# Introductory Algebra Student Workbook 

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## Lesson 1: Arithmetic Review

In this lesson we step back and review several key arithmetic topics that are extremely relevant to this course. As we work with algebraic expressions and equations, a good understanding of order of operations, fractions and signed numbers is critical.

## Mini-Lesson

Section 1.1: Order of Operations
Section 1.2: Fractions
Section 1.3: Operations on Fractions
Section 1.4: Signed Numbers

## Lesson 1 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 1

## Section 1.1: Order of Operations

## PEMDAS

If we are working with a mathematical expression that contains more than one operation, then we need to understand how to simplify. The acronym PEMDAS stands for Parentheses, Exponents, Multiplication, Division, $A$ ddition, Subtraction.

P Terms inside parenthesis () or brackets []
E Exponents and roots
MD Multiplication and division (from Left to Right).
AS Addition and subtraction (from Left to Right).

Use the order of operations to evaluate each of the following expressions.
Use your calculator to check your answers.
Example 1:
a. $(2 \cdot 5)^{2}$
b. $2 \cdot 5^{2}$
c. $10-7+1$
d. $10-(7+1)$

Example 2: $\quad 24 \div(4-2)^{3}$

Example 3: $\quad 4+5(1+12 \div 6)^{2}$

Example 4: $\frac{15-3}{1+5}$

## You Try

Use the order of operations to evaluate each of the following expressions. Use your calculator to check your answers.

1. $11-(12-2 \cdot 3) \div 3$
2. $\frac{6+8}{4-2}$

## Section 1.2: Fractions

## Improper Fractions and Mixed Numbers

To convert a mixed number to an improper fraction:

1. Multiply the denominator and the whole number.
2. Add the numerator.
3. Write the result over the denominator.

Example 1: Express as an improper fraction.
$3 \frac{2}{7}$
$12 \frac{1}{3}$

To convert an improper fraction to a mixed number:

1. Divide the numerator by the denominator.
2. The quotient becomes the whole number part of the mixed number.
3. Write the remainder over the denominator.

Example 2: Express an improper fraction as a mixed number.

$$
\frac{42}{5} \quad \frac{53}{9}
$$

Equivalent Fractions


Example 3: Find two fractions equivalent to $\frac{2}{7}$

## Fractions in Simplest Form

Example 4: Write the following fractions in simplest form.
$\frac{3}{18} \quad \frac{42}{54}$

## ONE and ZERO

Example 5: $\frac{1}{4}=\quad \frac{4}{1}=$

$$
\frac{4}{4}=\quad \frac{0}{4}=
$$

$$
\frac{4}{0}=
$$

## You Try

3. Reduce the fraction $\frac{24}{36}$ to lowest terms.
4. Rewrite the mixed number $4 \frac{1}{5}$ as an improper fraction.
5. Rewrite the improper fraction $\frac{35}{11}$ as a mixed number. $\qquad$
6. Find two fractions equivalent to $\frac{3}{5}$

## Section 1.3: Operations on Fractions

## Addition and Subtraction of Fractions

## Adding and Subtracting Fractions:

1. Rewrite mixed numbers and whole numbers as improper fractions.
2. Find a common denominator.
3. Rewrite the fractions as equivalent fractions with the common denominator.
4. Add or subtract the numerators.
5. Be sure to reduce your answer to simplest form!

Example 1: Perform the indicated operations.
a. $\frac{1}{2}+\frac{1}{3}$
b. $\frac{11}{15}-\frac{5}{12}$
c. $4 \frac{3}{5}-1 \frac{5}{6}$
d. $2-\frac{8}{5}$

## Multiplication and Division of Fractions

## Multiplying Fractions:

1. Rewrite mixed numbers and whole numbers as improper fractions.
2. Multiply straight across. (Multiply the numerators with the numerators, and the denominators with the denominators.) NOTE: There is no need to find a common denominator when multiplying.
3. Be sure to reduce your answer to simplest form!

Example 2: Multiply. Write your answers in simplest form.
a. $\frac{2}{3} \times \frac{3}{4}$
b. $\frac{12}{25} \times \frac{35}{48}$
c. $\frac{7}{8} \times 5$
d. $3 \frac{1}{5} \times 1 \frac{1}{9}$

## Dividing Fractions:

1. Rewrite mixed numbers and whole numbers as improper fractions.

NOTE: There is no need to find a common denominator when dividing.
2. Change the second fraction (the divisor) to its reciprocal.
3. Multiply.
4. Be sure to reduce your answer to simplest form!

Example 3: Divide. Write your answers in simplest form.
a. $\frac{1}{2} \div \frac{3}{5}$
b. $8 \div \frac{4}{5}$

## Order of Operations with Fractions

Example 4: Perform the indicated operations. $\frac{1}{2}+\frac{3}{2} \cdot \frac{2}{5}$

## You Try

7. Perform the indicated operations. Each answer must be written as a reduced fraction. Where appropriate, write your answer as both a mixed number and an improper fraction.
a. $\frac{3}{5}+\frac{2}{3}$
b. $\frac{3}{5}\left(\frac{2}{3}\right)$
c. $\frac{3}{5} \div \frac{2}{3}$
d. $3-\frac{12}{5}$
e. $\frac{3}{7} \div 5$
f. $\frac{3}{4} \div \frac{4}{5} \times \frac{5}{6}$

## Section 1.4: Signed Numbers

The Number Line


Absolute Value
The ABSOLUTE VALUE of a number is the distance that number is from 0 on the number line.
Example 1: Find the absolute value.
a. $|-3|$
b. $|3|$
c. $-|-3|$
d. $|0|$

## MATHEMATICAL OPERATIONS WITH SIGNED NUMBERS

## Some hints for working with signed numbers:

- Use () to separate numbers with negative signs.
- When two signs are given together, use these rules to resolve the signs:

$$
(-)(-)=+\quad(-)(+)=-\quad(+)(-)=-\quad(+)(+)=+
$$

- Use the number line to add and subtract.

Example 2: Perform the indicated operations.
a. $3+(-2)$
b. $-3+2$
c. $-3-(-2)$
d. $-3+(-2)$

Example 3: Multiply and divide.
a. $(-5)(-6)$
b. $3(-4)$
c. $\frac{-24}{8}$
d. $\frac{2}{3}\left(-\frac{1}{5}\right)$

Example 4: Evaluate the following exponents:
$(-5)^{2}=$ $\qquad$
$-5^{2}=$ $\qquad$
$(-5)^{3}=$ $\qquad$
$-5^{3}=$ $\qquad$

Example 5: Perform the indicated operations.

$$
-8 \div(-2)^{3}-(-3)-5^{2}
$$

## SIMPLIFIED FORM FOR A SIGNED FRACTION

The following fractions are all equivalent (meaning they have the same value):

$$
\frac{-1}{2}=\frac{1}{-2}=-\frac{1}{2}
$$

Notice that only the placement of the negative sign is different.
HOWEVER, only the last one, $-\frac{1}{2}$, is considered to be in simplest form.

## You Try

8. Find the absolute value:
a. $|-5|$
b. $-|-5|$
9. Perform the indicated operations. Show your work, and use your calculator to check.
a. $(-2)^{3}-2^{3}$
b. $6+12 \div 3 \cdot 4-(-2)-4^{2}$
$\qquad$

## Lesson 1 Practice Problems

## Skills Practice

1. Evaluate using the correct order of operations. Show all of your work. Use your graphing calculator to check your answer.
a. $8 \times 3^{2} \times 2 \div 4$
b. $24 \div(1+2)^{3}$
c. $20-(8-2) \div 3 \cdot 4$
d. $10 \cdot 3^{2}+\frac{15-3}{3 \cdot 2}$
e. $\left(\frac{8+2}{7-2}\right)^{2}$
f. $2+4 \times 8-(2+3)^{2}$
2. Express the following fractions as improper fractions. Write your answer in simplest form.
a. $2 \frac{3}{8}$
b. $-2 \frac{3}{4}$
c. $4 \frac{2}{6}$
3. Express the following fractions as mixed numbers. Write your answer in simplest form.
a. $\frac{43}{8}$
b. $\frac{38}{12}$
c. $\frac{70}{6}$
4. Write an equivalent fraction for each of the following:
a. $\frac{4}{9}=\frac{}{27}$
b. $\frac{6}{7}=\frac{36}{}$
c. $\frac{1}{3}=\frac{}{33}$
5. Write each of the following in simplest form.
a. $\frac{54}{72}$
b. $\frac{165}{345}$
c. $4 \frac{12}{28}$
6. Show the each step involved in evaluating each of the following. Write your answers in simplest form.
a. $\frac{1}{6}+\frac{2}{9}$
b. $\frac{5}{8}-\frac{6}{12}$
c. $\frac{1}{3}+\frac{2}{7}$
d. $\frac{8}{9}-\frac{6}{12}$
e. $2 \frac{3}{4}+3 \frac{4}{5}$
f. $2 \frac{2}{5}-1 \frac{1}{3}$
7. Evaluate each of the following. Show all steps. Write your answers in simplest form.
a. $\frac{24}{3} \cdot \frac{27}{8}$
b. $8 \times \frac{3}{24}$
c. $\frac{1}{4} \cdot \frac{3}{5} \cdot \frac{2}{9}$
d. $\frac{24}{3} \div \frac{8}{3}$
e. $\frac{3}{5} \div \frac{9}{15}$
f. $2 \frac{1}{3} \div 1 \frac{1}{2}$
8. Evaluate using the correct order of operations. Show all of your work. Use your graphing calculator to check your answer.
a. $(-2)^{2}-2^{2}$
b. $2(-3)^{3} \times 8 \div 4$
c. $-11(5-9)^{2}+(-4)(5)(-7) \div 2$
d. $-2(-5)^{2}-4^{2}-20 \div(-4)$
e. $(-4)^{2}+12 \div(-3)(9)$
f. $\frac{8-(1+3)^{2}}{4-(-5)}$

## Applications

9. A new 2012 Chevrolet Camaro convertible has a base MSRP of $\$ 35,000$. With good credit, and a $\$ 5,500$ down payment, you can finance this car for 60 months for $\$ 573$ per month. Calculate your total payment (including the down payment) over the 60 -month time period. Show all of your work. Write your answer in a complete sentence.
10. Abie makes $\$ 39,000$ a year, and spends about $\$ 250$ each month on entertainment. What fraction of her annual income is spent on entertainment? Show all of your work. Write your answer in a complete sentence.
11. There are 10 men and 14 women in Professor Smith's Introductory Algebra class. What fraction of the students in the class are women? Show all of your work. Write your answer in a complete sentence.
12. Michelle wants to make cupcakes for her daughter's birthday. The recipe calls for $3 / 4$ cup of brown sugar, $11 / 2$ cups of white sugar, and 2 cups of powdered sugar, and will make 12 cupcakes. How much sugar will be in each cupcake? Show all of your work. Write your answer in a complete sentence.
13. Judy took Jen and Bill to the casino. Bill and Jen each won $\$ 100$ playing the nickel slots. To say thanks, Jen gave Judy $1 / 7^{\text {th }}$ of her winnings and Bill gave Judy $1 / 9^{\text {th }}$ of his winnings. Who gave Judy more money? Explain your decision without using any calculations.
14. Jack and Jill each bought 100 pounds of cashews. Jack divided his cashews into 23 equal amounts and put them in paper bags. Jill divided her cashews into 18 equal amounts and put them in paper bags. To celebrate, each ate a bag of cashews. Now, Jack has $22 / 23$ of his paper bags and Jill has 17/18 of her paper bags. Who has more cashews left? Show all of your work. Write your answer in a complete sentence.

## Extension

15. Find the reciprocal of each of the numbers below.
a. $\frac{2}{3}$
b. $-\frac{7}{9}$
c. 8
d. -8
e. $5 \frac{1}{2}$
f. Why does zero not have a reciprocal?
16. At a store, there is a display of 240 boxes of cereal. Of the 240 boxes, $3 / 5$ are brand A and $2 / 5$ are brand B. How many boxes of brand B cereal must be added so that the display has $1 / 2$ of each brand. Explain.
$\qquad$ Date: $\qquad$

## Lesson 1 Assessment

1. Evaluate using the correct order of operations. Show all of your work. Use your graphing calculator to check your answer.
a. $\frac{1}{2} \div \frac{2}{3} \times \frac{3}{4}$
b. $\frac{8-(1-5)^{2}}{5 \times 2}$
2. Sara buys a bag of candy. In the bag, $1 / 2$ of the candies are red, $1 / 5$ are green, and the remainder are white. What fraction of the candies are white? Show all of your work. Write your answer in a complete sentence.
3. Fred and Wilma purchase a new home. They make a down payment of $\$ 20,000$ and take out a mortgage on the balance. To pay off the loan, they agree to make monthly payments of $\$ 1,022$ for the next thirty years. Calculate their total payment (including the down payment) over this thirty-year time period. Show all of your work. Write your answer in a complete sentence.
4. Below the tick marks on the graph, place the numbers -5 to 5 , in order from left to right. Place a dot on the graph for each of the numbers in the list and label above the dot with the number (exactly as it appears on the list).

$$
\frac{7}{2}, \quad-2^{2}, \quad-|-2|, \quad \frac{0}{8}
$$



## Lesson 2: Introduction to Variables

In this lesson we begin our study of algebra by introducing the concept of an unknown quantity, and how to represent that quantity as a variable in an algebraic expression. We then take a closer look at algebraic expressions to learn what they are comprised of, how they are defined, as well as how to evaluate and simplify them.

