic Properties of Exponents</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{x^n} = \frac{1}{\sqrt[n]{x}} )</td>
<td>( x^\frac{m}{n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m} )</td>
</tr>
</tbody>
</table>

**Example 1:** Rewrite each of the following as an equivalent expression with rational exponents.

a) \( \frac{3}{\sqrt{x}} \)

b) \( \frac{5}{\sqrt[3]{y^2}} \)

c) \( \sqrt[3]{x^8} \), for \( x \geq 0 \)

d) \( \frac{1}{\sqrt[3]{b^5}} \)

**Example 2:** Simplify.

a) \( 8^{4/3} = \)

b) \( 81^{3/4} = \)
Example 3: Use your calculator to compute each of the following in the real number system. Round to two decimal places as needed.

a) \( \sqrt{49} \)  
b) \( 8^{1/3} \)

c) \( \sqrt{-49} \)  
d) \( \frac{1}{\sqrt{-8}} \)

e) \(-25^{3/2}\)  
f) \((-25)^{3/2}\)

Section 9.4 – You Try

Complete the following problems

a. Rewrite each of the following as an equivalent expression with rational exponents.

\( \sqrt{x} = \) ___________  
\( \sqrt[2]{y} = \) ___________  
\( \frac{1}{\sqrt[n]{z}} = \) ___________

b. Evaluate the following using your graphing calculator. If there is no real solution, write “DNE”. Round answers to three decimal places if necessary.

\( \sqrt[6]{60} = \) ___________  
\( \sqrt[4]{-16} = \) ___________  
\( \sqrt[5]{-20} = \) ___________
Section 9.5: Simplifying Square Roots

Square Roots of Perfect Squares

\[
\begin{align*}
\sqrt{0} &= 0 & \sqrt{1} &= 1 & \sqrt{4} &= 2 & \sqrt{9} &= 3 & \sqrt{16} &= 4 & \sqrt{25} &= 5 \\
\sqrt{36} &= 6 & \sqrt{49} &= 7 & \sqrt{64} &= 8 & \sqrt{81} &= 9 & \sqrt{100} &= 10 & \sqrt{121} &= 11
\end{align*}
\]

**Example 1:** Simplify each of the following as much as possible. Leave answers in exact form.

**Simplifying Square Roots of Composite Numbers:**

\[
\begin{align*}
\sqrt{50} &= \\
\sqrt{18} &= \\
\sqrt{24} &=
\end{align*}
\]

**Simplifying Square Roots of Prime Numbers:**

\[
\begin{align*}
\sqrt{3} &= \\
\sqrt{47} &=
\end{align*}
\]

**Simplifying Square Roots with Fractions (I):**

\[
\begin{align*}
\frac{\sqrt{50}}{2} &= \\
\frac{\sqrt{18}}{6} &= \\
\frac{\sqrt{24}}{4} &=
\end{align*}
\]

**Simplifying Square Roots with Fractions (II):**

\[
\begin{align*}
\frac{2 + \sqrt{50}}{2} &= \\
\frac{2 - \sqrt{18}}{6} &=
\end{align*}
\]

\[
\begin{align*}
\frac{8 + \sqrt{24}}{4} &= \\
\frac{-4 + \sqrt{16}}{12} &=
\end{align*}
\]
Section 9.5 – You Try

Simplify each of the following as much as possible. Leave final answers in exact form.

a. $\sqrt{20}$

b. $\frac{\sqrt{300}}{5}$

c. $\sqrt{19}$

d. $\frac{4-\sqrt{8}}{4}$
Section 9.6: Complex Numbers

Complex Numbers are numbers of the form \( a + bi \) such that \( a \) and \( b \) are real numbers

\[
\begin{align*}
    i &= \sqrt{-1} \\
    i^2 &= -1
\end{align*}
\]

<table>
<thead>
<tr>
<th>Number Systems</th>
<th>Description and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Complex Numbers</strong></td>
<td>Include all Real Numbers and Integers. Complex numbers also include the imaginary unit ( i = \sqrt{-1} ).</td>
</tr>
<tr>
<td><strong>Real Numbers</strong></td>
<td>Include Integers, Rational and Irrational Numbers.</td>
</tr>
<tr>
<td><strong>Integers</strong></td>
<td>Include positive and negative Whole Numbers.</td>
</tr>
<tr>
<td><strong>Whole Numbers</strong></td>
<td>0, 1, 2, ...</td>
</tr>
<tr>
<td><strong>( \sqrt{2} )</strong></td>
<td>1.4</td>
</tr>
<tr>
<td><strong>( \frac{\pi}{4} )</strong></td>
<td>0.785</td>
</tr>
<tr>
<td><strong>( \frac{\pi}{6} )</strong></td>
<td>0.5</td>
</tr>
<tr>
<td><strong>( -5 )</strong></td>
<td></td>
</tr>
<tr>
<td><strong>( -80 )</strong></td>
<td></td>
</tr>
<tr>
<td><strong>( \frac{3 + \sqrt{-49}}{2} )</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Example 1:** Simplify each of the following leaving your final result in \( a + bi \) form.

a) \( \sqrt{-9} = \) 

b) \( -\sqrt{-7} \)

c) \( \frac{3 + \sqrt{-49}}{2} \)

**Example 2:** Simplify each of the following leaving your final result in exact \( a + bi \) form.

a) \( -4\sqrt{25} = \)

b) \( \sqrt{32} \)

c) \( \frac{5 - 2\sqrt{-25}}{10} \)
Example 3: Each of the following complex numbers is written in exact $a + bi$ form. Change each of them to an approximate $a + bi$ form rounded to the nearest thousandths place, if possible.

a. $3 + 4\sqrt{15}i$

b. $\frac{3}{7} + \left(\frac{1}{7}\right)i$

Section 9.6 – You Try

Complete the following problems

a. Simplify each of the following leaving your final result in exact $a + bi$ form.

\[ \sqrt{-121} \quad \sqrt{700} \]

\[ \sqrt{-12} \quad \frac{15 - \sqrt{-36}}{3} \]

b. Each of the following complex numbers is written in exact $a + bi$ form. Change each of them to an approximate $a + bi$ form rounded to the nearest thousandths place, if possible.

\[ 1 + \sqrt{2}i \quad \frac{5}{6} - \frac{\sqrt{6}}{6}i \]
Unit 9: Answers to You-Try Problems

Section 9.1:  
  a. \(4x^8\)  
  b. \(2x^6\)  
  c. \(40g^7\)  
  d. 2

Section 9.2:  
  a. \(\frac{9a^{20}}{49} = \frac{9}{49}a^{20}\)  
  b. \(\frac{2x^2y^3}{3} = \frac{2}{3}x^2y^3\)

Section 9.3:  
  a. \(\frac{7}{a^2}\)  
  b. \(7a^2\)  
  c. \(\frac{1}{49a^2}\)  
  d. \(\frac{a^2}{49}\)

Section 9.4:  
  a. \(\sqrt[3]{x} = x^{1/3}\)  
  b. \(\sqrt[4]{r^6} = r^{\frac{3}{2}}\)  
  c. \(\frac{1}{\sqrt[5]{n}} = n^{-1/4}\)  
  d. 1.979, DNE, -1.821

Section 9.5:  
  a. \(2\sqrt{5}\)  
  b. \(2\sqrt{3}\)  
  c. \(\sqrt{19}\)  
  d. \(1 - \frac{\sqrt{2}}{2}\)

Section 9.6:  
  a. \(11i, 10\sqrt{7}, 2\sqrt{3}i, 5 - 2i\)  
  b. \(1 + 1.414i\)  
  0.833 - 0.408i
Unit 9: Practice Problems

Skills Practice

Variables represent nonzero quantities

1. Simplify completely. Show all steps, and box your answers.
   
   a. \((2x)^3\)  
   b. \(5(3n)^2\)  
   c. \(y^3 \cdot y^7 \cdot y\)
   
   d. \((-2x)^3\)  
   e. \(5w(8w^3)\)  
   f. \((-2x^5)^2\)
   
   g. \((-5w^8)^2\)  
   h. \(3x^0 + 2x^0\)  
   i. \((5x - 7)^0\)

2. If possible, simplify each of the following by combining like terms or using properties of exponents.

   a. \(2n^5 + 3n^5 = \) \[\]  
   b. \(2n^5 \cdot 3n^5 = \) \[\]  
   c. \(3n^3 + 3n^5 = \) \[\]  
   d. \(3n^3 \cdot 3n^5 = \) \[\]

3. Evaluate the algebraic expression \(x^2\) given \(x = -7\). Show your work.

4. Evaluate the algebraic expression \(5x^3\) given \(x = -2\). Show your work.

5. Evaluate the algebraic expression \((5x)^2\) given \(x = -2\). Show your work.

6. Evaluate the algebraic expression \(5(2x)^2\) given \(x = -3\). Show your work.

7. Evaluate the algebraic expression \(\frac{6}{5x}\) given \(x = -2\). Show your work.
8. Evaluate the algebraic expression \( \frac{1}{4x^2} \) given \( x = -5 \). Show your work.

9. Simplify completely. Show all steps, and box your answers.

   a. \( 4p(-5p^3)^2 \)
   b. \( 3(-2x)^3 - 3x(-2)^3 \)

   c. \( 4w^5(3w^8)^2 \)
   d. \( 10p^3(-5p^7)^2 \)

   e. \( 2a^3b(3ab^5)^2 \)
   f. \( (3x^4)^3 - (5x^6)^2 \)

10. Simplify completely. Show all steps, and box your answers.

    a. \( \frac{x^8}{x^3} = \)
    b. \( \left(\frac{2}{5}\right)^4 = \)

    c. \( \frac{8n^8p^5}{12np^4} = \)
    d. \( \left(\frac{3a^5}{7b}\right)^2 = \)

11. Simplify completely. Show all steps, and box your answers. Use only positive exponents.

    a. \( 8n^{-2} = \)
    b. \( (8n)^{-2} = \)

    c. \( \left(\frac{2}{5}\right)^{-4} = \)
    d. \( \frac{5}{x^{-2}} = \)

    e. \( \frac{5}{6x^{-2}} = \)
    f. \( \frac{x^{-2}}{5} = \)

    g. \( g^2 \cdot g^{-6} \cdot g = \)
    h. \( \frac{d^{-4}}{d^{-7}} = \)

    i. \( \frac{2}{3} \left(\frac{5}{6}\right)^{-2} = \)
    j. \( \left(\frac{3a^{10}}{7}\right)^{-2} = \)
12. Evaluate the algebraic expression $x^{-2}$ given $x = 3$. Show your work.