## Mini-Lesson

Section 2.1: Evaluating Algebraic Expressions
Section 2.2: Some Vocabulary
Section 2.3: Like Terms
Section 2.4: The Distributive Property
Section 2.5: Simplifying Algebraic Expressions

## Lesson 2 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

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## Mini-Lesson 2

## Section 2.1: Evaluating Algebraic Expressions

## Evaluate the algebraic expression. Use your calculator to CHECK your answer.

Example 1: Evaluate $a^{2}-b^{2}$ given $a=-5$ and $b=-3$.

Example 2: Evaluate $-a^{2}-(b-c)$ given $a=-5, b=4$, and $c=-2$.

## Application

Example 3: The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you get the most benefits and reduce the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate (pulse) is about 80 percent of your maximum heart rate. The formula $M=0.8(220-A)$, gives the recommended maximum heart rate, $M$, in beats per minute, for a person who is $A$ years of age. What is the recommended maximum heart rate for a person who is 40 years old?

| GIVEN: | GOAL: |
| :--- | :--- |
| STRATEGY: | CHECK: |
| SOLUTION: |  |
|  |  |
|  |  |
| FINAL RESULT AS A COMPLETE SENTENCE: |  |

1. Evaluate $b^{2}-4 a c$ given $a=5, b=-1$, and $c=2$.
2. A golfer strikes a golf ball. The height, $H$ (in feet), of the ball above the ground after $t$ seconds is given by the equation $H=-16 t^{2}+80 t$. Determine the height of the ball after 3 seconds.

| GIVEN: | GOAL: |
| :--- | :--- |
| STRATEGY: | CHECK: |
| SOLUTION: |  |

## Section 2.2: Some Vocabulary

## Definitions

Terms: Parts of an algebraic expression separated by addition or subtraction (+ or - ) symbols.
Constant Term: A number with no variable factors. A term whose value never changes.

Example 1: Consider the algebraic expression $4 x^{5}+3 x^{4}-22 x^{2}-x+17$
a. List the terms. $\qquad$
b. Identify the constant term. $\qquad$

Factors: Numbers or variabless that are multiplied together.
Coefficient: The numerical factor in a term.

Example 2: Complete the table below.

|  | $-4 m$ | $-x$ | $\frac{1}{2} b h$ | $\frac{2 r}{5}$ |
| :---: | :--- | :--- | :--- | :--- |
| List the Factors |  |  |  |  |
| Identify the <br> Coefficient |  |  |  |  |

Example 3: Consider the algebraic expression $5 y^{4}-8 y^{3}+y^{2}-\frac{y}{4}-7$.
a. How many terms are there? $\qquad$
b. Identify the constant term. $\qquad$
c. What is the coefficient of the first term? $\qquad$
d. What is the coefficient of the second term? $\qquad$
e. What is the coefficient of the third term? $\qquad$
f. List the factors of the fourth term. $\qquad$

## You Try

3. Consider the algebraic expression $2 m^{3}+m^{2}-2 m-8$
a. How many terms are there?
b. Identify the constant term. $\qquad$
c. What is the coefficient of the first term? $\qquad$
d. What is the coefficient of the second term? $\qquad$
e. List the factors of the third term. $\qquad$

## Section 2.3: Like Terms

Terms whose variable factors (letters and exponents) are exactly the same are called LIKE TERMS.

## Identify the Like Terms

Example 1: Identify the like terms in each of the following expressions.
a. $3 a-6 a+10 a-a$
b. $5 x-10 y+6 z-3 x$
c. $7 n+3 n^{2}-2 n^{3}+8 n^{2}+n-n^{3}$

## Combine Like Terms

Example 2: Combine the like terms.
a. $3 a-6 a+10 a-a$
b. $5 x-10 y+6 z-3 x$
c. $7 n+3 n^{2}-2 n^{3}+8 n^{2}+n-n^{3}$

## You Try

4. Combine the like terms.
a. $3 x-4 x+x-8 x$
b. $-5+2 a^{2}-4 a+a^{2}+7$

## Section 2.4: The Distributive Property $a(b+c)=a b+a c$

Use the Distributive Property to Expand Each of the Following Expressions
Example 1: $5(2 x+4)$

Example 2: $-3\left(x^{2}-2 x+7\right)$

Example 3: $-\left(5 x^{4}-8\right)$

Example 4: $\frac{2}{5}\left(\frac{x}{4}-\frac{1}{3}\right)$

## You Try

5. Use the Distributive Property to expand the algebraic expression.

$$
-3\left(5 x^{2}-2 x+8\right)
$$

## Section 2.5: Simplifying Algebraic Expressions

Step 1: Simplify within parentheses.
Step 2: Use the distributive property to eliminate parentheses.
Step 3: Combine like terms.

## Simplify Completely

Example 1: Simplify the following algebraic expressions. Show all possible steps.
a. $-3(2 x-4)-(3 x+8)$
b. $3[2-(x-5)]-(4 x-10)$
c. $\frac{8-5 x}{2}$
d. $\frac{9-3(2 x-5)}{-6}$

Simplify completely. Show all steps.
6. $2\left(7 x^{2}+3 x+2\right)-\left(8 x^{2}-7\right)$
7. $\frac{2(x-6)+8}{2}$
$\qquad$

## Lesson 2 Practice Problems

## Skills Practice

1. Evaluate the following expressions for the given values. Show all of your work. Use your graphing calculator to check your answers.
a. $3 x^{2}-10$ for $x=-2$
b. $5-2 x$ for $x=-3$
c. $\frac{1}{2} b h$ for $b=8, h=4.5$
d. $3 x^{2}+2 x-1$ for $x=-1$
e. $x^{2}-y^{2}$ for $x=-3, y=-2$
f. $2(x-4)+3\left(y^{2}+2\right)$ for $x=5, y=3$
2. Complete the table below.

|  | $5 t$ | $-3 a b c$ | $-y$ | $x$ | $\frac{3}{5} x$ | $\pi d$ | $\frac{4 x}{7}$ | $\frac{m}{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Identify the <br> Coefficient |  |  |  |  |  |  |  |  |

3. Consider the algebraic expression $5 n^{8}-n^{5}+n^{2}+\frac{n}{8}-1$
4. How many terms are there? $\qquad$
5. Identify the constant term. $\qquad$
6. What is the coefficient of the first term? $\qquad$
7. What is the coefficient of the second term? $\qquad$
8. What is the coefficient of the third term? $\qquad$
9. List the factors of the fourth term. $\qquad$
10. Consider the algebraic expression $w^{3}-w^{2}-\frac{2 w}{3}+3$
a. How many terms are there? $\qquad$
b. Identify the constant term. $\qquad$
c. What is the coefficient of the first term? $\qquad$
d. What is the coefficient of the second term? $\qquad$
e. What is the coefficient of the third term? $\qquad$
11. Identify and combine the like terms.
a. $3 d-5 d+d-7 d$
b. $3 x^{2}+3 x^{3}-9 x^{2}+x-x^{3}$
c. $a-2 b+4 a+b-(-2 b)$
d. $\frac{2}{5} r-\frac{2}{3} r+r$
12. Apply the distributive property to expand the following expressions.
a. $6(4 x-8)$
b. $-5\left(6 w^{2}-3 w+1\right)$
c. $-\left(4 y^{2}+3 y-8\right)$
d. $\frac{3}{4}\left(\frac{2}{5} x+\frac{7}{12}\right)$
e. $\frac{1}{3}\left(\frac{3}{4} b-5\right)$
f. $-2\left(n^{2}-5 n+\frac{1}{4}\right)$
13. Simplify by using the distributive property and combining like terms. Show all steps.
a. $\left(5 x^{2}+3 x-6\right)-(3 x+6)$
b. $3\left(2 x^{2}-x+3\right)+2$
c. $2 a+3 a b-5 a+8 a b+3 b$
d. $12+3 x^{2}+4 x-2 x^{2}-x-6$
e. $5(2 x+3)+4(3 x-7)$
f. $-2\left(4 x^{2}+3 x-2\right)-\left(x^{2}-6\right)$
14. Simplify completely. Show all steps.
a. $\frac{12-9 x}{3}$
b. $\frac{21 m-18}{6}$
c. $\frac{3(4 a-8)+2}{2}$
d. $\frac{3(10 x-4)+6}{6}+3 x+1$

## Applications

9. The formula to convert from Fahrenheit to Celsius is $C=\frac{5}{9}(F-32)$. The temperature on a summer day in Phoenix, Arizona is $113^{\circ} \mathrm{F}$. What would this temperature be in degrees Celsius? Show all work, and write your answer in a complete sentence.
10. Isabel has a headache, and takes 500 mg of Tylenol. The amount, $A$, of Tylenol remaining in her body after $n$ hours is given by the formula $A=500(0.882)^{n}$. How much of the Tylenol remains in her body after 4 hours? Show all work, and round your answer to the nearest hundredth. Write your answer in a complete sentence.
11. A person's Body Mass Index (BMI) is given by the formula $B M I=\frac{703 W}{H^{2}}$, where $W$ is the weight of the person in pounds, and $H$ is the person's height, measured in inches. If a person is 5 feet 7 inches tall, and weighs 142 pounds, what is that person's BMI? Show all of your work. Round your answer to the nearest tenth. Write your answer in a complete sentence.
12. The formula for the volume, $V$, of a cylinder of radius $r$ and height $h$ is $V=\pi r^{2} h$. Determine the volume of a cylinder with radius 4 inches and height 10 inches. Use the $\pi$ key on your graphing calculator. Round your answer to the nearest hundredth, and include appropriate units in your answer.
13. The formula for the surface area, $S$, of a cylinder of radius $r$ and height $h$ is $S=2 \pi r^{2}+2 \pi r h$. Determine the surface area of a cylinder with radius 2.3 feet and height 4.2 feet. Use the $\pi$ key on your graphing calculator. Round your answer to the nearest hundredth, and include appropriate units in your answer.
14. Simple interest is given by the formula $A=P+\operatorname{Prt}$, where $A$ is the accrued value of the investment after $t$ years, and $P$ is the starting principal invested at an annual percentage rate of $r$, expressed as a decimal. Sally buys a $\$ 1,000$ savings bond that pays $4 \%$ simple interest each year. How much will the bond be worth after 5 years?
15. The formula for compound interest is $A=P(1+r)^{t}$, where $A$ is the accrued amount after $t$ years, $P$ is the starting principal, and $r$ is the annual interest rate expressed as a decimal. If you invest $\$ 1000$ at an interest rate of $7 \%$ and leave it there for 30 years, what would your ending balance be? Round your answer to the nearest cent.

## Extension

16. The formula when interest is compounded $n$ times per year is

$$
A=P\left(1+\frac{r}{n}\right)^{n t}
$$

where $A$ is the accrued amount after $t$ years, $P$ is the starting principal, and $r$ is the interest rate, expressed as a decimal, that is compounded $n$ times per year. If you invest $\$ 1000$ at an interest rate of $7 \%$, and leave it there for 30 years, determine your ending balance if the interest is compounded
a. Twice each year.
b. Monthly.
c. Daily.
d. Explain what happens to the ending balance as the number of compoundings increases. Why does this occur?
$\qquad$ Date: $\qquad$

## Lesson 2 Assessment

1. Evaluate the following expressions for the given values. Show all of your work. Use your graphing calculator to check your answers.
a. $4 x^{2}-x+3$ for $x=-5$
b. $x^{2}-y^{2}$ for $x=-5, y=3$
2. Simple interest is given by the formula $A=P+\operatorname{Prt}$, where $A$ is the accrued value of the investment after $t$ years, and $P$ is the starting principal invested at an annual percentage rate of $r$, expressed as a decimal. Amber buys a $\$ 5000$ savings bond that pays $3 \%$ simple interest each year. How much will the bond be worth after 20 years? Show all of your work and write your answer in a complete sentence.
3. Simplify by using the distributive property and combining like terms. Show all steps.
a. $3\left(a^{2}+5 a-1\right)-(12 a-3)$
b. $\frac{5(6 x+4)-9}{3}$

## Lesson 3: Polynomials and Exponents, Part 1

When working with algebraic expressions, variables raised to a power play a major role. In this lesson, we look in depth at variables with exponents and how to work with them. We then look at polynomials that contain variables with exponents.

## Mini-Lesson

Section 3.1: Polynomials
Section 3.2: Operations on Polynomials
Section 3.3: Properties of Exponents
Section 3.4: Multiplication of Polynomials
Section 3.5: Applications from Geometry

## Lesson 3 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 3

## Section 3.1: Polynomials

## Definitions

Polynomial: An algebraic expression composed of the sum of terms containing a single variable raised to a positive integer exponent.

Monomial: A polynomial consisting of one term.
Binomial: A polynomial consisting of two terms.
Trinomial: A polynomial consisting of three terms.
Leading Term: The term that contains the highest power of the variable in a polynomial.
Leading Coefficient: The coefficient of the leading term.
Degree: The highest exponent in a polynomial.