13. Evaluate the algebraic expression $5x^{-3}$ given $x = -2$. Show your work.

14. Evaluate the algebraic expression $(5x)^{-2}$ given $x = -3$. Show your work.

15. Evaluate the algebraic expression $5(2x)^{-2}$ given $x = -3$. Show your work.

16. Complete the table below. Each expression should be written in Radical Notation, written with Rational Exponents and evaluated using a calculator. Write DNE if the answer does not exist in the real number system. The first one is done for you.

<table>
<thead>
<tr>
<th></th>
<th>Written in Radical Notation</th>
<th>Written Using Rational Exponents</th>
<th>Evaluated Using the Calculator (rounded to two decimal places)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$\sqrt[3]{9}$</td>
<td>$9^{1/3}$</td>
<td>$9^{(1/3)} \approx 2.08$</td>
</tr>
<tr>
<td>b)</td>
<td>$\sqrt[5]{20}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>$\sqrt[3]{2^4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>$\sqrt[4]{7^{12}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>$\sqrt[3]{-8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td></td>
<td>$3^{1/4}$</td>
<td></td>
</tr>
<tr>
<td>g)</td>
<td></td>
<td>$11^{1/7}$</td>
<td></td>
</tr>
<tr>
<td>h)</td>
<td></td>
<td>$-4^{1/2}$</td>
<td></td>
</tr>
<tr>
<td>i)</td>
<td></td>
<td>$(-4)^{1/2}$</td>
<td></td>
</tr>
</tbody>
</table>
17. Complete the table below.

<table>
<thead>
<tr>
<th>Written in Radical Notation</th>
<th>Written Using Rational Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\sqrt{x}$</td>
<td></td>
</tr>
<tr>
<td>b) $\sqrt[4]{c^3}$</td>
<td></td>
</tr>
<tr>
<td>c) $\sqrt{w^7}$</td>
<td></td>
</tr>
<tr>
<td>d) $\frac{1}{\sqrt[5]{b}}$</td>
<td></td>
</tr>
<tr>
<td>e) $x^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>f) $y^{3/2}$</td>
<td></td>
</tr>
<tr>
<td>g) $k^{2/3}$</td>
<td></td>
</tr>
<tr>
<td>h) $d^{-3/5}$</td>
<td></td>
</tr>
</tbody>
</table>

18. Use the Product Property to simplify each of the following radical expressions. This first one has been done for you.

<table>
<thead>
<tr>
<th>Radical Expression</th>
<th>Simplified Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{20}$</td>
<td>$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \sqrt{5} = 2\sqrt{5}$</td>
</tr>
<tr>
<td>$\sqrt{75}$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{18}$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{40}$</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{600}$</td>
<td></td>
</tr>
</tbody>
</table>
19. Simplify the following square roots. If the result is a complex number, write it in the form $a + bi$.

\[
\sqrt{72} = \quad \frac{\sqrt{-50}}{5} = \quad \frac{8+\sqrt{4}}{2} = \\
\sqrt{-21} = \quad \frac{\sqrt{-16}}{2} = \quad \frac{2-\sqrt{-4}}{2} = 
\]

20. Rewrite each of the following, leaving your final answers WITH negative exponents (no variables in the denominator).

\[
\frac{8}{x^4} = \quad \left(\frac{5}{\sqrt{2}}\right)^2 = 
\]

Extension

21. Write each of the following numbers in scientific notation.

\[
45,613,000,000 = \quad 0.00000001534 = 
\]

22. Write each of the following numbers in standard form.

\[
2.35 \times 10^6 = \quad 8.09 \times 10^{-4} = \\
4.32 \text{ thousand} = \quad 9.1 \text{ billion} = 
\]

23. Perform the following operations on complex numbers. Remember that $i^2 = -1$

a. $(2 - 3i) + (4 + 5i)$

b. $(2 - 3i) - (4 + 5i)$

c. $(2 - 3i)(4 + 5i)$

d. $(2 - 3i)^2$
24. Given that $i^2 = -1$, evaluate the following.

\[ i^2 = -1 \]
\[ i^3 = \_\_\_\_ \]
\[ i^4 = \_\_\_\_ \]
\[ i^5 = \_\_\_\_ \]
\[ i^6 = \_\_\_\_ \]
\[ i^7 = \_\_\_\_ \]
\[ i^8 = \_\_\_\_ \]
\[ i^{20} = \_\_\_\_ \]
\[ i^{47} = \_\_\_\_ \]
\[ i^{242} = \_\_\_\_ \]

Describe the pattern that you see.
Unit 9: Review

1. If possible, simplify each of the following by combining like terms or using properties of exponents.
   
   a. \(8n^3 + 5n^3 = \) ________
   
   b. \(8n^3 \cdot 5n^3 = \) ________
   
   c. \(8n^3 + 8n^5 = \) ________
   
   d. \(8n^3 \cdot 8n^5 = \) ________

2. Simplify completely. Show all steps, and box your answers. Answers should include only positive exponents.

   a. \((-5x^5)^3\)
   
   b. \(4x^2(8x^2)\)
   
   c. \(2n^{-3} = \)
   
   d. \((2n)^{-3} = \)
   
   e. \((\frac{3}{8})^{-2} = \)
   
   f. \((\frac{5x}{8})^2 = \)
   
   g. \(\frac{8}{x^2} = \)
   
   h. \(\frac{24m^3}{18m^8} = \)
3. Rewrite each of the following as an equivalent expression with rational exponents.

\[ \sqrt[4]{x^5} = \underline{\phantom{0}} \quad \sqrt{x^8} = \underline{\phantom{0}} \quad \frac{1}{\sqrt{w}} = \underline{\phantom{0}} \]

4. Evaluate the following using your graphing calculator. If there is no real solution, write “DNE”. Round answers to three decimal places if necessary.

\[ \sqrt[6]{100} = \underline{\phantom{0}} \quad \sqrt[6]{60} = \underline{\phantom{0}} \quad \sqrt[3]{-36} = \underline{\phantom{0}} \]

5. Simplify the following square roots. If the result is a complex number, write it in the form \( a + bi \).

\[ \sqrt{80} = \underline{\phantom{0}} \quad \frac{\sqrt{45}}{6} = \underline{\phantom{0}} \quad \frac{6 + \sqrt{-4}}{2} = \underline{\phantom{0}} \]
\[ \sqrt{-3} = \underline{\phantom{0}} \quad \frac{\sqrt{16}}{4} = \underline{\phantom{0}} \quad \frac{9 - \sqrt{-49}}{2} = \underline{\phantom{0}} \]

6. Explain the difference between each pair of expressions.

a. \( 3x^2 \) and \((3x)^2\)

b. \( x^3 \) and \(-x^3\)

c. \( 3\sqrt{x} \) and \(3\sqrt[3]{x}\)

d. \( \sqrt{x + 3} \) and \( \sqrt{x} + 3 \)

e. \( -\sqrt{9} \) and \( \sqrt{-9} \)

f. \( \frac{x}{3} \) and \( \frac{3}{x} \)
Unit 10: Polynomials and Factoring

Section 10.1: Polynomials
Section 10.2: Operations on Polynomials
Section 10.3: Multiplication of Polynomials
Section 10.4: Division of Polynomials
Section 10.5: Factoring Quadratic Expressions Using GCF Method
Section 10.6: Factoring Quadratic Expressions Using Trial and Error
Section 10.7: Combining Factoring Methods
Section 10.8: Solving Equations by Factoring

UNIT 10 LEARNING OBJECTIVES

- Define the basic components polynomial expressions
- Add, Subtract, Multiply, and Divide polynomials
- Factor Quadratic Expressions using the GCF method
- Factor Quadratic Expressions using trial and error
- Solve equations by factoring

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial</td>
<td></td>
</tr>
<tr>
<td>Monomial</td>
<td></td>
</tr>
<tr>
<td>Binomial</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>Trinomial</td>
<td></td>
</tr>
<tr>
<td>Quadratic expressions</td>
<td></td>
</tr>
<tr>
<td>Factoring</td>
<td></td>
</tr>
<tr>
<td>Greatest Common Factor</td>
<td></td>
</tr>
<tr>
<td>The Zero Product Property</td>
<td></td>
</tr>
</tbody>
</table>
Unit 10: Media Lesson

Section 10.1: Polynomials

<table>
<thead>
<tr>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Polynomial</strong>: An algebraic expression composed of the sum of terms containing a single variable raised to a positive integer exponent.</td>
</tr>
<tr>
<td><strong>Monomial</strong>: A polynomial consisting of one term</td>
</tr>
<tr>
<td><strong>Binomial</strong>: A polynomial consisting of two terms</td>
</tr>
<tr>
<td><strong>Trinomial</strong>: A polynomial consisting of three terms</td>
</tr>
<tr>
<td><strong>Leading Term</strong>: The term that contains the highest power of the variable in a polynomial</td>
</tr>
<tr>
<td><strong>Leading Coefficient</strong>: The coefficient of the leading term</td>
</tr>
<tr>
<td><strong>Constant Term</strong>: A number with no variable factors. A term whose value never changes.</td>
</tr>
<tr>
<td><strong>Degree</strong>: The highest exponent in a polynomial</td>
</tr>
</tbody>
</table>

Examples of algebraic expressions that are polynomials and algebraic expressions that are not polynomials:

<table>
<thead>
<tr>
<th>Polynomials</th>
<th>NOT Polynomials</th>
</tr>
</thead>
</table>

The terms of a polynomial in one variable must be a constant or the product of a constant and a variable with a nonnegative integer exponent.
### Example 1: Complete the table.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Name</th>
<th>Leading Coefficient</th>
<th>Constant Term</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24a^6 + a^2 + 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2m^3 + m^2 - 2m - 8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5x^2 + x^3 - 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2x + 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Section 10.1 – You Try

Complete the table.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Name</th>
<th>Leading Coefficient</th>
<th>Constant Term</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^2 - 2n + 8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6x - 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 10.2: Operations on Polynomials