Example 1: Complete the table.

| Polynomial | Name | Leading <br> Coefficient | Constant Term | Degree |
| :---: | :---: | :---: | :---: | :---: |
| $24 a^{6}+a^{2}+5$ |  |  |  |  |
| $2 m^{3}+m^{2}-2 m-8$ |  |  |  |  |
| $5 x^{2}+x^{3}-7$ |  |  |  |  |
| $-2 x+4$ |  |  |  |  |
| $4 x^{3}$ |  |  |  |  |

1. Complete the table.

| Polynomial | Name | Leading <br> Coefficient | Constant Term | Degree |
| :---: | :---: | :---: | :---: | :---: |
| $3 n^{2}-2 n+8$ |  |  |  |  |
| $4 x$ |  |  |  |  |
| $x^{2}-7$ |  |  |  |  |

## Section 3.2: Operations on Polynomials

## Addition of Polynomials

Example 1: Add. $\left(3 n^{2}-2 n+8\right)+\left(3 n^{3}-7 n^{2}-n-9\right)$

## Subtraction of Polynomials

Example 2: Subtract. $\left(a^{3}+5 a+11\right)-\left(4 a^{3}+6 a^{2}-a+1\right)$

## Combine and Simplify

Example 3: Perform the indicated operations. Simplify.

$$
(3 x-1)-\left(x^{2}-x-9\right)+\left(4 x^{3}+x^{2}-7 x+2\right)
$$

## You Try

2. Perform the indicated operations. Simplify.

$$
(5 x+8)+\left(3 x^{2}-4 x-1\right)-\left(5 x^{3}+3 x^{2}-4 x+6\right)
$$

## Section 3.3: Properties of Exponents

## Given any real numbers $a, b, c, m$, and $n$

$n^{1}=$ $\qquad$
$1^{n}=$
$n^{0}=\underline{n \neq 0}$
$0^{n}=\frac{}{n \neq 0}$

Example 1: Evaluate and simplify the following expressions.
Assume $\mathrm{x} \neq 0, \mathrm{x} \neq-1 / 2, \mathrm{a} \neq 0, \mathrm{~b} \neq 0$, and $\mathrm{c} \neq 0$.
a. $5 x^{0}$
b. $(2 x+1)^{0}$
c. $a^{0}+b^{0}+c^{0}$

Example 2: Simplify the following expressions.
a. $n^{3} n^{9}$
b. $b^{5} \cdot b^{4} \cdot b$
c. $5 x^{2} y^{5}\left(7 x y^{9}\right)$

Example 3: Simplify the following expressions.
a. $\left(x^{3}\right)^{9}$
b. $5 b^{2}\left(b^{5}\right)^{8}$

```
Raising a Product to a Power: }(ab\mp@subsup{)}{}{n}=\mp@subsup{a}{}{n}\mp@subsup{b}{}{n
```

Example 4: Simplify the following expressions.
a. $(5 x)^{2}$
b. $\left(x^{3} y^{2}\right)^{9}$
c. $\left(-8 a b^{5}\right)^{2}$
d. $5\left(-2 w^{7}\right)^{3}$
e. $5 n^{4}\left(-3 n^{3}\right)^{2}$
3. Simplify the following expressions.
a. $3\left(-2 x^{4}\right)^{2}$
b. $2 x\left(-3 x^{2}\right)^{3}$
c. $g^{3} \cdot g^{4} \cdot g^{5}$
d. $2 n^{0}$

## Section 3.4: Multiplication of Polynomials

## Multiplication of Monomials

Example 1: Multiply and simplify.

$$
\left(3 x^{5}\right)\left(-2 x^{9}\right)
$$

The Distributive Property
Example 2: Expand and simplify.

$$
5 x^{3}\left(2 x^{5}-4 x^{3}-x+8\right)
$$

## Multiplication of Polynomials

Example 3: Multiply and simplify.
a. $(x+3)(x+4)$
b. $(m-5)(m-6)$
c. $(2 d-4)(3 d+5)$
d. $(x-2)\left(x^{2}+2 x-4\right)$

## Squaring a Binomial

Example 4: Multiply and simplify.
a. $(n+5)^{2}$
b. $(3-2 a)^{2}$
4. Multiply and simplify.
a. $-3 x^{2}\left(x^{5}+6 x^{3}-5 x\right)$
b. $(3 x-4)(5 x+2)$
c. $(2 p-5)^{2}$

## Section 3.5: Applications from Geometry

Example 1: Write a polynomial in simplest form that represents the area of the square.

## SOLUTION:



Region $A$ has area: $x \cdot x=x^{2}$
Regions $B$ and $C$ each have area: $x \cdot y=x y$

Region D has area: $y \cdot y=y^{2}$

$$
\begin{aligned}
\text { Total Area } & =x^{2}+x y+x y+y^{2} \\
& =y^{2}+x^{2}+2 x y
\end{aligned}
$$

Example 1 (another way): Write a polynomial in simplest form that represents the area of the square.

## SOLUTION:

The total length of each side is $x+y$.


$$
\begin{aligned}
\text { Total Area } & =(x+y)(x+y) \\
& =x^{2}+x y+y x+y^{2}
\end{aligned}
$$

Note that $x y$ and $y x$ are like terms: $x y+y x=2 x y$
Total Area $=x^{2}+2 x y+y^{2}$

Example 2: Write a polynomial in simplest form that represents the area of the shaded region.


SOLUTION: To find the area of the shaded region we find the area of the big square and subtract the area of the little square.

The big square has area: $y \cdot y=y^{2}$
Each little square has area: $x \cdot x=x^{2}$
Area of the shaded region $=y^{2}-4 x^{2}$
5. Write a polynomial in simplest form that represents the total area of the figure shown below.

6. Write a polynomial in simplest form that represents the area of the dark blue region of the figure shown below.

$\qquad$

## Lesson 3 Practice Problems

## Skills Practice

1. Complete the table below.

| Polynomial | Name | Leading <br> Coefficient | Constant <br> Term | Degree |
| :---: | :---: | :---: | :---: | :---: |
| $5 n^{8}-n^{5}+1$ |  |  |  |  |
| $x-5$ |  |  |  |  |
| $8 r^{2}$ |  |  |  |  |

2. Simplify completely. Show all steps, and box your answers.
a. $3 x^{0}+2 x^{0}$
b. $5(3 n)^{0}$
c. $y^{3} \cdot y^{7} \cdot y$
d. $4(-2 x)^{3}-4 x(-2)^{3}$
e. $4\left(-5 w^{8}\right)^{2}$
f. $10 p^{3}\left(-5 p^{7}\right)^{2}$
g. $2 a^{3} b\left(3 a b^{5}\right)^{2}$
h. $(5 x-7)^{0}$
i. $(-4 x)^{2}+4 x^{2}$
j. $\left(3 x^{4}\right)^{3}-\left(5 x^{6}\right)^{2}$
3. Multiply and simplify completely. Show all steps, and box your answers.
a. $4 x^{2}(3 x-5)$
b. $4 a^{2}\left(3 a^{2}-2 a-5\right)$
c. $(p+5)(p+7)$
d. $(x+2)(x-2)$
e. $(2 x-4)(3 x-5)$
f. $(5 w-8)(3 w+11)$
g. $(x+2)^{2}$
h. $(2 x-4)^{2}$
i. $(x-4)\left(x^{2}+x-5\right)$
j. $3(x+2)(x+4)$
k. $4(x+2)^{2}$
4. $(q-2)^{3}$
5. Evaluate the algebraic expression $x^{2}$ given $x=-7$. Show your work.
6. Evaluate the algebraic expression $5 x^{3}$ given $x=-2$. Show your work.
7. Evaluate the algebraic expression $(5 x)^{2}$ given $x=-2$. Show your work.
8. Evaluate the algebraic expression $5(2 x)^{2}$ given $x=-3$. Show your work.

## Applications

8. Write an expression that represents the total area of the figure shown below. Simplify completely.

9. Write an expression that represents the area of the shaded region of the figure shown below. Simplify completely.

10. Write an expression that represents the total area of the figure shown below. Simplify completely.

11. Write an expression that represents the perimeter of the figure shown below. Simplify completely.

12. Write a polynomial in simplest form that represents the volume of the figure shown below.


## Extension

13. If possible, simplify each of the following by combining like terms or using properties of exponents.
a. $2 n^{5}+3 n^{5}=$
b. $2 n^{5} \cdot 3 n^{5}=$
c. $3 n^{3}+3 n^{5}=$ $\qquad$ d. $3 n^{3} \cdot 3 n^{5}=$ $\qquad$

## Lesson 3 Assessment

1. Simplify completely. Show all steps, and box your answers.
a. $(-5 x)^{2}-5 x^{2}$
b. $4 x^{2}\left(8 x^{2}-5 x-3\right)$
c. $(3-5 x)^{2}$
d. $4 p\left(-5 p^{3}\right)^{2}$
2. Write an algebraic expression that represents the total area of the figure shown below. Simplify completely. Show your work.

3. Consider the polynomial $n^{2}-7 n-11$
4. Is this a monomial, binomial, or trinomial?
5. Identify the constant term. $\qquad$
6. What is the leading coefficient? $\qquad$
7. What is the degree of this polynomial? $\qquad$
8. Identify the coefficient of the second term. $\qquad$

## Lesson 4: Polynomials and Exponents, Part 2

When working with algebraic expressions, variables raised to a power play a major role. In this lesson, we look in depth at variables with exponents and how to work with them. We then look at polynomials that contain variables with exponents.

## Mini-Lesson

Section 4.1: Division Properties of Exponents
Section 4.2: Negative Exponents
Section 4.3: Division of Polynomials
Section 4.4: Scientific Notation

## Lesson 4 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

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$\qquad$

## Mini-Lesson 4

## Section 4.1: Division Properties of Exponents

$$
\text { The Division Property: } \quad \frac{a^{m}}{a^{n}}=a^{m-n} \quad a \neq 0
$$

Example 1: Simplify the following expressions. Variables represent nonzero quantities.
a. $\frac{x^{50}}{x^{4}}$
b. $\frac{4 a^{10} b^{5}}{6 a b^{2}}$

Raising a Quotient to a Power: $\left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}} \quad b \neq 0$
Example 2: Simplify the following expressions. Variables represent nonzero quantities.
a. $\left(\frac{5}{7}\right)^{2}$
b. $\left(\frac{x^{5}}{y^{3}}\right)^{4}$
c. $\left(\frac{-4 t^{10}}{u^{6}}\right)^{2}$

## You Try

1. Simplify the following expressions. Variables represent nonzero quantities.
a. $\left(\frac{3 a^{10}}{7}\right)^{2}$
b. $\frac{6 x^{3} y^{8}}{9 x y^{5}}$

## Section 4.2: Negative Exponents

| For any real numbers $a \neq 0, b \neq 0$, and $m$ : |
| :--- |
| $\qquad\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m} \quad a^{-m}=\frac{1}{a^{m}} \quad \frac{1}{a^{-m}}=a^{m}$ |

Example 1: Rewrite each of the following with only positive exponents.
Variables represent nonzero quantities.
a. $x^{-3}$
b. $\frac{1}{x^{-3}}$
c. $2^{-3}$
d. $\left(\frac{4}{5}\right)^{-2}$
e. $3 x^{-4}$
f. $(3 x)^{-4}$

Example 2: Simplify the following expressions. Variables represent nonzero quantities.
Write your answer with only positive exponents.
a. $p^{-4} \cdot p^{2} \cdot p$
b. $\frac{2}{3} a^{-5} b^{-3} c^{2}$
c. $\frac{d^{-2}}{d^{-7}}$
d. $\frac{4 t^{-10} u}{6 t^{-3} u^{-1}}$

## You Try

2. Simplify the following expressions. Write your answers with only positive exponents. Variables represent nonzero quantities.
a. $\frac{7}{a^{-2}}$
b. $n^{-2} \cdot n^{-3} \cdot n^{8}$
c. $\frac{4 w^{3} x}{6 w x^{-2}}$
d. $2\left(3 x^{2}\right)^{-3}$

## Section 4.3: Division of Polynomials

Simplify the following expressions. Write your answer with only positive exponents. Variables represent nonzero quantities.
Example 1: $\frac{-6 w^{8}}{30 w^{3}}$

Example 2: $\frac{3 x-6}{2}$

Example 3: $\frac{6 x^{3}+2 x^{2}-4}{4 x}$

Example 4: $\frac{20 a^{2}+35 a-4}{-5 a^{2}}$

## You Try

3. Simplify the following expressions. Write your answer with only positive exponents. Variables represent nonzero quantities.
a. $\frac{11 x-15}{3}$
b. $\frac{3 x^{2}+5 x-12}{3 x^{2}}$

## Section 4.4: Scientific Notation

Scientific notation is the way that scientists easily handle very large numbers or very small numbers. For example, instead of writing 0.00000000000000092 , we write $9.2 \times 10^{-16}$.