Addition of Polynomials

**Example 1:** Add. \((3n^2 - 2n + 8) + (3n^3 - 7n^2 - n - 9)\)

Subtraction of Polynomials

**Example 2:** Subtract. \((a^3 + 5a + 11) - (4a^3 + 6a^2 - a + 1)\)

Combine and Simplify

**Example 3:** Perform the indicated operations. Simplify.

\((3x - 1) - (x^2 - x - 9) + (4x^3 + x^2 - 7x + 2)\)

Section 10.2 – You Try

**You Try:** Perform the indicated operations. Simplify completely. Show all steps.

\((5x + 8) + (x^2 - x - 1) - (x^3 + 3x^2 - 4x + 8)\)
Section 10.3: Multiplication of Polynomials

<table>
<thead>
<tr>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Polynomial</strong>: An algebraic expression composed of the sum of terms containing a single variable raised to a non-negative integer exponent.</td>
</tr>
<tr>
<td><strong>Monomial</strong>: A polynomial consisting of one term</td>
</tr>
<tr>
<td><strong>Binomial</strong>: A polynomial consisting of two terms</td>
</tr>
<tr>
<td><strong>Trinomial</strong>: A polynomial consisting of three terms</td>
</tr>
</tbody>
</table>

**Example 1**: Multiply and simplify.

\[(3x^5)(-2x^9)\]

**Example 2**: Expand and simplify.

\[5x^3(2x^5 - 4x^3 - x + 8)\]

**Example 3**: Multiply and simplify.

a. \((x + 3)(x + 4)\)  
b. \((m - 5)(m - 6)\)  
c. \((2d - 4)(3d + 5)\)  
d. \((x - 2)(x^2 + 2x - 4)\)

**Example 4**: Multiply and simplify

a. \((n + 5)^2\)  
b. \((3 - 2a)^2\)
Example 5: Multiply.

a. $5(2x - 4)(3x + 2)$

b. $(x + 3)^2 - (x - 1)^2$

c. $(x - 2)^3$

Section 10.3 – You Try

Multiply and simplify. Show all steps as in the media examples.

a. $-3x^2(x^5 + 6x^3 - 5x)$

b. $(3x - 4)(5x + 2)$

c. $(2p - 5)^2$

d. $4(2x + 5)(x - 3)$
Section 10.4: Division of Polynomials

Simplify the following expressions. Write your answer with only positive exponents. Variables represent nonzero quantities.

Example 1: \[-\frac{6x^8}{30x^3}\]

Example 2: \[\frac{3x-6}{2}\]

Example 3: \[\frac{6x^3+2x^2-4}{4x}\]

Example 4: \[\frac{20a^2+35a-4}{-5a^2}\]

Section 10.4 – You Try

Simplify the following expressions. Write your answer with only positive exponents. Variables represent nonzero quantities.

a. \[\frac{11x-15}{3}\]

b. \[\frac{3x^2+5x-12}{3x^2}\]
Section 10.5: Factoring Quadratic Expressions (GCF Method)

A **Quadratic Expression** is a second degree polynomial of the form $ax^2 + bx + c$, where $a$, $b$, and $c$ are real number coefficients. Examples of quadratic expressions are:

- $3x^2 - x + 27$ Here, $a = 3$, $b = -1$, and $c = 27$
- $x^2 - 3$ Here, $a = 1$, $b = 0$, and $c = -3$
- $2x^2 + 5x$ Here, $a = 2$, $b = 5$, and $c = 0$
- $2.3x^2$ Here, $a = 2.3$, $b = 0$, and $c = 0$
- $3x + 8 - x^2$ Here, $a = -1$, $b = 3$, and $c = 8$

**REVIEW: Factoring Whole Numbers**

**Worked Example:** To factor 60, there are many possibilities, some of which are below:

- $60 = 1 \cdot 60$ (not very interesting, but true)
- $60 = 2 \cdot 30$
- $60 = 3 \cdot 20$
- $60 = 4 \cdot 3 \cdot 5$
- $60 = 2 \cdot 2 \cdot 3 \cdot 5$ (This one is also called the **prime factorization** of 60 because it is made of only prime factors.)

When we factor quadratic expressions, we use a similar process. This process involves factoring expressions such as $24x^2$. To factor $24x^2$ completely, we would first find the prime factorization of 24 and then factor $x^2$.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 \quad \text{and} \quad x^2 = x \cdot x$$

Putting these factorizations together, we obtain the following: $24x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$

**Example 1:** Completely factor the quadratic expressions below. Check your final results.

- **a.** $11a^2 - 4a$
- **b.** $55w^2 + 5w$
Complete the following problems.

1. Identify the coefficients $a$, $b$, and $c$ for each of the following quadratic expressions.
   
   a. $x^2 - 7x + 4$  
      $a = ______$, $b = ______$, $c = ______$
   
   b. $8x^2 - x$  
      $a = ______$, $b = ______$, $c = ______$
   
   c. $3x^2 + 43$  
      $a = ______$, $b = ______$, $c = ______$

2. Completely factor the quadratic expressions below. Check your final results.
   
   a. $64b^2 - 24b$  
   b. $11c^2 + 7c$  
   c. $12v^2 - 3v$
Section 10.6: Factoring Quadratic Expressions \((a = 1)\)

Quadratic expressions with 3 terms are called \textit{trinomials}. The Trial and Error Method to factor trinomials is shown below.

<table>
<thead>
<tr>
<th>Factoring Quadratic Expressions of the form (x^2 + bx + c) by Trial and Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + bx + c = (x + p)(x + q)), where (b = p + q) and (c = p \cdot q)</td>
</tr>
</tbody>
</table>

**Worked Example:** Completely factor the quadratic expression \(x^2 + 5x - 6\).

**Step 1:** Look to see if there is a common factor in this expression. If there is, then use the GCF method to factor out the common factor.

The expression \(x^2 + 5x - 6\) has no common factors.

**Step 2:** For this problem, \(b = 5\) and \(c = -6\). We need to identify \(p\) and \(q\). These are two numbers whose product is \(-6\) and sum is \(5\). A helpful method to identify \(p\) and \(q\) is to list different numbers whose product is \(-6\) and then see (i.e. trial and error) which pair adds to \(5\).

<table>
<thead>
<tr>
<th>Product = (-6)</th>
<th>Sum = (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3 \cdot 2)</td>
<td>No</td>
</tr>
<tr>
<td>(3 \cdot -2)</td>
<td>No</td>
</tr>
<tr>
<td>(-1 \cdot 6)</td>
<td>YES</td>
</tr>
<tr>
<td>(1 \cdot -6)</td>
<td>No</td>
</tr>
</tbody>
</table>

**Step 3:** Write in factored form

\[x^2 + 5x - 6 = (x + (-1))(x + 6)\]

\[x^2 + 5x - 6 = (x - 1)(x + 6)\]

**Step 4:** Always check your result!

\[(x - 1)(x + 6) = x^2 + 6x - x - 6\]

\[= x^2 + 5x - 6 \ \text{CHECKS!}\]
Example 1: Completely factor the following quadratic expressions. Check your final results.

a. \(a^2 + 7a + 12\)  
b. \(w^2 + w - 20\)  
c. \(x^2 - 36\)

Section 10.6 – You Try

Completely factor the following quadratic expressions. Check your final results.

a. \(n^2 + 8n + 7\)  
b. \(r^2 + 3r - 70\)  
c. \(m^2 - 4\)
Section 10.7: Combining Factoring Methods

Example 1: Completely factor the following quadratic expressions.
   a. $2x^2 + 10x - 12$  
   b. $6x^2 - 21x - 12$

Example 2: Completely factor the following quadratic expressions.
   a. $2x^3 - 50x$  
   b. $3x^3 + 9x^2 - 84x$

Section 10.7 – You Try

Completely factor the following expressions.
   a. $5n^2 + 40n + 35$  
   b. $r^3 + 3r^2 - 70r$  
   c. $6m^2 - 24$
Section 10.8: Solving Equations by Factoring

The Zero Product Property

If two numbers \( a \) and \( b \) are multiplied together and the resulting product is 0, then at least one of the numbers must be zero.

\[
\text{If } a \cdot b = 0 \\
\text{then } a = 0 \text{ or } b = 0, \\
\text{or both } a = 0 \text{ and } b = 0.
\]

Example 1: Solve the following equations

\[
x(x - 3) = 0 \\
4x(x + 5) = 0 \\
(x - 2)(x + 7) = 0 \\
(2x + 3)(5x - 1) = 0 \\
x(x - 1)(6x + 11) = 0
\]

To solve a quadratic equation by factoring:

Step 1: Make sure the quadratic equation is in standard form: \( ax^2 + bx + c = 0 \).
Step 2: Write the left side in completely factored form.
Step 3: Apply the Zero Product Principal to set each linear factor = 0 and solve for \( x \).
Example 2: Solve by factoring.
   a. $12x^2 + 18x = 0$  
   b. $4x^2 = 20x$

Example 3: Solve by factoring.
   a. $x^2 + 9x + 18 = 0$  
   b. $x^2 + 24 = 11x$

Section 10.8 – You Try

Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work.
   a) Solve $x^2 + 3x = 10$  
   b) Solve $3x^2 = 15x$
Unit 10: Answers to You-Try Problems

Section 10.1:

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Name</th>
<th>Leading Coefficient</th>
<th>Constant Term</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^2 - 2n + 8$</td>
<td>Trinomial</td>
<td>1</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$4x^3$</td>
<td>Monomial</td>
<td>4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$6x - 7$</td>
<td>Binomial</td>
<td>6</td>
<td>-7</td>
<td>1</td>
</tr>
</tbody>
</table>

Section 10.2: $-x^3 - 2x^2 + 8x - 1$

Section 10.3: a. $-3x^7 - 18x^5 + 15x^3$  
   b. $15x^2 - 14x - 8$  
   c. $4p^2 - 20p + 25$  
   d. $8x^2 - 4x - 60$

Section 10.4: a. $\frac{11}{3}x - 5$  
   b. $1 + \frac{5}{3x} - \frac{4}{x^2}$

Section 10.5:  
   1. a. $a = 1, b = -7, c = 4$  
      b. $a = 8, b = -1, c = 0$  
      c. $a = 3, b = 0, c = 43$
   