## Powers of Ten

| $10^{4}$ | 10,000 |
| :---: | :---: |
| $10^{3}$ | 1,000 |
| $10^{2}$ | 100 |
| $10^{1}$ | 10 |
| $10^{0}$ | 1 |
| $10^{-1}$ | .1 |
| $10^{-2}$ | .01 |
| $10^{-3}$ | .001 |
| $10^{-4}$ | .0001 |

Scientific Notation Standard Form
$3.21 \times 10^{4}=32,100$
$3.21 \times 10^{3}=3,210$
$3.21 \times 10^{2}=321$
$3.21 \times 10^{1}=32.1$
$3.21 \times 10^{0}=3.21$
$3.21 \times 10^{-1}=0.321$
$3.21 \times 10^{-2}=0.0321$
$3.21 \times 10^{-3}=0.00321$
$3.21 \times 10^{-4}=0.000321$

## Writing Numbers in Scientific Notation and Standard Form

Scientific Notation
$3.21 \times 10^{4}$
Standard Form
32,100
$3.21 \times 10^{-2}$
0.000321

Example 1: Write the following numbers in standard form.
a. $5.9 \times 10^{5}$
b. $8.3 \times 10^{-7}$

Example 2: Write the following numbers in scientific notation.
a. $8,140,000$
b. 0.0000000091

## On Your Calculator

Example 3: Evaluate the following on your calculator. Write in standard form.
a. $850^{6}$
b. $0.25^{8}$

## You Try

4. Write the following numbers in standard form.
a. $4.9 \times 10^{5}$
b. $1.5 \mathrm{E}-3$
5. Write the following numbers in scientific notation.
a. 0.00000061
b. $5,430,000,000$
$\qquad$
$\qquad$

## Lesson 4 Practice Problems

## Skills Practice

## Variables represent nonzero quantities.

1. Simplify completely. Show all steps, and box your answers.
a. $\frac{8 n^{8} p^{5}}{12 n p^{4}}$
b. $\left(\frac{-5 a^{3}}{7 b^{5}}\right)^{2}$
2. Simplify completely. Show all steps, and box your answers. Use only positive exponents.
a. $8 n^{-2}$
b. $(8 n)^{-2}$
c. $g^{2} \cdot g^{-6} \cdot g$
d. $\left(\frac{5}{6}\right)^{-2}$
e. $5 w^{-3} x^{4} y^{-5} z$
f. $\frac{15 a b^{-3}}{24 a^{-4} b^{-1}}$
g. $5\left(2 v^{4}\right)^{-3}$
h. $\frac{d^{-5} d}{d^{-7}}$
i. $5 n\left(-2 n^{-4}\right)^{3}$
j. $\left(\frac{3 a^{10}}{7}\right)^{-2}$
3. Evaluate the algebraic expression $x^{-2}$ given $x=3$. Show your work.
4. Evaluate the algebraic expression $5 x^{-3}$ given $x=-2$. Show your work.
5. Evaluate the algebraic expression $(5 x)^{-2}$ given $x=-3$. Show your work.
6. Evaluate the algebraic expression $5(2 x)^{-2}$ given $x=-3$. Show your work.
7. Simplify completely. Show all steps, and box your answers. Use only positive exponents.
a. $\frac{15 n-12}{6}$
b. $\frac{4 x+2}{4 x}$
c. $\frac{8 d^{2}-5 d+11}{5 d^{2}}$
d. $\frac{w^{3}-4 w^{2}+6 w-9}{-2 w}$
8. Write the following numbers in scientific notation.
a. $45,600,000,000$
b. 0.0000000000238
9. Write the following numbers in standard form.
a. $2.35 \times 10^{9}$
b. $8.09 \times 10^{-5}$

## Applications

10. Complete the table below. Write each number in standard form and in scientific notation.

|  | Standard Form | Scientific Notation |
| :---: | :---: | :---: |
| A company's profit is <br> $\$ 3.24$ million |  |  |
| Your age, in days |  |  |
| A decimeter is one tenth of a |  |  |
| meter |  |  |$\quad$| A nanoliter is one billionth |
| :--- |
| of a liter |$\quad$|  |
| :--- |
| Disney purchased Lucasfilm <br> for \$4.05 billion |
| There are approximately 315 <br> million people in the U.S. |

## Extension

11. Complete the table below. You may need to look these up on the internet. Write each number in standard form and in scientific notation.

|  | Standard Form | Scientific Notation |
| :---: | :---: | :---: |
| The age of the earth, <br> in years |  |  |
| The diameter of the earth at <br> the equator, in meters |  |  |
| The diameter of an atom, <br> in meters |  |  |
| The distance from the earth <br> to the sun, in miles |  |  |
| The world population |  |  |

## Lesson 4 Assessment

1. Simplify completely. Show all steps, and box your answers. Answers should include only positive exponents.
a. $\frac{2}{3}\left(\frac{5}{6}\right)^{-2}$
b. $4 b\left(-5 b^{-5}\right)^{2}$
c. $\frac{15 d^{3}-3 d^{2}}{3 d^{2}}$
2. Evaluate the algebraic expression $4(7 x)^{-2}$ given $x=2$. Show your work. Your answer should be simplified and include only positive exponents.
3. For each of the following, circle the larger number.
a. $1.02 \times 10^{8}$
$5.9 \times 10^{7}$
b. $9.5 \times 10^{3}$
$9.5 \times 10^{-2}$
c. $3.4 \times 10^{-6} \quad 3.4 \times 10^{-3}$
d. $7.3 \mathrm{E}-2$
7.3

## Lesson 5: Solving Equations

Up to this point, our study of algebra has involved a deep look at algebraic expressions and operations on those expressions. We've learned how to characterize, write, and simplify algebraic expressions, and we have also learned how to evaluate expressions given specific values for the variables.

We now extend our study of algebra to include algebraic equations. This involves introducing the equal sign ( $=$ ) to connect an algebraic expression to a value, variable, or another expression. In this lesson, we will look at how an algebraic equation is defined and methods for solving algebraic equations.

## Mini-Lesson

Section 5.1: Algebraic Equations
Section 5.2: Solving One-Step Equations
Section 5.3: Solving Two-Step Equations
Section 5.4: Solving Multi-Step Equations
Section 5.5: Solving Equations - Applications

## Lesson 5 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$

## Mini-Lesson 5

## Section 5.1: Algebraic Equations

DEFINITION: An algebraic equation is a mathematical sentence connecting one expression to another expression with an equal sign $(=)$.

## Verify that a given value is a solution to an equation.

DEFINITION: The solution to an equation is the value, or values, that makes the equation true.
Example 1: Verify that $x=-3$ is a solution to the algebraic equation $5 x-2=8 x+7$.

Example 2: Is $m=-1$ a solution to the algebraic equation $m+9=3 m+5$ ?

Example 3: Is $a=5$ a solution to the algebraic equation $-4(a+1)=6(1-a)$ ?

## Equivalent Equations

DEFINITION: Equivalent equations are two or more equations that have the same solution.
Example 4: Verify that $x=2$ is a solution to the following equations.

$$
8 x-5=x+9 \quad 7 x-5=9 \quad 7 x=14
$$

## YOU TRY

1. Verify that $p=-9$ is a solution to the algebraic equation $p-4=2 p+5$.
2. Is $x=2$ is a solution to the algebraic equation $2(5 x-12)=1-5(x-1)$ ?

## Section 5.2: Solving One-Step Equations

## Properties of Equality

The Addition/Subtraction Property of Equality:

$$
\text { If } a=b, \text { then } a+c=b+c \quad \text { If } a=b, \text { then } a-c=b-c
$$

The Multiplication/Division Property of Equality:
If $a=b$, then $a \times c=b \times c$
If $a=b$ and $c \neq 0$, then $\frac{a}{c}=\frac{b}{c}$

## Solving an Equation

DEFINITION: To solve an equation means to "undo" all the operations of the equation, leaving the variable by itself on one side. This in known as isolating the variable.

Solve for the variable in each of the following equations. Check your answers.
Example 1: $x+7=18$
Example 2: $r-4=-5$

Example 3: $-4+b=45$
Example 4: 3=19+m

Example 6: $\frac{x}{6}=-5$

Example 7: $\frac{3}{4} a=8$
Example 8: $17=-x$

## You Try

3. Solve for the variable in each equation and check your answer. Show all steps as in the MiniLesson examples.
a. $12+x=-40$
b. $\frac{3}{5} n=-2$
c. $14=-x$
d. $-3=\frac{w}{5}$

## Section 5.3: Solving Two-Step Equations

## STEPS FOR SOLVING A LINEAR TWO-STEP EQUATION

1. Apply the Addition/Subtraction Property of Equality.
2. Apply the Multiplication/Division Property of Equality to isolate the variable.
3. Check by substituting your answer into the original equation.

Solve for the variable in each of the following equations. Check your answers.
Example 1: Solve: $2 b-4=12$
Check:

Example 2: Solve: $4+3 r=5$
Check:

Example 3: Solve: 3=19-2m
Check:

Check:

Example 5: Solve: $3+\frac{3}{5} x=12$
Check:

## You Try

4. Solve for the variable in each equation and check your answer. Show all steps as in the MiniLesson examples.
a. Solve: $14-3 x=-40$
Check:
b. Solve: $\frac{3}{4} w-8=-2$

Check:
c. Solve: $14=2-x$

Check:

## Section 5.4: Solving Multi-Step Equations

## STEPS FOR SOLVING A LINEAR EQUATION

1. Simplify each side of the equation. Remove parenthesis if necessary. Collect like terms.
2. Add or subtract terms on each side of the equation so that all terms containing the variable are on one side and all constant terms are on the other side.
3. Simplify each side of the equation by combining like terms.
4. Apply the Multiplication/Division Property of Equality to isolate the variable.
5. Check by substituting the solution into the original equation.

Solve for the variable in each of the following equations. Check your answers.
Example 1: Solve $x-5=4 x+7$
Check:

Example 2: Solve $3(4 n-2)=5(n+3)$ Check:

Example 3: Solve $4-(2 y-1)=2(5 y+9)+y$ Check:
5. Solve for the variable in each equation and check your answer. Show all steps as in the MiniLesson examples.
a. Solve $m-5=8 m+2$
b. Solve $2(5 x-12)=-(5 x-6)$

Check:

Check:

## Section 5.5: Solving Equations - Applications

## For this type of problem, first determine the Givens and the Goal, then form a Strategy, Solve, and Check. Write your answer in a complete sentence.

Example 1: The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate (pulse) is about $70 \%$ percent of your maximum heart rate. The formula $T=0.7(220-a)$, gives the target heart rate, $T$, in beats per minute, for a person who is $a$ years of age. Determine the age of a person whose target heart rate is 135 beats per minute.

| GIVEN: | GOAL: |  |
| :--- | :--- | :--- |
|  |  |  |
| STRATEGY: | CHECK: |  |
| SOLUTION: |  |  |
| FINAL RESULT AS A COMPLETE SENTENCE: |  |  |

## You Try

For this problem, identify the Givens the Goal. Form a strategy, solve, check, and write your answer in a complete sentence.
6. The cost of tuition at SCC is given by the equation $C=76 n$, where C represents the total cost of tuition and $n$ represents the number of credits taken. If you have $\$ 800$ dollars to spend on tuition, how many credits can you take?

| GIVEN: | GOAL: |
| :--- | :--- |
|  |  |
| STRATEGY: |  |
| SOLUTION: |  |

$\qquad$

## Lesson 5 Practice Problems

## Skills Practice

1. Verify that $a=-1$ is a solution to $4-a=6 a+11$. Show all work.
2. Verify that $x=-5$ is a solution to $3(2 x+4)=8(x+2)+6$. Show all work.
3. Is $x=8$ a solution to the equation $-16=\frac{3}{4} x-10$ ? Answer yes or no, and show all supporting work.
4. Is $x=-3$ a solution to the equation $3(6+2 x)=8+(x-5)$ ? Answer yes or no, and show all supporting work.
5. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.
a. $8 x-2=22$

Check:
b. $-x-2=22$

Check:
c. $-\frac{1}{2} x-4=8$

## Check:

d. $\frac{2}{3} x+3=15$

Check:
e. $4 x-8=-x+7$
f. $\frac{3}{4} x-\frac{1}{2}=\frac{9}{8} x+\frac{3}{2}$
g. $6 x-4(-2 x+8)=10$
h. $-2(4 x-2)=-(2 x-8)$
i. $(2 x-7)-(4 x+8)=4(x+6)$

Check:

## Applications

For each of the following, underline the Givens and circle the Goal of the problem. Form a Strategy, Solve, and Check. Show all work, and write your answer in a complete sentence.
6. John is a door to door vacuum salesman. His weekly salary, S , is $\$ 200$ plus $\$ 50$ for each vacuum he sells. This can be written as $\mathrm{S}=200+50 v$, where $v$ is the number of vacuums sold. If John earns $\$ 1000$ for a week's work, how many vacuums did he sell?

| STRATEGY: |  |
| :--- | :--- |
| SOLUTION: | CHECK: |
|  |  |
|  |  |
|  |  |

FINAL RESULT AS A COMPLETE SENTENCE:
7. Paul is planning to sell bottled water at the local Lollapalooza. He buys 2 crates of water ( 2000 bottles) for $\$ 360$ and plans on selling the bottles for $\$ 1.50$ each. Paul's profit, $P$ in dollars, from selling $b$ bottles of water is given by the formula $P=1.5 b-360$. How many bottles does Paul need to sell in order to break even?