   2. a. $8b(8b - 3)$  
      b. $c(11c + 7)$  
      c. $3v(4v - 1)$

Section 10.6: a. $(n + 7)(n + 1)$  
   b. $(r - 7)(r + 10)$  
   c. $(m - 2)(m + 2)$

Section 10.7: a. $5(n + 7)(n + 1)$  
   b. $r(r - 7)(r + 10)$  
   c. $6(m - 2)(m + 2)$

Section 10.8: a. $x = -5$ and $2$  
   b. $x = 0$ and $5$
Unit 10: Practice Problems

1. Simplify completely. Show all steps, and box your answers.

a. \(4x^2(3x - 5)\)   
b. \(4a^2(3a^2 - 2a - 5)\)   
c. \((p + 5)(p + 7)\)   
d. \((x + 2)(x - 2)\)   
e. \((2x - 4)(3x - 5)\)   
f. \((5w - 8)(3w + 11)\)   
g. \((x + 2)^2\)   
h. \((2x - 4)^2\)   
i. \((x - 4)(x^2 + x - 5)\)   
j. \(3(x + 2)(x + 4)\)   
k. \(4(x + 2)^2\)   
l. \((q - 2)^3\)

2. Completely factor each of the following expressions using the GCF Method.

a) \(5x^2 - 10x\)   
b) \(3x^7 + 12x^5\)   
c) \(-x^2 - 7x\)   
d) \(4x^2 - 7x\)   
e) \(6x^3 + 2x\)   
f) \(x^2 - x\)   
g) \(-20x^2 - 5x\)   
h) \(x^5 + 8x^7\)   
i) \(6x^3 + 15x^2\)

3. Completely factor each of the following expressions using Trial and Error.

a) \(x^2 + x - 20\)   
b) \(x^2 - 12x + 11\)   
c) \(x^2 + 7x + 6\)

d) \(x^2 + 5x + 6\)   
e) \(x^2 - 5x - 6\)   
f) \(x^2 + 5x - 6\)   
g) \(x^2 - 36\)   
h) \(x^2 - 9\)   
i) \(2x^2 + 14x + 24\)   
j) \(3x^3 - 6x^2 + 24x\)   
k) \(5x^2 - 20\)   
l) \(-4x^3 + 8x^2 - 4x\)

4. Write each quadratic equation in standard form. Identify coefficients \(a\), \(b\) and \(c\).

a) \(3x^2 - 4x + 3 = 7\)   
b) \(5x^2 - x + 6 = 3 - x\)   
c) \((x + 1)(x - 4) = 8\)

d) \(2(x - 3)(x + 7) = 0\)   
e) \((x + 2)^2 = 5\)   
f) \(2(x - 4)^2 - 9 = 0\)   
g) \(5(x + 1)^2 + 3 = 1\)
5. Solve the following quadratic equations by factoring. Verify your results by graphing
   a) \(5x^2 - 10x = 0\)  
   b) \(3x^2 + 12x + 7 = 7\)  
   c) \(x^2 = 7x\)  
   d) \(x^2 + x = 20\)  
   e) \(x^2 = 12x - 11\)  
   f) \(x^2 + 8x + 2 = x - 4\)  
   g) \(x^2 = 36\)  
   h) \(2x^2 = 18\)  
   i) \(2x^2 + 14x + 28 = 4\)

### Applications

6. The function \(H(t) = 96t - 16t^2\) gives the height (in feet) of golf ball after \(t\) seconds. Use **factoring** to determine when \(H(t) = 0\). Show all algebraic steps. Round your answers to the nearest tenth. Write a sentence explaining the meaning of your answers.

7. A company’s revenue earned from selling \(x\) items is given by the function \(R(x) = 30x\), and the cost to sell the same \(x\) items is given by \(C(x) = 63 + 3x^2\). Round answers to two decimal places as needed. All answers must include appropriate units of measure.
   a) Write a function, \(P(x)\), that represents the company’s profit from selling \(x\) items.
   b) Identify the vertical intercept of \(P(x)\). Write it as an ordered pair and interpret its meaning in a complete sentence.
   c) Identify the horizontal intercepts of \(P(x)\). Write it as an ordered pair and interpret its meaning in a complete sentence.
   d) Determine \(P(5)\). Write your answer as an ordered pair and interpret its meaning in a complete sentence.

### Extension

8. Simplify the following rational expressions. Begin by determining the domain.
   a) \(\frac{4x}{6-2x}\)  
   b) \(\frac{5x}{20x-15}\)  
   c) \(\frac{3x+5}{12x+20}\)  
   d) \(\frac{6x-8}{3x-4}\)  
   e) \(\frac{12x+21}{8x+14}\)  
   f) \(\frac{6x}{3x^2-6x}\)  
   g) \(\frac{x+2}{3(x+2)(x-5)}\)  
   h) \(\frac{x-3}{x^2-5x+6}\)  
   i) \(\frac{x+5}{2x^2-50}\)
Unit 10: Review

1. Multiply and simplify completely. Show all steps, and box your answers.
   a. \(4x^2(8x^2 - 5x - 3)\)  
   b. \((2x - 3)(5x + 11)\)  
   c. \((3 - 5x)^2\)  
   d. \(2(x + 5)^2 - 9\)

2. Explain the difference between the two expressions below.
   \((2x - 3)(4x + 5)\)  
   \(2x - 3(4x + 5)\)

3. Explain the difference between the three expressions below.
   \(2x^2\)  
   \((2x)^2\)  
   \((2 + x)^2\)
4. Factor each of the following quadratic expressions. Write your answers in factored form.
   a. \( x^2 - 6x + 8 \)  
   b. \( 15x^5 - 3x^3 \)  
   c. \( x^2 - 9 \)  
   d. \( 3x^2 - 3x - 36 \)  
   e. \( x^3 - 3x^2 + 2x \)  
   f. \( x^8 + 6x^7 + 9x^6 \)  

5. Solve each of the following equations by factoring.
   a. \( x(x + 8)(3x - 1) = 0 \)  
   b. \( x^2 + 2x - 11 = 4 \)  
   c. \( 7x^2 = 9x \)  
   d. \( x^2 + 12x + 8 = 23x - 2 \)  
   e. \( 2x^2 - 8x = 10 \)  
   f. \( 3x^3 = 6x^2 - 24x \)
Unit 11: Quadratic Functions and Equations

Section 11.1: Characteristics of Quadratic Functions
Section 11.2: Horizontal Intercepts of Quadratic Functions
Section 11.3: Sketch the Graph of a Quadratic Function
Section 11.4: Solving Quadratic Equations by Graphing
Section 11.5: Solving Quadratic Equations by Factoring
Section 11.6: The Quadratic Formula
Section 11.7: Solving Quadratic Equations with Complex Solutions
Section 11.8: Choosing Solution Methods
Section 11.9: Applications

UNIT 11 LEARNING OBJECTIVES
- Identify characteristics of quadratic functions and graphs
- Determine domain and range of quadratic functions
- Sketch the graph of a quadratic function by hand and on the graphing calculator.
- Solve quadratic equations by graphing, factoring, and quadratic formula

KEY TERMS AND CONCEPTS
Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Function</td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Axis of Symmetry</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td></td>
</tr>
</tbody>
</table>
A quadratic function is a function which can be written in the form \( f(x) = ax^2 + bx + c \).

- \( f(x) = ax^2 + bx + c \) has three distinct terms each with its own coefficient:
  - \( ax^2 \) is the first term and has coefficient \( a \)
  - \( bx \) is the second term and has coefficient \( b \)
  - \( c \) is the third term, called the constant term (We say that this term has coefficient \( c \))
- Note: If any term is missing, the coefficient of that term is 0
- \( a, b, \) and \( c \) can be any real numbers. Note that \( a \) cannot be 0.
- The graph of \( f(x) = ax^2 + bx + c \) is called a parabola, is shaped like a U and opens either up or down.
- \( a \) determines which direction the parabola opens (\( a > 0 \) opens up, \( a < 0 \) opens down)
- \( c \) is the vertical intercept with coordinates \((0, c)\)
- The domain of a quadratic function is \((-\infty, \infty)\)

**Graph of the Basic Quadratic Function:** \( f(x) = x^2 \)


Example 1: Given the Quadratic Function \( f(x) = x^2 - 2x + 3 \)

Identify the coefficients \( a, b, c \) ________________________________

Which direction does the parabola open? Why? ___________________________

What is the vertical intercept? Plot and label on the graph. ________________
Quadratic Functions: Vertex and Axis of Symmetry

Given a quadratic function, \( f(x) = ax^2 + bx + c \)

The vertex is the lowest or highest point of the associated parabola and is always written as an ordered pair \( \left( -\frac{b}{2a}, f \left( -\frac{b}{2a} \right) \right) \)

The axis of symmetry equation is the equation of the vertical line \( x = -\frac{b}{2a} \) that passes through the vertex and divides the parabola in half.

**Example 2:** Given the quadratic function \( f(x) = x^2 - 2x + 3 \).

Identify the coefficients \( a, b, c \) __________________________

Determine the coordinates of the vertex.

Identify the axis of symmetry equation. __________________________

Graph the function. Plot and label the vertex and axis of symmetry on the graph.
**Example 3:** Sketch the graph and find and label the vertex ordered pair for \( f(x) = -2x^2 - 6 \). Then, use the graph to help you determine the domain and range. Write your domain and range answers in interval notation and inequality notation.

\[
\text{Domain of } f(x)
\]

\[
\text{Range of } f(x)
\]

---

**Section 11.1 – YOU TRY**

Given the quadratic function \( f(x) = -x^2 + 8x - 6 \)

a. Identify the coefficients \( a, b, c \) ________________________________

b. Which direction does the parabola open? Why? __________________________

c. Graph \( f(x) \) on your calculator. Use the standard viewing window.

\[
\text{Graph of } f(x)
\]

d. What is the vertical intercept? Plot and label on the graph. ______________

e. Determine the coordinates of the vertex. Label this point on the graph.

\[
\text{Vertex of } f(x)
\]

f. Identify the axis of symmetry equation. ______________ Draw the axis of symmetry on the graph.

g. Determine the domain of \( f(x) \). Use interval notation. __________________________

h. Determine the range of \( f(x) \). Use interval notation. __________________________
Section 11.2: Horizontal Intercepts of Quadratic Functions

The quadratic function, \( f(x) = ax^2 + bx + c \), will have horizontal intercepts if its parabola crosses the \( x \)-axis (i.e. if \( f(x) = 0 \)). These intercepts are labeled as \( G \) and \( H \) on the graphs above.