| STRATEGY: |  |
| :--- | :--- |
| SOLUTION: | CHECK: |
|  |  |
|  |  |
|  |  |

FINAL RESULT AS A COMPLETE SENTENCE:
8. Ringo has $\$ 100$ in the bank and is adding $\$ 50$ each week in savings. George has $\$ 250$ in the bank, and is adding $\$ 40$ each week in savings. Their plan is to wait until their savings are equal and then buy a Magic Yellow Bus and take a road trip. They figure out that the equation can be written as $50 w+100=40 w+250$, where $w$ is the number of weeks. How long will it take for their savings to be equal?

| STRATEGY: |  |
| :--- | :--- |
| SOLUTION: | CHECK: |
|  |  |
|  |  |

FINAL RESULT AS A COMPLETE SENTENCE:
9. The formula to convert from Celsius to Fahrenheit is $F=\frac{9}{5} C+32$. The temperature on a summer day in Phoenix, Arizona is $113^{\circ} \mathrm{F}$. What would this temperature be in degrees Celsius? Show all work, and write your answer in a complete sentence.

| STRATEGY: | CHECK: |
| :--- | :--- |
| SOLUTION: |  |
|  |  |
|  |  |

FINAL RESULT AS A COMPLETE SENTENCE:
9. Suppose you want to accumulate $\$ 1,000,000$ for your retirement in 30 years. You decide to put money into an account that earns $3 \%$ interest compounded annually. How much should you deposit? The formula for compound interest is $A=P(1+r)^{t}$, where $A$ is the accrued amount after $t$ years, $P$ is the starting principal, and $r$ is the annual interest rate expressed as a decimal. Round your answer $u p$ to the nearest cent.

| STRATEGY: |  |
| :--- | :--- |
| SOLUTION: | CHECK: |
|  |  |
|  |  |
|  |  |
|  |  |
| FINAL RESULT AS A COMPLETE SENTENCE: |  |

10. Fred and Wilma just had a baby! They want to start a college fund for their baby, and decide to put money into an investment that is expected to earn $4.2 \%$ simple interest each year. How much would they have to deposit now to pay for their newborn's $\$ 100,000$ college in 18 years? The formula for simple interest is $A=P+\operatorname{Prt}$, where $A$ is the accrued value of the investment after $t$ years, $r$ is the interest rate (expressed as a decimal), and $P$ is the starting principal invested. Round your answer $u p$ to the nearest cent.

| STRATEGY: |  |
| :--- | :--- |
| SOLUTION: | CHECK: |
|  |  |
|  |  |
|  |  |
|  |  |
| FINAL RESULT AS A COMPLETE SENTENCE: |  |

## Extension

11. Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.
a. $2(4 x+3)=8 x+1$
b. $5(x+6)-x=4(x+7)+2$
12. Solve the following nonlinear equations.
a. $x^{2}=25$
b. $x^{3}=27$
c. $|x|=3$
d. $\sqrt{x}=7$
e. $\sqrt[3]{x}=2$
13. Write a story problem for the equation shown below. Solve the problem, and write your answer in a complete sentence.

$$
300-50 x=0
$$

## Lesson 5 Assessment

1. Solve the following equations for $x$. Show your work. Reduce, simplify and CHECK your answers!
a. $7-(a-3)=3(2 a-6)$
b. $-31=\frac{3}{5} x-10$
2. The formula for the area, $A$, of a triangle with base $b$ and height $h$ is $A=\frac{1}{2} b h$. Determine the height of a triangle with a base of 18 inches and area 84.6 square inches. Round your answer to the nearest tenth, and include appropriate units in your answer.
3. You decide to invest $\$ 7000$ into an account that pays $5 \%$ simple interest each year. How long will it take for the investment to double in value?

The formula for simple interest is $A=P+\operatorname{Prt}$, where $A$ is the accrued value of the investment after $t$ years, $r$ is the interest rate (expressed as a decimal), and $P$ is the starting principal invested.

Show all steps, and write your answer in a complete sentence.

## Lesson 6: Linear Equations: Real World Applications

In this lesson, we investigate real world problems that can be modeled by and solved using algebraic equations. In addition to writing and solving linear equations, we will also work with problems involving proportions and percents.

## Mini-Lesson

Section 6.1: Writing Equations
Section 6.2: Proportions
Section 6.3: Percent Equations
Section 6.4: More Percent Problems

## Lesson 6 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 6

## Section 6.1: Writing Equations

Step 1: Read and understand the problem. Underline the givens and circle the goal.
Step 2: Form a strategy to solve the problem.
Step 3: Choose a variable to represent the unknown quantity.
Step 4: Read every word in the problem, and translate the given information into an algebraic equation.

Step 5: Solve the equation.
Step 6: Write your answer in a complete sentence.

Example 1: The cost of leasing a new Ford Mustang is $\$ 2,311$ for a down payment and processing fee plus $\$ 276$ per month. For how many months can you lease this car with $\$ 10,000$ ?

[^0]
## You Try

1. Your yard is a mess, and you decide to hire a landscaper. The Garden Pros charges a $\$ 50$ consultation fee plus $\$ 36$ per hour for the actual work. If the total cost is $\$ 212$, how many hours did the landscapers work?
a. Write an equation to represent this situation. Clearly indicate what the variable represents.
b. Solve the equation. Show all work, and write your answer in a complete sentence. Your answer must include correct units of measure.

## Section 6.2: Proportions

## Definitions

A $\qquad$ is the quotient of two quantities with the same unit of measure.

A $\qquad$ is the quotient of two quantities with different units of measure.

A $\qquad$ is a mathematical statement that two ratios or two rates are equal.

## Solving Proportions

Example 1: Solve for the variable in each of the following proportions.

$$
\frac{2}{3}=\frac{t}{42} \quad \frac{r}{3}=\frac{5}{2} \quad \frac{7}{12}=\frac{35}{x}
$$

Example 2: The recommended daily allowance (RDA) of protein for active adults 19 years of age and older is based primarily on body weight. In general, the RDA of protein for adults is 0.8 grams for every kilogram (about 2.2 pounds) of body weight. If you weigh 150 pounds, how many grams of protein should you consume each day? Round your answer to the nearest tenth.

## You Try

2. Solve the proportion $\frac{1.2}{t}=\frac{3.2}{5.8}$
3. Last week, Liam earned $\$ 225$ for working 12 hours. If he works 20 hours this week, how much will he earn if he is paid at the same rate?
a. Use the information given in the problem to set up a proportion representing this situation. Clearly indicate what the variable represents.
b. Solve, showing all steps. Write your answer in a complete sentence.

## Section 6.3: Percent Equations

## Creating and Solving Percent Equations

When working with situations involving percents, the most reliable solution method is to translate the given problem into an equation.

Look for:

1. The unknown - Always start by identifying what it is you are trying to find.
2. The percent - If given, you will need to convert this to decimal form before doing any calculations. If you are asked to determine the percent, then you will need to convert your answer from decimal form to percent form.
3. Multiplication - Replace the word "of" with multiplication.
4. Equals - Look for words like "is," "becomes," etc... and replace with an equal sign.

Example 1: For each of the following, first translate the given statement into a percent equation, then solve the equation.
a. What is $12 \%$ of 20 ?
b. $60 \%$ of what is 15 ?
c. What percent of 140 is 3.5 ?

Example 2: A lender requires a minimum down payment of $16 \%$ of the value of the home.
a. What is the required down payment for a $\$ 180,000$ home?
b. You have $\$ 23,500$ cash available to use as a down payment toward a home. Determine the maximum home value that you can finance.

You Try
For each of the following, first translate the given statement into a percent equation, then solve the equation.
4. What is $18 \%$ of $\$ 75.23$ ? Round to the nearest cent.
5. $18 \%$ of what is $\$ 75.23$ ? Round to the nearest cent.

## Section 6.4: More Percent Problems

Example 1: A $\$ 750$ watch is on sale for $15 \%$ off. Find the sale price.

Example 2: A salesman tells you that the $\$ 140$ earrings are already marked $20 \%$ off. What was the original price?

[^1]
## You Try

6. A salesman is paid a monthly salary of $\$ 2,300$ plus $7 \%$ commission on his monthly sales. Determine the amount of sales required for his total monthly income to be $\$ 3,000$.
$\qquad$

## Lesson 6 Practice Problems

## Skills Practice

1. Solve the proportions. Simplify your answers. Show all work.
a. $\frac{28}{x}=\frac{3.5}{5}$
b. $\frac{p}{5}=\frac{12}{50}$
c. $\frac{11}{20}=\frac{m}{6}$
d. $\frac{\frac{4}{5}}{8}=\frac{10}{w}$
2. Complete the missing parts of the table.

| Decimal | Percent |
| :---: | :---: |
| 0.08 |  |
| 0.625 | $41 \%$ |
|  | $100 \%$ |
| 3.5 |  |
|  | $0.7 \%$ |

3. For each of the following, first translate the given statement into a percent equation, then solve the equation. Show all steps.
a. $45 \%$ of what number is 27 ?
b. 50 is what percent of 80 ?

Equation: $\qquad$
Solve:
c. 67 is $80 \%$ of what number?

Equation: $\qquad$
Solve:

Equation:
Solve:
d. 300 is what percent of 48 ?

Equation:
Solve:
4. Calculate the percent change for each of the following. Show all work.
a. A quantity decreases from 15 to 10
b. A quantity increases from 10 to 15

## Applications

For each of the following, underline the Givens and circle the Goal of the problem. Form a Strategy, Solve, and Check. Show all work, and write your answer in a complete sentence.
5. Amber is baking cupcakes for a school fundraiser, and wants to sell them for five times the cost of making them. The ingredients cost $\$ 11.22$, and the recipe makes 24 cupcakes. Write an equation to represent this situation, and use it to determine the amount of money Amber should charge for each cupcake.
6. A new Sony $55^{\prime \prime} 3 \mathrm{D}$ television set costs $\$ 2,499$. You are going to pay $\$ 600$ as a down payment, and pay the rest in equal monthly installments for one year. Write an equation to represent this situation, and use it to determine how much you should pay each month.
7. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $\$ 20$ consultation fee plus $\$ 11$ per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $\$ 15$ per hour for the actual work. Write an equation that will help you determine the number of hours at which the two companies charge the same. Solve the equation, and write your answer in a complete sentence.
8. The scale on a map is 1 inch to 75 miles. Find the actual distance between two towns that are $4 \frac{1}{2}$ inches apart on the map. Set up a proportion to represent this situation. Solve, and write your answer in a complete sentence.
9. An 8 fluid ounce serving of PowerAde contains 72 calories. How many calories are in 30 fluid ounces of PowerAde? Set up a proportion to represent this situation. Solve, and write your answer in a complete sentence.
10. Megan makes $\$ 32,500$ per year. Every day, she stops by Starbucks and spends an average of $\$ 4$ on coffee. What percent of her income is spent on Starbucks? Round your answer to the nearest tenth of a percent.
11. One banana contains about 425 mg of potassium. That is about $13 \%$ of the dailyrecommended amount of potassium.
a. How much potassium (in mg ) should be consumed daily?
b. How many bananas would you need to eat each day in order to consume this much potassium?
12. Jon has 892 points in his math class. To earn an A in the class, he must have $90 \%$ of the 1180 points possible by the end of the term. How many more points must he earn by the end of the term to receive an A in the class?
13. The bill for dinner (after tax) was $\$ 85.20$. You decide to leave a $15 \%$ tip. Calculate the total amount paid.
14. A clothing store is having a $40 \%$ off sale on all its merchandise. A customer bought an item that originally cost $\$ 80$.
a. What was the sale price of the item?
b. Calculate the total amount the customer paid after a $9 \%$ sales tax was added to the purchase.

## Extension

15. Justin receives a $10 \%$ pay cut followed by a $10 \%$ pay raise. His salary after the raise is
a. More than his original salary
b. The same as his original salary
c. Less than his original salary.

Explain your reasoning.
16. It is the day after Thanksgiving (Black Friday!), and April is at Kohls, standing in the very long line waiting to check out. She has two coupons, the first is for $10 \%$ off her entire purchase. The second is for $\$ 10$ off her entire purchase. April estimates that she has well over $\$ 200$ worth of merchandise in her basket.
a. Assuming that only one of her coupons can be applied to her purchase, which one should she use? Show all mathematical work and explain your answer.
b. Assuming that both of her coupons can be applied to her purchase, which one should she show to the cashier first? Show all mathematical work and explain your answer.
17. Consider the proportion $\frac{1}{15}=\frac{4}{x}$
a. Write a story problem for the proportion above.
b) Solve your problem showing all possible steps. Write your answer in a complete sentence.
$\qquad$ Date: $\qquad$

## Lesson 6 Assessment

1. A $2 / 3$ cup serving of cereal contains 140 calories. Set up a proportion to determine the number of calories that would be in a $1 / 2$ cup serving. Solve, and write your answer in a complete sentence.
2. Jackson's grandma gave him a $\$ 20$ gift card to Toys $R$ Us for his birthday. Sales tax is currently $9.8 \%$. Determine the price of the most expensive toy Tommy can buy with the $\$ 20$ gift card. Show all steps, and write your answer in a complete sentence.
3. A lender requires a down payment of $18 \%$ of the value of the home.
a. What is the required down payment for a $\$ 200,000$ home? Show all steps, and write your answer in a complete sentence.
b. You have $\$ 25,000$ cash available to use as a down payment toward a home. Determine the price of the most expensive home you can buy. Show all steps, and write your answer in a complete sentence.