**Example 1:** For each of the following functions, draw a sketch of the graph then use the Graphing /Intersection Method on your TI 83/84 calculator to identify the horizontal intercepts rounded to 2 decimal places. If these exist, label them on the graph. If there are no intercepts, indicate that as well.

a. \( f(x) = -2x^2 + 6x + 3 \)

b. \( g(x) = x^2 - x + 2 \)

c. \( h(x) = x^2 - 6x + 9 \)

**Section 11.2 – YOU TRY**

For the quadratic functions below, sketch the graph then use the Graphing /Intersection Method on your TI 83/84 calculator to identify the horizontal intercepts rounded to 2 decimal places. If these exist, label them on the graph. If there are no intercepts, indicate that as well.

a. \( f(x) = 2x^2 - 5 \)

b. \( f(x) = x^2 + 2x + 1 \)

c. \( f(x) = x^2 - 4x + 7 \)
Section 11.3: Sketch the Graph of a Quadratic Function

Example 1: Provide a detailed sketch of the quadratic function \( g(x) = 3x^2 + 96x - 478 \). Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________ Graph
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s): ____________
- Domain: ______________
- Range: ______________

Example 2: Provide a detailed sketch of the quadratic function \( f(x) = -4x^2 + 72x \). Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________ Graph
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s): ____________
- Domain: ______________
- Range: ______________
Example 3: Provide a detailed sketch of the quadratic function $h(x) = x^2 + 5$. Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________  
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s): ________________
- Domain: ______________
- Range: ______________

Section 11.3 – You Try

Provide a detailed sketch of the quadratic function $f(x) = -2x^2 + 44x + 230$. Identify the characteristics of the graph and determine the domain and range.

- Vertical intercept: ___________  
- Vertex: ____________
- Axis of Symmetry: _____________
- Horizontal intercept(s): ________________
- Domain: ______________
- Range: ______________
Section 11.4: Solving Quadratic Equations by Graphing

Example 1: Solve \( x^2 - 10x + 1 = 4 \). Plot and label the graphs and intersection points that are part of your solution process. Identify the final solutions clearly. Round to 2 decimal places.

\[
\begin{align*}
X_{\text{min}} &= \underline{\quad} \\
X_{\text{max}} &= \underline{\quad} \\
Y_{\text{min}} &= \underline{\quad} \\
Y_{\text{max}} &= \underline{\quad}
\end{align*}
\]

Solution: \( \underline{\quad} \)

NOTE: The Graphing/Intersection Method will provide us only with approximate solutions to a quadratic equation when decimal solutions are present. To find exact solution values, you will need to use an algebraic approach. This will be covered in a later lesson.

Section 11.4 – You Try

Solve \( x^2 + 9x - 18 = 32 \). Plot and label the graphs and intersection points that are part of your solution process. Round your answers to two decimal places. Identify the final solutions clearly.

\[
\begin{align*}
X_{\text{min}} &= \underline{\quad} \\
X_{\text{max}} &= \underline{\quad} \\
Y_{\text{min}} &= \underline{\quad} \\
Y_{\text{max}} &= \underline{\quad}
\end{align*}
\]

Solution: \( \underline{\quad} \)
Section 11.5: Solving Equations by Factoring

The Zero Product Property
If $a \cdot b = 0$, then $a = 0$ or $b = 0$

To solve a quadratic equation by factoring:
- **Step 1:** Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$.
- **Step 2:** Write the left side in completely factored form.
- **Step 3:** Apply the Zero Product Principal to set each linear factor $= 0$ and solve for $x$.
- **Step 4:** Verify result by graphing and finding the intersection point(s).

**Example 3:** Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work. Be sure to write your final solutions using proper notation. Verify your answer by graphing.

a) Solve by factoring: $-2x^2 = 8x$

b) Solve by factoring: $x^2 = 3x + 28$
Section 11.5 – You Try

Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work.

a) Solve \( x^2 + 4x - 11 = 10 \)

Graph

b) Solve \( 3x^2 = 15x \)

Graph
Section 11.6: The Quadratic Formula

The Quadratic Formula can be used to solve any quadratic equations written in standard form:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

To solve a quadratic equation using the Quadratic Formula:

1. Make sure the quadratic equation is in standard form: \( ax^2 + bx + c = 0 \) and write down the coefficients \( a, b \) and \( c \).
2. Graph the quadratic part of the equation and see how many times the graph crosses the x-axis. Identify the number and type of solutions. If the graph crosses the x-axis, enter 0 as \( Y2 \) then find the intersection point(s) and round to the appropriate number of decimal places. Write the x-values down. These will be your approximate solutions.
3. Substitute the coefficient values into the Quadratic Formula.
4. Simplify your result completely, leaving in the form indicated by the directions.

**Example 1:** Solve the equation \( x^2 - 3 = 3x \) using the Quadratic Formula. Leave your solution(s) in exact form and in approximate form rounded to the thousandths place.
Example 2: Solve the equation \(-x^2 + 3x + 10 = 0\) using the Quadratic Formula. Verify your result by graphing and using the Intersection Method.

Example 3: Solve the equation \(2x^2 - 4x = 3\) using the Quadratic Formula. Verify your result by graphing and using the Intersection Method.
Section 11.6 – You Try

Solve the equation $3x^2 + x = 2x + 5$ using the Quadratic Formula. Include a sketch of the graph. Write your answer(s) in exact form and in approximate form rounded to the thousandths place.

Step 1: Make sure the quadratic equation is in standard form: $ax^2 + bx + c = 0$ and write down the coefficients $a$, $b$ and $c$.

Step 2: Graph the quadratic part of the equation and see how many times the graph crosses the x-axis. Identify the number and type of solutions. If the graph crosses the x-axis, enter 0 as Y2 then find the intersection point(s) and round to the appropriate number of decimal places. Write the x-values down. These will be your approximate solutions.

Step 3: Substitute the coefficient values into the Quadratic Formula

Step 4: Simplify your result completely, leaving in the form indicated by the directions.
Example 1: Solve the equation $x^2 + 4x + 8 = 1$. Leave your results in the form of a complex number, $a + bi$.

Section 11.7 – You Try

Solve the equation $x^2 + 8x + 25 = 0$ using the Quadratic Formula. Leave your results in the form of a complex number, $a + bi$. Sketch the graph in the space provided.
Section 11.8: Choosing Solution Methods

We have learned several methods to solve quadratic equations. Which one should you use when? The pros and cons of each method are presented below and the problems in this lesson will provide practice for all solution methods.

### Solving Quadratic Equations by Graphing

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Good starting point for all quadratic equations as graphing will provide information about the number and types of solutions</td>
<td>• Non-integer solution values will be approximate, not exact</td>
</tr>
<tr>
<td>• Not necessary to put the equation in standard form to use this method</td>
<td>• Window adjustments are often necessary in order to view necessary parts of the graph</td>
</tr>
</tbody>
</table>

### Solving Quadratic Equations by Factoring

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Can be the fastest solution path if you quickly recognize how to factor the quadratic part of your equation</td>
<td>• Equation must be in standard form to apply this method</td>
</tr>
<tr>
<td></td>
<td>• Does not work well for non-integer solutions and large numbers</td>
</tr>
<tr>
<td></td>
<td>• Can be time consuming if there are many choices for the Trial and Error Method</td>
</tr>
</tbody>
</table>

### Solving Quadratic Equations by Quadratic Formula

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Can be used to solve any quadratic equation.</td>
<td>• Equation must be in standard form to apply this method</td>
</tr>
<tr>
<td>• Best way to find exact form solutions.</td>
<td>• Must remember the formula.</td>
</tr>
<tr>
<td></td>
<td>• Must pay close attention to details in order to avoid computational errors</td>
</tr>
</tbody>
</table>
Example 1: Given the quadratic equation $2x^2 - x = 4$, solve using the methods indicated below, leaving all solutions in exact form and in approximate form rounded to the thousandths place. If solutions are complex, leave them in the form $a + bi$.

a) Solve by graphing (if possible). Sketch the graph on a good viewing window (the vertex, vertical intercept and any horizontal intercepts should appear on the screen). Mark and label the solutions on your graph.

b) Solve by factoring (if possible). Show all steps.

c) Solve using the Quadratic Formula. Show all steps.
Example 2: Given the quadratic equations below, solve for $x$ using an appropriate method of your choice leaving all solutions in exact form and approximate form to the thousandths place. Clearly identify each method that you attempt. If solutions are complex, leave them in the form $a + bi$.

a) $x^2 - 3x - 10 = 0$

b) $x^2 - 6x = -10$
Given the quadratic equation $x^2 + 3x - 7 = 3$, solve using the methods indicated below leaving all solutions in exact form. Clearly identify your solutions in all cases.

a) Solve by graphing (if possible). Sketch the graph on a good viewing window (the vertex, vertical intercept, and any horizontal intercepts should appear on the screen). Mark and label the solutions on your graph.

b) Solve by factoring (if possible). Show all steps.

c) Solve using the Quadratic Formula. Show all steps.
Section 11.9: Applications

Example 1: The height of a train tunnel is modeled by the quadratic function
\[ h(x) = -0.35x^2 + 25 \]
where \( x \) is the distance, in feet, from the center of the tracks and \( h(x) \) is the height of the tunnel, also in feet. Assume that the high point of the tunnel is directly in line with the center of the train tracks.

a. Draw a complete diagram of this situation. Find and label each of the following: vertex, horizontal intercept (positive side), and vertical intercept.

b. How wide is the base of the tunnel?

c. A train with a flatbed car 6 feet off the ground is carrying a large object that is 15 feet high. How much room will there be between the top of the object and the top of the tunnel?
Example 2: The function $h(t) = -16t^2 + 100t + 3$, where $h(t)$ is the height in feet after $t$ seconds, models the path of a baseball hit from home plate into the playing field.

a. Begin by drawing an accurate representation of the graph of $h(t)$ including: starting height, time to maximum height and maximum height and time to hit the ground.

b. One day during a game, player Scott hit a ball whose path followed the model above. The stadium in which the game was played had a design flaw that prevented the fans in the middle sections from being able to view the ball during the times when its height was more than 100 feet. For how many seconds could the fans in this game not see the ball that Scott had hit? Write your answer in a complete sentence.
An archer stands on a platform and fires an arrow. The height of the arrow above the ground is modeled by the quadratic function \( h(t) = -16t^2 + 80t + 130 \), where \( h(t) \) is height (in feet) of the arrow after \( t \) seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a. Draw a detailed sketch of this situation

b. How tall was the launching pad?

c. What is the maximum height of the rocket?

d. How long does it take for the rocket to hit the ground?

e. Determine the practical domain of \( h(t) \). Use inequality notation and include units.

f. Determine the practical range of \( h(t) \). Use inequality notation and include units.
Unit 11 Answers to You-Try Problems

**Section 11.1:**  
- a. \( a=-1, b=8, c=-6 \)  
- b. Opens down (because \( a<0 \))  
- d. \( (0,-6) \)  
- e. \( (4,10) \)  
- f. \( x=4 \)  
- g. \( (-\infty,\infty) \)  
- h. \( (-\infty,10] \)

**Section 11.2:**  
- a) Horizontal Intercepts: \((-1.58,0)\) & \((1.58,0)\)  
- b) Horizontal Intercept: \((-1,0)\)  
- c) Horizontal Intercepts: None

**Section 11.3:**  
- Vertical Intercept: \((0,230)\)  
- Vertex: \((11,472)\)  
- Axis of Symmetry: \(x = 11\)  
- Horizontal Intercepts: \((-4.36,0), (26.36,0)\)  
- Domain: \((\infty,\infty)\)  
- Range: \((-\infty,472]\)

**Section 11.4:**  
- \( x \approx 3.88 \) and \( x \approx -12.88 \)

**Section 11.5:**  
- a) \( x = -7 \) and \( 3 \)  
- b) \( x = 0 \) and \( 5 \)
Section 11.6: \[ x_1 = \frac{1 + \sqrt{61}}{6} \approx 1.468 \quad x_2 = \frac{1 - \sqrt{61}}{6} \approx -1.135 \]

Section 11.7: \[ x_1 = -4 + 3i \quad x_2 = -4 - 3i \]

Section 11.8: \[ x = -5 \text{ and } 2 \]

Section 11.9:

b. 130 feet
c. 230 feet
d. 6.29 seconds
e. \( 0 \leq t \leq 6.29 \) seconds
f. \( 0 \leq h(t) \leq 230 \) feet
Unit 11: Practice Problems

Skills Practice

1. For each of the following quadratic functions:
   - Identify the coefficients $a$, $b$ and $c$.
   - Determine if the parabola opens up or down and state why.
   - Identify the vertical intercept as an ordered pair.
   - Identify the horizontal intercept(s)
   - Determine the vertex
   - Determine the equation of the axis of symmetry
   - Determine the domain
   - Determine the range
   - Sketch the graph of this quadratic function. Include the intercepts and axis of symmetry on your sketch
   - Graph the function on your calculator using an appropriate viewing window.

   a) $f(x) = 2x^2 - 4x - 4$  
   b) $g(x) = -8x^2 + 6x - 4$

   c) $h(x) = x^2 - 9$  
   d) $K(x) = -3x^2 + 7x$

2. For each of the graphs below.
   - Determine the vertex
   - Determine the axis of symmetry equation and draw the axis of symmetry on the graph.
   - Identify the domain and range using interval notation.
3. The graph of f(x) is shown below. Write DNE if the answer does not exist.

![Graph of f(x) with x and y axes labeled]

a. f(x) = 3 when x = ______________

b. f(x) = 8 when x = ______________

c. f(x) = 11 when x = ______________

d. f(x) = 12 when x = ______________

e. f(x) = 13 when x = ______________

4. Solve each equation using your calculator and the Graphing/Intersection Method. Draw the graph and plot/label the point(s) of intersection. Clearly identify the final solution(s). Round to two decimal places as needed.

a. x^2 - x - 6 = 0

b. x^2 - 9x + 10 = -4

c. x^2 - 8 = 1

d. -x^2 + 6x - 4 = -10

5. Draw a graph that represents the type of function indicated.

- Quadratic Function with TWO horizontal intercepts
- Quadratic Function with ONE horizontal intercept
- Quadratic Function with ZERO horizontal intercept

6. Solve each equation using the indicated method. Round your answers to the nearest hundredth as needed.

- Using the Quadratic Formula.
- Using the Graphing/Intersection Method.

a) 3x^2 - 4x + 3 = 7

b) 5x^2 - x + 6 = 3 - x

c) (x + 1)(x - 4) = 8

d) 2(x - 3)(x + 7) = 0

e) (x + 2)^2 = 5

f) 2(x - 4)^2 - 9 = 0

g) 5(x + 1)^2 + 3 = 1
7. Solve the following quadratic equations by factoring. Verify your results by graphing.

   a) \( 5x^2 - 10x = 0 \)  
   b) \( 3x^2 + 12x + 7 = 7 \)  
   c) \( x^2 = 7x \)  
   d) \( x^2 + x = 20 \)  
   e) \( x^2 = 12x - 11 \)  
   f) \( x^2 + 8x + 2 = x - 4 \)  
   g) \( x^2 = 36 \)  
   h) \( 2x^2 = 18 \)  
   i) \( 2x^2 + 14x + 28 = 4 \)  

8. Solve the following equations using ANY method. Write your answers in exact form AND approximate form, rounded to the nearest thousandth. Complex solutions must be written in the form \( a+bi \).

   a. \( 3x^2 + 2x = 5 \)  
   b. \( x^2 + 11x + 16 = 3x \)  
   c. \( x^2 + 4 = 0 \)  
   d. \( x^2 + 2x = 3 \)  
   e. \( x^2 = 4x - 4 \)  
   f. \( -x^2 + x = 2 \)  
   g. \( -2x^2 = 8x \)  
   h. \( -x^2 = 12x + 36 \)  
   i. \( 2x^2 + 3x + 5 = 0 \)  
   j. \( x^2 - 3x = 10 \)  
   k. \( x^2 + 1 = 2x \)  
   l. \( 5x^2 + 6x + 5 = 2x^2 - x \)  

Applications

9. In the Angry Bird electronic game, birds are shot at different targets using a slingshot. The function \( B(t) = -0.2t^2 + 1.3t + 15 \) models the height of one of the birds, in feet, after \( t \) seconds. Round answers to two decimal places as needed. All answers must include appropriate units.

   a) Draw a GOOD graph of this situation. Label the axes of your graph appropriately. Find and label each of the following: vertex, horizontal intercept (positive only) and vertical intercept.

   b) How high above the ground was the bird when it was launched? _______________

   c) After how many seconds does the bird reach its highest point? _______________

   d) How high is the angry bird at its highest point? _______________

   e) After how many seconds does the angry bird hit the ground? _______________

   f) Determine the practical domain of this function. Write your answer in interval notation.

   g) Determine the practical range of this function. Write your answer in interval notation.
10. A company’s revenue earned from selling \( x \) items is given by the function \( R(x) = 680x \), and the cost to sell the same \( x \) items is given by \( C(x) = 10,000 + 2x^2 \). Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a) Write a function, \( P(x) \), that represents the company’s profit from selling \( x \) items.

b) Identify the vertical intercept of \( P(x) \). Write it as an ordered pair and interpret its meaning in a complete sentence.

c) Identify the horizontal intercepts of \( P(x) \). Write it as an ordered pair and interpret its meaning in a complete sentence.

d) Determine \( P(170) \). Write your answer as an ordered pair and interpret its meaning in a complete sentence.

e) Use your graphing calculator to graph the function \( P(x) \). Use viewing window \( x[0,350]; y[-10000, 50000] \). In the space below, draw what you see on your calculator screen.

- Mark the point on the graph that would result in the maximum profit.
- Mark the point(s) on the graph where this company would break even.

11. Solve the following quadratic inequalities by factoring. Verify your results by graphing

a) \( 5x^2 - 10x > 0 \)  

b) \( 3x^2 + 12x + 7 < 7 \)  

c) \( x^2 \geq 7x \)

d) \( x^2 + x \leq 20 \)  

e) \( x^2 > 12x - 11 \)  

f) \( x^2 + 8x + 2 < x - 4 \)
8. In the space below, draw a graph that represents the type of function indicated.

- Quadratic Function with TWO horizontal intercepts
- Quadratic Function with ONE horizontal intercept
- Quadratic Function with ZERO horizontal intercept

9. Fill out the following table. Always use proper notation. Round to two decimal places.

<table>
<thead>
<tr>
<th>Function</th>
<th>Vertical Intercept</th>
<th>Horizontal Intercept(s)</th>
<th>Vertex</th>
<th>Axis of Symmetry Equation</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 2x^2 - 4x - 30 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = 5 - x^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 5x^2 - 4x + 17 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Provide a detailed SKETCH of the functions below. Your sketches must show all interesting and identifying features of the graph.

- \( f(x) = 5x^2 + 70x - 445 \)
- \( f(x) = 230 - 8x^2 \)
4. Given the quadratic equation \( x^2 + 4x - 3 = 2 \), solve using the processes indicated below.

a. Solve by GRAPHING. Use an appropriate viewing window that shows all important features of the graph (the vertex, vertical intercept, intersection points, and any horizontal intercepts should appear on the screen).

\[
\begin{array}{c}
X_{\text{min}} = \underline{\phantom{0}} \\
X_{\text{max}} = \underline{\phantom{0}} \\
Y_{\text{min}} = \underline{\phantom{0}} \\
Y_{\text{max}} = \underline{\phantom{0}} \\
\end{array}
\]

Solutions: \( x = \underline{\phantom{0}} \)
\( x = \underline{\phantom{0}} \)

b. Solve by FACTORING. You must show all algebraic steps for full credit. Box your answers.

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]

\[x^2 + 4x - 3 = 2\]
Unit 12: Toolkit Functions

Section 12.1: Introduction to Basic Toolkit Functions (Graphs)
Section 12.2: Finding Vertical Intercepts of Toolkit Functions
Section 12.3: Domain and Range of Toolkit Functions
Section 12.4: Determining Domain from a Formula
Section 12.5: Average Rates of Change
Section 12.6: Average Rates of Change of Functions and Graphs

UNIT 12 LEARNING OBJECTIVES

- Identify basic Toolkit Functions based on their graphs.
- Identify basic Toolkit Functions based on their formula.
- Determine the Domain and Range of basic Toolkit Functions given the graph.
- Determine the Domain of basic Toolkit Functions given the formula.
- Find Vertical Intercepts of Toolkit functions
- Determine and interpret average rates of change

KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the Media Lesson. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the Media Lesson.