## Lesson 7: Literal Equations, Inequalities, and Absolute Value

In this lesson, we first look at literal equations, which are equations that have more than one variable. Many of the formulas we use in everyday life are literal equations.

We then look at algebraic inequalities, how they are written, how to solve them and how to represent the solution set graphically and using interval notation.

Finally, we look at algebraic expressions, equations, and inequalities that involve absolute value.

## Mini-Lesson

Section 7.1: Literal Equations
Section 7.2: Linear Inequalities
Section 7.3: Solving Linear Inequalities
Section 7.4: Solving Inequalities - Applications
Section 7.5: Compound Inequalities
Section 7.6: Absolute Value

## Lesson 7 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 7

## Section 7.1: Literal Equations

## LITERAL EQUATIONS

What are literal equations? $\qquad$

What does it mean to "solve" a literal equation? $\qquad$

Example 1: Solve for $b$ in each of the following equations.
$2 b=8 \quad a b=c$
$5+b=9$
$a+b=c$
$2 b+1=13$
$a b+c=d$

## SOLVING LITERAL EQUATIONS

Example 2: Solve the following equation for $c: 4 a b c=32$

Example 3: Solve the following equation for $B: \quad A=B+C+D$

Example 4: Solve the following equation for $x: y=m x+b$

Example 5: Solve the following equation for $y$ : $3 x+4 y=20$

Example 6: Solve the following equation for $y: x-y=5$

Example 7: Solve the following equation for $C$ : $F=\frac{9}{5} C+32$

## You Try

1. Solve the following equation for $y: 3 x y z=9$
2. Solve the following equation for $y: 5 x-y=2$

## Section 7.2: Inequalities

DEFINITION: An algebraic inequality is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign.

| Symbol | In words | Examples |
| :---: | :---: | :---: |
| $<$ |  |  |
| $\leq$ |  |  |
| $>$ |  |  |
| $\geq$ |  |  |
| $\neq$ |  |  |

Verify that a given value is a solution to the inequality.
DEFINITION: A solution to an inequality is a value that makes the inequality true.

l
Example 1: Determine whether the number 4 is a solution to the following inequalities.

$$
x>1 \quad x<1 \quad x \leq 9 \quad x>4 \quad x \geq 4
$$

THE SOLUTION SET OF A LINEAR INEQUALITY

| Inequality | Graph | Interval Notation |
| :---: | :---: | :---: |
| $x>2$ | $-\infty<\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad \mid>\infty$ |  |
| $x \geq 2$ | $-\infty<1$ \| | | | | | | | | $\mid$ \| |  |
| $x<2$ | $-\infty<1,\|,\|,\|, 1,1,\|-1>\infty$ |  |
| $x \leq 2$ | $-\infty<\|,\|,\|,\|,\|, 1,\|,\|>\infty$ |  |

## Translate a statement into an inequality.

Example 2: Write an inequality to represent the following situation. Clearly indicate what the variable represents.
a. In order to go on the ride, a child must be more than 48 inches tall.
b. Jordan can spend at most $\$ 10$ on lunch.

## You Try

3. Which of the following values are in the solution set for $n<5$ ?

$$
n=-3 \quad n=0 \quad n=4.99 \quad n=5 \quad n=12
$$

4. Translate the statement into an inequality.

Children age 2 and under are free at Disneyland.
5. Complete the table below:

| Inequality | Graph | Interval <br> Notation |
| :---: | :---: | :---: |
| $x \geq-3$ |  |  |
|  | $-\infty<1,\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad \mid>\infty$ | $(-\infty, 11]$ |
|  | $-\infty<\underset{-3}{1}$ |  |

## Section 7.3: Solving Linear Inequalities

## STEPS FOR SOLVING A LINEAR INEQUALITY

1. Simplify each side of the inequality. Remove parenthesis if necessary. Collect like terms.
2. Add or subtract terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side.
3. Simplify each side of the inequality by combining like terms.
4. Multiply or divide on both sides to isolate the variable. CAUTION!!! If you multiply or divide both sides of an inequality by a negative number, you have to reverse the inequality sign.
5. Check by substituting the solution (endpoint and a value from the solution set) into the original inequality.

Solve the inequality, check your answer, and graph the solution on a number line.
Example 1: Solve the inequality, check your answer, and graph the solution on a number line.

$$
3 x>x+6
$$

Graph:


Interval Notation: $\qquad$

Example 2: Solve the inequality and graph the solution on a number line.

$$
3-5 a \leq 2(a+5)
$$

Graph:


Interval Notation: $\qquad$

Example 3: Solve the inequality and graph the solution on a number line.

$$
-5(x+2) \geq-3(x+4) \quad \text { Graph: }
$$



Interval Notation: $\qquad$

## You Try

Solve the inequality, check your answer, and graph the solution on a number line.
6. $7-4 x \geq-5$

Graph:


Interval Notation: $\qquad$
7. $6 x+13<5(2 x-3)$

Graph:


Interval Notation: $\qquad$

## Section 7.4: Solving Inequalities - Applications

For each problem, underline the Givens and circle the Goal. Form a Strategy, Solve, and Check. Write your answer in a complete sentence.

Example 1: The cost of tuition is $\$ 76$ per credit hour. Write an inequality that can be used to determine the number of credit hours a student can take for under $\$ 1000$. Solve the inequality, and write your answer in a complete sentence.

[^2]
## You Try

To solve this problem, first underline the Givens and circle the Goal. Form a strategy, solve, check, and write your answer in a complete sentence.
8. Gasoline costs $\$ 3.79$ per gallon. Write an inequality that can be used to determine how many gallons of fuel can be purchased for under $\$ 20$. Solve the inequality, and write your answer in a complete sentence.

## Section 7.5: Compound Inequalities

## THE SOLUTION SET OF A COMPOUND INEQUALITY

| Inequality | Graph | Interval Notation |
| :---: | :---: | :---: |
| $-1<x<2$ | $-\infty<\|\|\|\| \|>\infty$ |  |
| $-1 \leq x \leq 2$ | $-\infty<1$ \| | | | | | | | | |  |
| $-1 \leq x<2$ | $-\infty<1,\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad \mid>\infty$ |  |

## Verify that a given value is a solution to the inequality.

Example 1: Which of the following values are in the solution set for $-3 \leq n<5$ ?

$$
n=-5 \quad n=-3 \quad n=0 \quad n=4.99 \quad n=5 \quad n=12
$$



## Translate a statement into an inequality.

Example 2: Write a compound inequality to represent the following situation. Clearly indicate what the variable represents.
a. A number is greater than or equal to 5 but less than 8 .
b. My car's tank can hold a maximum of 20 gallons of gas.

## You Try

9. Which of the following values are in the solution set for $-8<w<2$ ?

$$
w=-11 \quad w=-8 \quad w=-5 \quad w=0 \quad w=2 \quad w=2.1
$$

10. Translate the statement into a compound inequality.

A number is greater than 0 , and less than or equal to 8 .
11. Complete the table below:

| Inequality | Graph | Interval Notation |
| :---: | :---: | :---: |
| $5<x<11$ |  |  |
|  | $-\infty<1 \text { \| }\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\|>\infty$ | $(-3,1]$ |
|  | $-\infty<\begin{array}{cccccccccccc} 1 & - & - & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}>\infty$ |  |

## Section 7.6: Absolute Value



Example 1: Evaluate the following: $|2|=$
$|-2|=$

## Absolute Value Equations

Determine the solution to each of the following equations.
CExample 2: $|x|=2$
$|x|=3$
$|x|=-4$

| Absolute Value Inequalities |  |
| :---: | :---: |
| $\|x\|<2$ | List some values in the solution set: $\qquad$ |
| $\|x\| \leq 2$ | List some values in the solution set: $-\infty<\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\|>\infty$ |
| $\|x\|>2$ | List some values in the solution set: |

12. Determine the solution to the equation $|x|=8$.
13. Absolute Value Inequalities:

| $\|x\|<3$ | List some values in the solution set: $\qquad$ |
| :---: | :---: |
| $\|x\| \geq 3$ | List some values in the solution set: |

$\qquad$

## Lesson 7 Practice Problems

## Skills Practice

1. Solve the following equations for the given variable. Show all steps. Simplify your answers.
a. $I=P r t \quad$ Solve for $t$.
b. $2 x+3 y=6$ Solve for $y$.
c. $A=B(C+D) \quad$ Solve for $D$.
d. $A=p+p r t \quad$ Solve for $r$.
e. $6 x-y=11 \quad$ Solve for $y$.
f. $A=P(1+r)^{t} \quad$ Solve for $P$.
g. $r=\frac{C}{2 \pi} \quad$ Solve for $C$.
h. $3 x-5 y=8 \quad$ Solve for $y$.
i. $P=A-B-C \quad$ Solve for $B$.
j. $a x^{2}+b x+c=0 \quad$ Solve for $b$.
2. For each of the following, circle all correct answers.
a. Which of the given values are in the solution set for $x<3$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=3 \quad x=5 \quad x=-\frac{5}{3}
$$

b. Which of the given values are in the solution set for $x \geq-1$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=3 \quad x=5 \quad x=-\frac{5}{3}
$$

c. Which of the given values are in the interval $(-2, \infty)$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=3 \quad x=5 \quad x=-\frac{5}{3}
$$

d. Which of the given values are in the interval $(-1,5]$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=3 \quad x=5 \quad x=-\frac{5}{3}
$$

e. Which of the given values are in the interval $-5<x \leq 3$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=3 \quad x=5 \quad x=-\frac{5}{3}
$$

3. Complete the table below:

| Inequality | Graph | Interval Notation |
| :---: | :---: | :---: |
| $x>8$ | $-\infty<1\| \|\|\perp\|>\infty$ |  |
| $x \leq-1$ | $-\infty<1$ \| | | | | | | | $\mid$, |  |
| $8 \leq x<12$ | $-\infty<\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\|>\infty$ |  |
|  | $-\infty<\|-\|\quad\| \quad\| \quad\|\quad\| \quad\|\quad\| \quad \mid>\infty$ | $(-2, \infty)$ |
|  | $-\infty<\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\|>\infty$ | $(-\infty, 6]$ |

4. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

$$
4 x \leq 2 x+12
$$

Interval Notation: $\qquad$

Graph:

5. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.
$\qquad$

## Graph:


6. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

$$
5(-2 a-8) \leq-9 a+4 \quad \text { Interval Notation: }
$$

## Graph:


7. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

$$
-2 d>2-(4-2 d)+d
$$

Interval Notation: $\qquad$

## Graph:


8. Solve the equation $|x|=11$
9. Solve the equation $|x|=-11$
10. For each of the following, circle all correct answers.
a. Which of the given values are in the solution set for $|x|<5$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=-7 \quad x=3 \quad x=5 \quad x=9
$$

b. Which of the given values are in the solution set for $|x| \geq 5$ ?

$$
x=0 \quad x=-1 \quad x=-5 \quad x=-7 \quad x=3 \quad x=5 \quad x=9
$$

11. Graph the solution set for the inequalities shown below.
a. $|x|<1$

b. $|x| \geq 4$


## Applications

12. The area of a triangle is given by the formula $\mathrm{A}=\frac{1}{2} b h$. Solve this equation for $h$. Show your work. Simplify your answer.
13. The volume of a cylinder is given by the formula $V=\pi r^{2} h$. Solve this equation for $h$. Show your work. Simplify your answer.
14. The surface area of a cylinder is given by the formula $S=2 \pi r h+2 \pi r^{2}$. Solve this equation for $h$. Show your work. Simplify your answer.
15. Relative change is given by the formula $R=\frac{N-C}{C}$. Solve for $N$. Show your work. Simplify your answer.
16. Translate each of the given statements into an algebraic inequality.
a. You must be at least 13 years of age in order to view a PG-13 movie. Let $a$ represent your age.
b. Your car's gas tank can hold up to 25 gallons of gas. Let $g$ represent the number of gallons in your gas tank.
c. A company must sell more than 850 items in order to make a positive profit. Let $n$ represent the number of items sold.
d. The maximum heart rate, M , is the highest heart rate achieved during maximal exercise. In general, you gain the most benefits and lessen the risks when you exercise within your target heart rate zone. Usually this is when your exercise heart rate is between 60 and 80 percent of your maximum heart rate. Let T represent your target heart rate.
17. You have $\$ 1200$ for your trip to the beach. You estimate that it will cost $\$ 160$ a day for food, entertainment and hotel, plus $\$ 230$ round trip air fair.
a. Write an inequality that can be used to determine the maximum number of days you can stay at the beach. Clearly indicate with the variable represents.
b. Solve the inequality, and interpret your answer in a complete sentence.
18. Let $p$ represent the marked price of an item at Toys R Us. Bella's aunt gave her a $\$ 50 \mathrm{gift}$ card to Toys R Us for her birthday.
a. If sales tax is currently $9 \%$, set up an algebraic inequality to express how much she can spend using her gift card. Clearly indicate what the variable represents.
b. Solve the inequality, and interpret your answer in a complete sentence.
19. Your car is worth $\$ 1000$ at most. It is old. You find out that it needs repairs to pass inspection. The auto shop tells you that the parts cost a total of $\$ 520$, and the labor cost is $\$ 68$ per hour. If the repairs are more than the car is worth, you are going to donate the car to charity.
a. Write an inequality that can be used to determine the maximum number of hours the mechanic can spend working on your car to help you decide to repair it or donate it. Clearly indicate what the variable represents.
b. Solve the inequality, and interpret your answer in a complete sentence.