<table>
<thead>
<tr>
<th>Key Term / Concept</th>
<th>Definition and examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toolkit Functions</td>
<td></td>
</tr>
<tr>
<td>Linear Function</td>
<td></td>
</tr>
<tr>
<td>Identity Function</td>
<td></td>
</tr>
<tr>
<td>Constant Function</td>
<td></td>
</tr>
<tr>
<td>Polynomial Function</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>Quadratic Function</td>
<td></td>
</tr>
<tr>
<td>Cubic Function</td>
<td></td>
</tr>
<tr>
<td>Rational Function</td>
<td></td>
</tr>
<tr>
<td>Radical Function</td>
<td></td>
</tr>
<tr>
<td>Absolute Value Function</td>
<td></td>
</tr>
<tr>
<td>Average Rate of Change</td>
<td></td>
</tr>
</tbody>
</table>
Example 1: Sketch the graph and fill in a table of values for each of the functions below.

| Constant Function $f(x) = c$ or $y = c$ | Identity Function $f(x) = x$ or $y = x$ | Absolute Value Function $f(x) = |x|$ or $y = |x|$ |
|-----------------------------------------|----------------------------------------|--------------------------------------|
| $x$ | $y$ | $x$ | $y$ | $x$ | $y$ |
| 4   | 4   | 4   | 4   | 4   | 4   |
| 3   | 3   | 3   | 3   | 3   | 3   |
| 2   | 2   | 2   | 2   | 2   | 2   |
| 1   | 1   | 1   | 1   | 1   | 1   |
| -2  | -2  | -2  | -2  | -2  | -2  |
| -3  | -3  | -3  | -3  | -3  | -3  |
| -4  | -4  | -4  | -4  | -4  | -4  |

<table>
<thead>
<tr>
<th>Quadratic Function $f(x) = x^2$ or $y = x^2$</th>
<th>Cubic Function $f(x) = x^3$ or $y = x^3$</th>
<th>Square Root Function $f(x) = \sqrt{x}$ or $y = \sqrt{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
<td>$x$</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
<td>9</td>
<td>-3</td>
</tr>
<tr>
<td>-4</td>
<td>16</td>
<td>-4</td>
</tr>
</tbody>
</table>

| $x$ | $y$ | $x$ | $y$ | $x$ | $y$ |
| 4   | 2   | 4   | 2   | 4   | 2   |
| 3   | 3   | 3   | 3   | 3   | 3   |
| 2   | 2   | 2   | 2   | 2   | 2   |
| 1   | 1   | 1   | 1   | 1   | 1   |
| -2  | 2   | -2  | -2  | -2  | 2   |
| -3  | 3   | -3  | -3  | -3  | 3   |
| -4  | 4   | -4  | -4  | -4  | 4   |


Unit 12: Toolkit Functions

Cube Root Function
\[ f(x) = \sqrt[3]{x} \quad \text{or} \quad y = \sqrt[3]{x} \]

Reciprocal Function
\[ f(x) = \frac{1}{x} \quad \text{or} \quad y = \frac{1}{x} \]

Reciprocal Squared Function
\[ f(x) = \frac{1}{x^2} \quad \text{or} \quad y = \frac{1}{x^2} \]

Section 12.1 – YOU TRY

Identify the TYPE of function shown below.

a. \( f(x) = -38 \)  
   b. \( p(t) = 5t + 8 \)  
   c. \( h(x) = \sqrt{x + 9} \)  
   d. \( g(x) = x \)

---

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Section 12.2: Finding Vertical Intercepts of Toolkit Functions

The vertical intercept (y-intercept) is the point at which the graph crosses the vertical axis.

The input value of the vertical intercept is always ________________

The coordinates of the vertical intercept will be ________________

To determine the vertical intercept of any function:

Example 1: Determine the vertical intercept of each of the functions below. Use your graphing calculator to check your answers. If the vertical intercept does not exist, write DNE.

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2 + 3x - 10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 5x + 7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt{36 - 3x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt{2x - 8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>Expression</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>( f(x) = \sqrt[3]{4x - 6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 19 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = x^3 - 4x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = \frac{x + 1}{x - 6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) =</td>
<td>x</td>
<td>- 3 )</td>
</tr>
</tbody>
</table>
### Section 12.2 – YOU TRY

Identify the vertical intercept for each of the following functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^3 + 6x^2 - x + 8 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p(t) =</td>
<td>3x - 4</td>
<td>)</td>
</tr>
<tr>
<td>( h(x) = \sqrt[3]{5x - 7} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = \frac{1}{2x - 16} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 12.3: Domain and Range of Toolkit Functions

Example 1: Determine the domain and range of each of the functions below.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Function $f(x) = b$ or $y = b$</td>
<td>$\mathbb{R}$</td>
<td>${b}$</td>
</tr>
<tr>
<td>Identity Function $f(x) = x$ or $y = x$</td>
<td>$\mathbb{R}$</td>
<td>$\mathbb{R}$</td>
</tr>
<tr>
<td>Absolute Value Function $f(x) =</td>
<td>x</td>
<td>$ or $y =</td>
</tr>
<tr>
<td>Quadratic Function $f(x) = x^2$ or $y = x^2$</td>
<td>$(-\infty, \infty)$</td>
<td>$[0, \infty)$</td>
</tr>
<tr>
<td>Cubic Function $f(x) = x^3$ or $y = x^3$</td>
<td>$(-\infty, \infty)$</td>
<td>$(-\infty, \infty)$</td>
</tr>
<tr>
<td>Square Root Function $f(x) = \sqrt{x}$ or $y = \sqrt{x}$</td>
<td>$[0, \infty)$</td>
<td>$[0, \infty)$</td>
</tr>
</tbody>
</table>
Example 2: Consider the graphs shown below. Write your answers in interval notation.

a) The graph of \( g(x) \) is shown below.

Domain: __________________

Range: _________________

Over what interval(s) is \( g(x) \) increasing?

Over what interval(s) is \( g(x) \) decreasing?
b) The graph of $f(x)$ is shown below.

**Domain:** ___________________

**Range:** __________________

Over what interval(s) is $f(x)$ increasing?

Over what interval(s) is $f(x)$ decreasing?

---

**Section 12.3 – YOU TRY**

The Function $f(x)$ is shown below. Write your answers in Interval Notation.

**Domain:** ___________________

**Range:** __________________

Over what interval(s) is $f(x)$ increasing?

Over what interval(s) is $f(x)$ decreasing?
Section 12.4: Determining Domain from a Formula

The domain of a function is the set of all possible inputs. If we have a function of $x$, the domain is the set of all possible $x$-values. To determine the domain of our Toolkit functions, there are two rules to consider:

1. We cannot divide by 0.
2. When dealing with the set of real numbers we cannot take the square root of a negative number.

Example 2: Finding the domain of a Rational function: $f(x) = \frac{1}{x-3}$

<table>
<thead>
<tr>
<th>Table</th>
<th>Graph</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$f(x)$</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>-0.13</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-0.2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-0.33</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Undefined</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

Example 3: Finding the domain of a Square Root function: $f(x) = \sqrt{x + 2}$

<table>
<thead>
<tr>
<th>Table</th>
<th>Graph</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$f(x)$</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>Undefined</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>Undefined</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>Undefined</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1.4142</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.7321</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.2361</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.4495</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.6458</td>
<td></td>
</tr>
</tbody>
</table>
**Example 4:** Finding the domain of a **Quadratic function**.

Using the graph and table as a reference, find the domain of the function $f(x) = \frac{1}{4}x^2 - 4$.

<table>
<thead>
<tr>
<th>Table</th>
<th>Graph</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>$f(x)$</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>-1.75</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-3.75</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-3.75</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-1.75</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.25</td>
<td></td>
</tr>
</tbody>
</table>

**Example 5:** Identify the domain of the following functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Domain in Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2 + 3x - 10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 5x + 7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt{36 - 3x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt{2x - 8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt[3]{4x - 6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 19$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = x^3 - 4x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{x+1}{x-6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) =</td>
<td>x</td>
<td>- 3$</td>
</tr>
</tbody>
</table>
Identify the domain of the following functions. Write your answers in Interval Notation.

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Domain in Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 2x^3 + 8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p(t) =</td>
<td>5t</td>
<td>$</td>
</tr>
<tr>
<td>$h(x) = \sqrt{7 - x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(x) = \frac{3x + 1}{5x - 40}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 12.5: Average Rates of Change

Given any two points \((x_1, f(x_1))\) and \((x_2, f(x_2))\), the average rate of change between the points on the interval \(x_1\) to \(x_2\) is determined by computing the following ratio:

\[
\text{Average rate of change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}
\]

Units for the Average Rate of Change are always: \(\frac{\text{Output Units}}{\text{Input Unit}}\)

Units are read as “output units per input unit”

A function is linear if the average rate of change is constant.

Example 1: The function \(C(t)\) below gives the average cost, in dollars, of a gallon of gasoline \(t\) years after 2000. Find the average rate of change for each of the following time intervals. Interpret the meaning of your answers.

<table>
<thead>
<tr>
<th>(t)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C(t))</td>
<td>1.47</td>
<td>1.69</td>
<td>1.94</td>
<td>2.30</td>
<td>2.51</td>
<td>2.30</td>
<td>3.01</td>
<td>2.14</td>
</tr>
</tbody>
</table>

a) 2003 to 2008

b) 2006 to 2009

c) 2005 to 2007
Example 2: The total sales, in thousands of dollars, for three companies over 4 weeks are shown below. Total sales for each company are clearly increasing, but in very different ways. To describe the behavior of each function, we can compare its average rate of change on different intervals.

What do you notice about the values above for each function? What do these values tell you about each function?
1. Jose recorded the daily high temperature, D(n) (in degrees Fahrenheit), for his science class during the month of May in Scottsdale, Arizona. The table below represents the highest temperature on the nth day of May for that year. (n = 1 is May 1, n = 2 is May 2, and so on.) Show all of your work for each of the following and be sure to include correct units in your answers.

<table>
<thead>
<tr>
<th>n</th>
<th>D(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97</td>
</tr>
<tr>
<td>2</td>
<td>99</td>
</tr>
<tr>
<td>4</td>
<td>83</td>
</tr>
<tr>
<td>6</td>
<td>97</td>
</tr>
<tr>
<td>9</td>
<td>104</td>
</tr>
<tr>
<td>11</td>
<td>102</td>
</tr>
</tbody>
</table>

a. Find the average rate of change between n = 1 and n = 11. Write a sentence explaining the meaning of your answer.

b. Find the average rate of change between n = 9 and n = 11. Write a sentence explaining the meaning of your answer.

c. Find the average rate of change between n = 1 and n = 6. Write a sentence explaining the meaning of your answer.

2. The graph of the function H(t) shown below represents the height of a toy rocket over time. Determine the Average Rate of Change of the rocket over the following time intervals. Include units in your answers.

   ![Graph of a toy rocket height vs. time]

   a. [0, 1]
   b. [1, 3.5]
   c. [3.5, 6]
   d. [1, 6]
Section 12.6: Average Rates of Change of Functions and Graphs

Example 1: Use the graph to determine the Average Rate of Change over the given intervals.

Average Rate of Change from \( x = 0 \) to \( x = 1 \)

Average Rate of Change from \( x = 2 \) to \( x = 5 \)

Example 2: The function \( H(t) \) gives the height in feet of a hot air balloon after \( t \) seconds. Find the average rate of change from \( t = 1 \) to \( t = 9 \). Interpret the results.

\[ H(t) = 10\sqrt{t} \]
Complete the following problems. Show your work and simplify your answers.

a. The function $g(x)$ is shown below. Find the average rate of change of $g(x)$ over the interval $[0, 3]$.

![Graph of function g(x)]

b. Find the average rate of change of the function $f(x) = -x^2 + 6x - 1$ from $x = -1$ to $x = 2$. 


Unit 12: Answers to You-Try Problems

Section 12.1: a. Constant  b. Linear  c. Square Root  d. Identity  e. Constant  
f. Absolute Value  g. Quadratic

Section 12.2:

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Vertical Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x) = x^3 + 6x^2 - x + 8</td>
<td>Cubic</td>
<td>(0,8)</td>
</tr>
<tr>
<td>p(t) =</td>
<td>3x - 4</td>
<td></td>
</tr>
<tr>
<td>h(x) = 3√5x - 7</td>
<td>Cube Root (Radical)</td>
<td>(0, 3√-7)</td>
</tr>
<tr>
<td>g(x) = 1/(2x-16)</td>
<td>Rational</td>
<td>(0, -1/16)</td>
</tr>
</tbody>
</table>

Section 12.3: Domain (-∞,∞)  Range [-4,∞)  Increasing (-1,∞)  Decreasing (-∞,-1)

Section 12.4:

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Type</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x) = 2x^3 + 8</td>
<td>Cubic</td>
<td>(-∞,∞)</td>
</tr>
<tr>
<td>p(t) =</td>
<td>5t</td>
<td></td>
</tr>
<tr>
<td>h(x) = √7 - x</td>
<td>Radical (Square Root)</td>
<td>(-∞,7]</td>
</tr>
<tr>
<td>g(x) = 3x + 1</td>
<td>Rational</td>
<td>(-∞,8)U(8,∞)</td>
</tr>
</tbody>
</table>

Section 12.5:

1. 
   a. 0.5° F; Between May 1 and May 11, the daily high temperature increased at an average rate of 0.5° F each day.
   b. -1° F; Between May 9 and May 11, the daily high temperature decreased at an average rate of 1° F each day
   c. 0° F; Between May 1 and May 6, the daily high temperature remained constant

2. 
   a. 50 feet per second
   b. 20 feet per second
   c. -20 feet per second
   d. 0 feet per second

Section 12.6:  
   a. -2/3  
   b. 5
1. Complete the following table. Function Types: Quadratic, Cubic, Constant, Rational, Linear, Radical (Square Root), Radical (Cube Root), Identity, Exponential, and Absolute Value.

<table>
<thead>
<tr>
<th>Function Formula</th>
<th>Type of Function</th>
<th>Graph on Standard Viewing Window</th>
<th>Domain in interval notation</th>
</tr>
</thead>
</table>
| $f(x) = x^2 + 4x - 5$ | Type: Quadratic  
Reason: | | |
| $f(x) = x^3 - 4x^2 + 7$ | Type: Cubic  
Reason: | | |
| $f(x) = 2|x + 3| - 6$ | Type: Absolute Value  
Reason: | | |
| $f(x) = \sqrt{8 - 2x}$ | Type: Radical (Square Root)  
Reason: | | |
| $f(x) = \frac{1}{4}x - 6$ | Type: Rational  
Reason: | | |
| $f(x) = \frac{5}{x + 3}$ | Type: Rational  
Reason: | | |
2. For each of the following functions,
   - Identify the type of function
   - Determine the domain of the function
   - Determine the vertical intercept of the function.
   - Find the Average Rate of Change of the function from \( x = -2 \) to \( x = 3 \).

   a. \( f(x) = 5\sqrt{x} + 3 \)      
   b. \( f(x) = \sqrt{7 - 2x} \)

   c. \( f(x) = \frac{3}{\sqrt{x} + 3} \)   
   d. \( f(x) = 3x + 4 \)

   e. \( f(x) = \frac{4}{x-6} \)        
   f. \( f(x) = 5x^3 - 6x^2 + x - 11 \)

   g. \( f(x) = 3|x + 8| - 5 \)       
   h. \( f(x) = \frac{4x-12}{x^2-x-20} \)

3. The data below represent the number of times your friend’s embarrassing YouTube video has been viewed per hour since you uploaded it.

<table>
<thead>
<tr>
<th>Time, ( t ), in hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Views, ( V(t) ), in thousands of views</td>
<td>0</td>
<td>6200</td>
<td>11200</td>
<td>18600</td>
<td>26600</td>
</tr>
</tbody>
</table>

For each of the following, show all of your work and be sure to include correct units in your answer. Write a sentence explaining the meaning of your answer.

a) Determine the average rate of change from hours 0 to 1.

b) Determine the average rate of change from hours 1 to 2.

c) Determine the average rate of change from hours 0 to 4.

4. In the year 1987, an investment was worth $30,200. In the year 1996, this investment was worth $43,700. Determine the average rate of change in this situation. Write a sentence explaining its meaning.

5. In the year 1998, the surface elevation of Lake Powell was 3,843 feet above sea level. In the year 2001, the surface elevation of Lake Powell was 3,609 feet above sea level. Determine the average rate of change in this situation. Write a sentence explaining its meaning.
6. The graph of $A(m)$ below shows the amount of water in a play pool after $m$ minutes. Use the graph to determine the rate of change over the indicated time intervals. Show all of your work and include correct units in your answers. Write a sentence explaining the meaning of each answer.

- a) $[0,1]$  
- b) $[2,4]$  
- c) $[5,10]$  
- d) $[0,12]$  
- e) What do you notice about the values above? What do these values tell you about $A(m)$?

7. The graph of $D(t)$ below shows the distance Sally is from home (in miles) after $t$ minutes. Use the graph to determine the rate of change over the indicated time intervals. Show all of your work and include correct units in your answers. Write a sentence explaining the meaning of each answer.

- a) $[0,8]$  
- b) $[8,16]$  
- c) $[16,25]$
8. The graph of $H(t)$ below shows the height (in feet) of a toy rocket $t$ seconds after being launched into the air. Use the graph to determine the rate of change over the indicated time intervals. Show all of your work and include correct units in your answers. Write a sentence explaining the meaning of each answer.

a) $[0,1]$  

b) $[1,3.5]$  

c) $[3.5,6]$  

d) $[6,7]$  

e) $[1,6]$  

f) $[0,7]$  

9. The graph of $P(t)$ below shows the population (in thousands of people) of a town $t$ years after 2008. Use the graph to determine the rate of change over the indicated time intervals. Show all of your work and include correct units in your answers. Write a sentence explaining the meaning of each answer.

a) 2008 to 2017  

b) 2008 to 2010  

c) 2010 to 2015  

d) 2015 to 2017  

e) What do you notice about the values above? What do these values tell you about $P(t)$?

Extension

10. Find the Average Rate of Change of each function from $x = a$ to $x = a+h$. Simplify your answers.

a. $f(x) = 3x + 4$  

b. $f(x) = x^2 - 3x + 5$
Unit 12: Review

1. Complete the table. Write intercepts as ordered pairs and use interval notation for domain. Write your answers in exact form (no decimals). Write “DNE” if the answer does not exist.

<table>
<thead>
<tr>
<th></th>
<th>Vertical Intercept</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational</td>
<td>$f(x) = \frac{2x + 6}{x - 7}$</td>
<td></td>
</tr>
<tr>
<td>Cube Root</td>
<td>$f(x) = \frac{3}{\sqrt{x - 8}}$</td>
<td></td>
</tr>
<tr>
<td>Quadratic</td>
<td>$f(x) = x^2 + x - 12$</td>
<td></td>
</tr>
<tr>
<td>Square Root</td>
<td>$f(x) = \sqrt{3x + 15}$</td>
<td></td>
</tr>
<tr>
<td>Exponential</td>
<td>$f(x) = 17(1.085)^x$</td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>$f(x) = \frac{3}{7}x + 5$</td>
<td></td>
</tr>
<tr>
<td>Absolute Value</td>
<td>$f(x) =</td>
<td>x</td>
</tr>
</tbody>
</table>

2. The graph below shows the population of a town in the years after 2004. Use the graph to determine the average rate of change over the given time intervals.

a) 2004 to 2013

b) 2011 to 2013
3. In the space below, draw a graph that represents the type of function indicated.

![Graphs of identity, quadratic, and cube root functions]

4. The graph of $H(a)$ is given below. Use interval notation.

![Graph of $H(a)$]

<table>
<thead>
<tr>
<th>Domain: ____________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range: ______________</td>
</tr>
<tr>
<td>Increasing: __________</td>
</tr>
<tr>
<td>Decreasing: __________</td>
</tr>
</tbody>
</table>

5. Find the Average Rate of Change of each function over the interval $[-3, 5]$.
   a. $f(x) = 8 - 4x^2$
   
   b. $g(x) = 11 - 3x$
   
   c. $h(x) = 28$