## Extension

20. CONVERSION FORMULAS:

Complete the table below. Show your work. Simplify your answers.

| $F=\frac{9}{5} C+32$ | Fahrenheit to Celsius |
| :--- | :--- |
| $K=C+273.15$ | Kelvin to Celsius |
| Celsius to Kelvin |  |
|  |  |

$\qquad$ Date: $\qquad$

## Lesson 7 Assessment

1. Solve the following equations for the given variable. Show all steps. Simplify your answers.
a. $\quad A=\frac{2}{3} B C \quad$ Solve for $B$.
b. $5 x-2 y=8$ Solve for $y$.
2. Solve the inequality, showing all steps. Write your answer as an inequality and in interval notation, then graph the solution set on the number line.

$$
1-3 x>14-(4-6 x) \quad \text { Interval Notation: }
$$

$\qquad$

Graph:

3. Complete the table below.

| Inequality | Graph | Interval Notation |
| :---: | :---: | :---: |
| $x<0$ |  |  |
| $-2<x \leq 1$ | $-\infty<1 \text { \| }\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad\|\quad\| \quad \mid>\infty$ |  |
|  | $-\infty<1$ \| | | | | | | | | | $[-3, \infty)$ |
|  | $-\infty<\underset{-3}{1} \begin{array}{lllllllllllll} \mid & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline \end{array}$ |  |

## Lesson 8: Graphs and Graphing Linear Equations

In this chapter, we will begin looking at the relationships between two variables. Typically one variable is considered to be the input, and the other is called the output. The input is the value that is considered first, and the output is the value that corresponds to or is matched with the input. The input/output designation may represent a cause/effect relationship, but that is not always the case.

A critical skill required for the study of algebra is the ability to construct and interpret graphs. In this lesson we will learn how the Cartesian plane is used for constructing graphs and plotting data. We will interpret the points and behavior of a graph with respect to the input and output variables. We will also learn the rules/ guidelines for constructing good graphs so that the graphs we create can be easily read and understood.

## Mini-Lesson

Section 8.1: The Cartesian Plane
Section 8.2: Constructing Good Graphs from Data
Section 8.3: Linear Equations - Two Variables
Section 8.4: Graphing Linear Equations by Plotting Points
Section 8.5: Intercepts
Section 8.6: Horizontal and Vertical Lines

## Lesson 8 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$

## Mini-Lesson 8

## Section 8.1: The Cartesian Plane

In this chapter, we will begin looking at the relationships between two variables. Typically one variable is considered to be the INPUT, and the other is called the OUTPUT. The input is the value that is considered first, and the output is the value that corresponds to or is matched with the input. The input/output designation may represent a cause/effect relationship, but that is not always the case.
Ordered Pairs

| Input | Output | Ordered Pairs (input, output) |
| :---: | :---: | :---: |
| 4 | -3 |  |
| 5 | 8 |  |
|  |  | $(0,-4)$ |
|  |  | $(-2,6)$ |

Example 2: The Rectangular Coordinate System (Cartesian Coordinate System)



## Quadrants

| Quadrant | Coordinates |
| :---: | :---: |
| I | $(+,+)$ |
| II | $(-,+)$ |
| III | $(-,-)$ |
| IV | $(+,-)$ |

1. 



Plot and label the points.
A. $(6,-3)$
B. $(1,9)$
C. $(-4,0)$
D. $(-2,-8)$
E. $(0,5)$

## Section 8.2: Constructing a Graph from Data

## Criteria for a Good Graph

1. The horizontal axis should be properly labeled with the name and units of the input variable.
2. The vertical axis should be properly labeled with the name and units of the output variable.
3. Use an appropriate scale.
a. Start at or just below the lowest value.
b. End at or just above the highest value.
c. Scale the graph so the adjacent tick marks are equal distance apart.
d. Use numbers that make sense for the given data set.
e. The axes meet at $(0,0)$. Use a "//" between the origin and the first tick mark if the scale does not begin at 0 .
4. All points should be plotted correctly, and the graph should make use of the available space.

Example 1: The table below shows the total distance (including reaction time and deceleration time) it takes a car traveling at various speeds to come to a complete stop.

| Speed (miles per hour) | 15 | 25 | 35 | 45 | 50 | 60 | 75 | 80 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stopping Distance (ft) | 44 | 85 | 135 | 196 | 229 | 304 | 433 | 481 |



## You Try

2. Consider the following data set.

| Elapsed time (seconds) | 1 | 1.5 | 2.4 | 3 | 3.6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Height of Golf Ball (feet) | 43 | 52.5 | 50 | 62 | 46 |

a. What is the input variable? $\qquad$
b. What was the height of the ball after 3 seconds? $\qquad$
c. After how many seconds was the ball 50 feet in the air? $\qquad$
d. In a complete sentence, interpret the meaning of the ordered pair $(1,43)$.
e. Construct a good graph of this data.


# Section 8.3: Linear Equations - Two Variables 

## Linear Equations

Example 1: Verify that the ordered pairs below satisfy the equation $y=2 x+3$.

$$
\begin{equation*}
(-2,-1) \tag{0,3}
\end{equation*}
$$



Example 2: Verify that the ordered pairs below satisfy the equation $3 x+2 y=6$.
$(-2,6)$
$(0,3)$
$(2,0)$


## You Try

3. Verify that the ordered pairs below satisfy the equation $y=3 x-2$ ?
$(-2,-8)$
$(3,7)$
$(0,-2)$

## Section 8.4: Graphing Linear Equations - Two Variables

## Graphing Linear Equations by Plotting Points

Step 1: Choose two or more values for the input variable, and list them in a table of values.
Step 2: Substitute each input value into the equation and compute the corresponding output values. List these values in the table.
Step 3: Write each input-output pair in the table as an ordered pair.
Step 4: Plot the ordered pairs, connect them, and extend the line beyond the points to show that the pattern continues

Example 1: Graph the equation $y=3 x-2$

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



Example 2: Graph the equation

$$
y=\frac{2}{3} x-1
$$

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |



Example 3: Graph the equation $4 x+2 y=10$

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



## You Try

4. Graph the equation $y=\frac{1}{2} x+3$

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |



## Section 8.5: Intercepts

Vertical and Horizontal Intercepts
The vertical intercept $(y$-intercept $)$ is the point at which the graph crosses the vertical axis.


The input value of the vertical intercept is always $\qquad$
The coordinates of the vertical intercept will be $\qquad$
To determine the vertical intercept:
The horizontal intercept ( $x$-intercept) is the point at which the graph crosses the horizontal axis.


The output value of the horizontal intercept is always $\qquad$
The coordinates of the horizontal intercept will be $\qquad$
To determine the horizontal intercept:

Example 1: Determine the vertical and horizontal intercepts for $y=3 x-2$.

Example 2: Determine the vertical and horizontal intercepts for $4 x-2 y=10$.
$\square$ You Try
5. Complete the table below. Write the intercepts as ordered pairs.

| Equation | Vertical Intercept | Horizontal Intercept |
| :---: | :---: | :---: |
|  |  |  |
| $y=24-6 x$ |  |  |
|  |  |  |
| $5 x-3 y=30$ |  |  |
|  |  |  |
|  |  |  |

## Section 8.6: Horizontal and Vertical Lines

## Horizontal Lines $y=b$, where $b$ is a real constant

Example 1: Graph the equation $y=2$.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



6.

Graph the equation $y=-2$.


Graph the equation $x=4$.

$\qquad$

## Lesson 8 Practice Problems

## Skills Practice

1. Plot and label the points.

A. $(8,2)$
B. $(0,0)$
C. $(0,5)$
D. $(10,-10)$
E. $(-4,4)$
F. $(-9,-1)$
G. $(-5,0)$
H. $(2,-8)$
2. Give the coordinates of each of the points shown below.

A. $\qquad$
B. $\qquad$
C. $\qquad$
D. $\qquad$
E. $\qquad$
F. $\qquad$
3. Identify the graph that best represents the speed of a car coming to a stop at a red light.
a.

b.

c.

4. Identify the graph that best represents the height of an arrow that has been shot straight up in the air, and lands on the ground.
a.

b.

c.

5. Identify the graph that best represents the distance traveled by a car driving at a constant speed.
a.

b.

c.

6. Which of the following ordered pairs satisfy the equation $y=-2 x-4$. Circle all that apply, and show all supporting work
$(9,-22)$
$(6,-5)$
$(-9,14)$
$(2,0)$
$(-4,0)$
7. Which of the following ordered pairs satisfy the equation $3 x-2 y=8$. Circle all that apply, and show all supporting work
$(2,-1)$
$(-4,0)$
$(-2,-7)$
$(-16,-8)$
8. Which of the following ordered pairs satisfy the equation $y=1-x$. Circle all that apply, and show all supporting work
$(-7,8)$
$(3,-2)$
$(-1,0)$
$(-20,21)$
9. Which of the following ordered pairs satisfy the equation $y=-2 x$. Circle all that apply, and show all supporting work
$(6,-12)$
$(-1,2)$
$(4,-8)$
$(0,-2)$
$(0,0)$
10. Graph the equation $y=-4 x+2$.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


11. Graph the equation $y=\frac{2}{5} x-3$.

| $x$ | $y$ | Ordered Pair |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |


12. Graph the equation $y=3-x$.

| $x$ | $y$ | Ordered Pair |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


13. Graph the equation $4 x-2 y=12$.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


14. Graph the equation $x-y=4$.

| $x$ | $y$ | Ordered Pair |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |


15. Graph the equation $y=x$.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |


16. Graph the equation $y=\frac{2}{3} x$.

| $x$ | $y$ | Ordered Pair |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


17. Graph the equation $y=-4$.

| $x$ | $y$ | Ordered Pair |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |


18. Graph the equation $x=3$.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |


19. Complete the table below. Write the intercepts as ordered pairs.

| Equation | Vertical Intercept | Horizontal Intercept |
| :---: | :---: | :---: |
| $y=5 x-3$ |  |  |
| $y=4-x$ |  |  |
| $y=4 x$ |  |  |
| $y=3$ |  |  |
| $x=5$ |  |  |
| $x+2 y=8$ |  |  |
| $3 x-4 y=24$ |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Applications

20. The graph below shows Sally's distance from home over a 25 minute time period.

a. What is the input variable? $\qquad$
b. What are the units of the input variable? $\qquad$
c. What is the output variable? $\qquad$
d. What are the units of the output variable? $\qquad$
e. Sally is 4 miles from home after $\qquad$ minutes.
f. After 15 minutes, Sally is $\qquad$ miles from home.
g. Interpret the meaning of the ordered pair $(10,12)$.
21. Consider the following data set.

| Years Since 1980 | Sales (in millions of dollars) |
| :---: | :---: |
| 0 | 3.19 |
| 5 | 2.40 |
| 10 | 1.91 |
| 15 | 1.28 |
| 21 | 1.86 |
| 25 | 2.62 |
| 26 | 3.48 |

a. What is the input variable? $\qquad$
b. What is the output variable? $\qquad$
c. What were the sales in 2001 ? $\qquad$
d. In what year did sales total $\$ 1,280,000$ ? $\qquad$
e. In a complete sentence, interpret the meaning of the ordered pair (10, 1.91).
f. Use the values in the table to construct a properly scaled and labeled graph of the data.

22. The following data set gives the value of a car over time.

| Years since purchase | Value in Dollars |
| :---: | :---: |
| 0 | 20,025 |
| 1 | 17,822 |
| 2 | 15,862 |
| 3 | 14,117 |
| 5 | 11,182 |
| 8 | 7,883 |

a. What was the purchase price of the car?
b. After one year the car will be worth what percent of its original value?
c. After five years the car will be worth what percent of its original value?
d. Use the values in the table to construct a properly scaled and labeled graph of the data.

23. A pebble falls from a bridge into the river below.

| Time (seconds) | Height above the water (feet) |
| :---: | :---: |
| 0 | 132 |
| 0.5 | 128 |
| 1.0 | 116 |
| 1.5 | 96 |
| 2.0 | 68 |
| 2.5 | 32 |

a. What is the input variable? $\qquad$
b. What is the output variable? $\qquad$
c. How far did the pebble fall during the first second?
d. In a complete sentence, interpret the meaning of the ordered pair $(2,68)$.
e. Use the values in the table to construct a properly scaled and labeled graph of the data.

24. Jordan is saving money for emergencies (or a trip to Europe). She has $\$ 420$ under her mattress, and is adding $\$ 60$ to it each week.
a. Let $A$ represent the total amount of money under her mattress, and $w$ represent the number of weeks. Write an algebraic equation to represent this situation.
b. Use the equation in part a. to complete the table below.

| $w$ | 0 | 8 |  |  | 37 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ |  |  | 1800 | 2220 |  | 3000 |

c. Interpret the meaning of the ordered pair $(18,1500)$.
d. Use the values in the table to construct a properly scaled and labeled graph of the linear equation found in part a.


## Extension

25. Which of the following ordered pairs satisfy the inequality $y>2 x+1$.

## Circle all that apply.

$(1,8)$
$(3,7)$
$(-1,-10)$
$(-2,9)$
$(0,0)$
26. The graph below shows the distance traveled by a car. Draw a graph to represent the speed of the car during the same time period.

27. The graph below shows the speed of a car. Draw a graph to represent the distance traveled by the car during the same time period.


28. Draw a graph to represent each situation.
a. The height above the ground of a child swinging on a swing.

b. Bill is walking to school when he realizes that he forgot his math book. He runs home to get it, and then jogs to school.

c. The speed of a car stuck morning traffic.

$\qquad$

## Lesson 8 Assessment

1. Complete the table below. Show all work. Write the intercepts as ordered pairs.

| Equation | Vertical Intercept | Horizontal Intercept |
| :---: | :---: | :---: |
| $y=2-5 x$ |  |  |
| Ordered Pair: |  |  |
| Ordered Pair: |  |  |

2. The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you get the most benefits and reduce the risks when you exercise near your target heart rate. Usually this is when your exercise heart rate (pulse) is about $80 \%$ percent of your maximum heart rate. For adults 19 years of age and older, the formula $T=176-0.8 a$, gives the target heart rate, $T$, in beats per minute, for a person who is $a$ years of age.
a. Complete the table below.

| Age (years) | 20 | 25 | 38 |  | 70 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Target Heart <br> Rate (bpm) | 160 | 156 | 145.6 | 132 |  |

b. In a complete sentence, interpret the meaning of the ordered pair $(25,156)$.
c. Use the values in the table to construct a properly scaled and labeled graph of this linear equation.

## Lesson 9: Introduction to Functions

In this lesson we are introduced to the concept of a Function. We begin the study by learning about specific definitions and concepts related to functions, and examine different ways that functions can be represented. We then look at how functions are written using Function Notation.

## Mini-Lesson

Section 9.1: Relations and Functions
Section 9.2: Function Notation
Section 9.3: Formulas in Function Notation
Section 9.4: Domain and Range
Section 9.5: Practical Domain and Range

## Lesson 9 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 9

## Section 9.1: Relations and Functions

## Definitions

A RELATION is any set of ordered pairs.
A FUNCTION is a relation in which every input value is paired with exactly one output value.

## Table of Values

One way to represent the relationship between the input and output variables in a relation or function is by means of a table of values.

Example 1: Which of the following tables represent functions?

| Input | Output |
| :---: | :---: |
| 1 | 5 |
| 2 | 5 |
| 3 | 5 |
| 4 | 5 |

Yes
No

| Input | Output |
| :---: | :---: |
| 1 | 8 |
| 2 | -9 |
| 3 | 7 |
| 3 | 12 |

Yes No

| Input | Output |
| :---: | :---: |
| 2 | 4 |
| 1 | -5 |
| 4 | 10 |
| -3 | -87 |
| Yes |  | No

## Ordered Pairs

A relations and functions can also be represented as a set of points or ordered pairs.

Example 2: Which of the following sets of ordered pairs represent functions?

$$
\begin{aligned}
& A=\{(0,-2),(1,4),(-3,3),(5,0)\} \\
& B=\{(-4,0),(2,-3),(2,-5)\} \\
& C=\{(-5,1),(2,1),(-3,1),(0,1)\} \\
& D=\{(3,-4),(3,-2),(0,1),(2,-1)\} \\
& E=\{(1,3)\}
\end{aligned}
$$

## The Vertical Line Test

Example 3: On the graphs below, plot the points for A, B, C, and D from Example 2, then circle the "problem points."


## THE VERTICAL LINE TEST

a. If all vertical lines intersect the graph of a relation at only one point, the relation is also a function. One and only one output value exists for each input value.
b. If any vertical line intersects the graph of a relation at more than one point, the relation "fails" the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Example 4: Use the Vertical Line Test to determine which of the following graphs are functions.



Behavior of Graphs

| Increasing | Decreasing | Constant |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |

## Dependent and Independent Variables

In general, we say that the output depends on the input.

$$
\begin{aligned}
& \text { Output variable = Dependent Variable } \\
& \text { Input Variable = Independent Variable }
\end{aligned}
$$

If the relation is a function, then we say that the output is a function of the input.

## You Try

1. Is it a function? Circle "Yes" or "No" for each of the following.
a. Yes or No
b. Yes or No
c. Yes or No


| Input | Output |
| :---: | :---: |
| 4 | 12 |
| 6 | 12 |
| 8 | 12 |
| 10 | 12 |

$(2,-3),(-5,2),(-3,1)$

## Section 9.2: Function Notation: $f$ (input) $=$ output

Example 1: The function $V(m)$ represents value of an investment (in thousands of dollars) after $m$ months. Explain the meaning of $V(36)=17.4$.

## Ordered Pairs

## Example 2:

| Ordered Pair <br> (input, output) | Function Notation <br> $f$ (input) $=$ output |
| :---: | :---: |
| $(2,3)$ | $f(2)=3$ |
| $(-4,6)$ | $f\left(\_\right)=$ |
| $(\square,-)$ | $f(5)=-1$ |

Example 3: Consider the function: $f=\{(2,-4),(5,7),(8,0),(11,23)\}$

$$
f(5)=
$$

$f\left(\_\right)=0$

## Table of Values

Example 4: The function $B(t)$ is defined by the table below.

| $\boldsymbol{t}$ | 1 | 3 | 12 | 18 | 22 | 31 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{B}(\boldsymbol{t})$ | 70 | 64 | 50 | 39 | 25 | 18 |

$B(12)=$ $\qquad$ $B(t)=18$ when $t=$ $\qquad$

## Graph

Example 5: Consider the graph $g(x)$ of shown below.

$g(2)=$ $\qquad$

Ordered pair: $\qquad$
$g(0)=$ $\qquad$
Ordered pair: $\qquad$


Ordered pair: $\qquad$
$g\left(\_\quad\right.$ _ $)=1$
Ordered pair: $\qquad$
2. Complete the table.

| Ordered Pair | Function Notation |
| :---: | :---: |
| $(8,1)$ | $f(\ldots)=$ |
| $(\ldots, \ldots)$ | $f(0)=11$ |

3. The function $k(x)$ is defined by the following table:

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k(x)$ | 8 | 2 | -9 | 4 | 6 | 1 | 0 |

a. $k(2)=$ $\qquad$ b. $k(x)=1$ when $x=$ $\qquad$
4. At an ice cream factory, the total cost production is a function of the number of gallons of ice cream produced. The function $C(g)$, gives the cost, in dollars, to produce $g$ gallons of ice cream. Explain the meaning of $C(580)=126$ in terms of ice cream production.

# Section 9.3: Formulas in Function Notation 

Example 1: Let $f(x)=x^{2}-2 x+11$
a. Determine $f(-3)$.
b. Determine $f(0)$.

Example 2: Let $h(x)=2 x-5$
a. Determine $h(4)$.
b. For what value of $x$ is $h(x)=17$ ?

Example 3: Let $g(x)=71$
a) Determine $g(5)$.
b) Determine $g(-40)$.

## You Try

5. Let $r(a)=4-5 a$. Determine $r(-2)$. Write each answer using function notation and as an ordered pair.
6. Let $r(a)=4-5 a$. For what value of $a$ is $r(a)=19$ ? Write each answer using function notation and as an ordered pair.

## Section 9.4: Domain and Range

## Definitions

The DOMAIN of a function is the set of all possible values for the input (independent) variable.
The RANGE of a function is the set of all possible values for the output (dependent) variable.

## Domain and Range

Example 1: Consider the function below

| $x$ | -2 | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $k(x)$ | 3 | -7 | 11 | 3 | 8 |

Input values $\qquad$
Domain: $\qquad$ \}

Output values: $\qquad$
Range: $\qquad$

Example 2: Consider the function: $B=\{(2,-4),(5,7),(8,0),(11,23)\}$
Input values $\qquad$
Domain: \{ $\qquad$ \}

Output values: $\qquad$
Range: $\qquad$

Example 3: Consider the graph of $f(x)$ shown below:


Domain: $\qquad$ $\leq x \leq$ $\qquad$
Range: $\qquad$ $\leq f(x) \leq$ $\qquad$

Example 4: Determine the Domain and Range of each of the following graphs:

Domain


Range


Range

## You Try

Determine the Domain and Range of the functions below.
7.

| Input | Output |
| :---: | :---: |
| 4 | 12 |
| 6 | 12 |
| 8 | 12 |
| 10 | 12 |

Domain:

Range:
8. The graph of $f(x)$ is shown below:


Domain:

Range:

## Section 9.5: Practical Domain and Range

## Definitions

The Practical Domain of a function is the set of all possible values for the input variable that make sense in a given situation.

The Practical Range of a function is the set of all possible values for the output variable that make sense in a given situation.

Example 1: The gas station is currently charging $\$ 3.83$ per gallon for gas. The cost, $\mathrm{C}(n)$, in dollars, to fill up your car depends on the number of gallons, $n$, that you pump. Your car's tank can hold a maximum of 20 gallons of gas.

- In this situation, the input variable is $\qquad$ .
- The practical domain of this function is $\qquad$ .
- The output variable in this situation is $\qquad$ .
- The practical range of this function is $\qquad$ .


## You Try

9. A student's final course grade is a function of the number of points earned during the semester. The function $\mathrm{G}(n)$ gives the final course grade (as a percent) for a student who has earned $n$ out of the 1500 points possible throughout the semester.
a. In this situation, the input variable is $\qquad$ .
b. The practical domain of this function is $\qquad$ .
c. The output variable in this situation is $\qquad$ .
d. The practical range of this function is $\qquad$ .
$\qquad$

## Lesson 9 Practice Problems

## Skills Practice

1. In the space below, draw a graph that represents a function, and a graph that does NOT represent a function.


2. Are these functions? Circle yes or no.

| Input | Output |
| :---: | :---: |
| 1 | 5 |
| 2 | 5 |
| 3 | 5 |
| 4 | 5 |
| Yes | No |$\quad$| Input | Output |
| :---: | :---: |
| 1 | 3 |
| 1 | 4 |
| 2 | 5 |
| 2 | 6 |
| Yes | No |$\quad$| Input | Output |
| :---: | :---: |
| -23 | 695 |
| 6 | 85 |
| 302 | -80 |
| 12 | 0 |

3. Are these functions? Circle yes or no.
a. $\{(2,-4),(6,-4),(0,0),(5,0)\} \quad$ Yes No
b. $\{(1,1),(2,2),(3,3),(4,4)\} \quad$ Yes No
c. $\{(1,-8),(5,2),(1,6),(7,-3)\} \quad$ Yes No
4. Are these functions? Circle yes or no.



Yes No

5. The function $r(x)$ is defined by the following table of values:

| $x$ | 3 | 5 | 6 | 9 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $r(x)$ | -9 | 3 | 2 | 2 | 1 |

a. $r(9)=$ $\qquad$ b. $r(3)=$ $\qquad$
c. $r(\square)=1$
d. $r(\square)=3$
e. The domain of $r(x)$ is $\{$ $\qquad$ \}
f. The range of $r(x)$ is \{ $\qquad$ \}
6. Consider the function $g=\{(2,5),(0,6),(5,8),(-3,7)\}$
a. $g(0)=$
b. $g(5)=$ $\qquad$
c. $g($ $\qquad$ ) $=7$
d. $g($ $\qquad$ ) $=5$
e. The domain of $g$ is $\{$ $\qquad$ \}
f. The range of $g$ is $\{$ $\qquad$ \}
7. Given $f(4)=8, f(3)=11, f(0)=6$
a. The domain of $f$ is $\qquad$ \}
b. The range of $f$ is $\{$ $\qquad$ \}
c. Write the function $f$ as a set of ordered pairs.
8. Let $W(p)=4 p^{2}-9 p+1$. Show all steps. Write each answer in function notation and as an ordered pair.
a. Determine $W(5)$.
b. Determine $W(0)$.
c. Determine $W(-1)$.
d. Determine $W(-10)$.
9. Let $k(m)=8-3 m$. Show all steps. Write each answer in function notation and as an ordered pair.
a. Determine $k(5)$.
b. Determine $k(-3)$.
c. For what value of $m$ is $k(m)=29$ ?
d. For what value of $m$ is $k(m)=0$ ?
10. Let $R(t)=1500+40 t$. Show all steps. Write each answer in function notation and as an ordered pair.
a. Determine $R(18)$.
b. For what value of $t$ is $R(t)=3000$ ?
11. Let $h(x)=4$. Show all steps. Write each answer in function notation and as an ordered pair.
a. Determine $h(5)$.
b. Determine $h(81)$.
12. Let $p(x)=\frac{40}{2 x}$. Show all steps. Write each answer in function notation and as an ordered pair.
a. Determine $p(5)$.
b. Determine $p(-4)$.
c. For what value of $x$ is $p(x)=\frac{1}{4}$ ?
d. For what value of $x$ is $p(x)=20$ ?
13. Graph the function $S(t)=t+4$.

| $t$ | $S(t)$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


14. Graph the function $f(x)=4-2 x$.

| $x$ | $f(x)$ | Ordered Pair |
| :---: | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


15. Graph the function $p(r)=3$.

| $r$ | $p(r)$ | Ordered Pair |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



## Applications

16. The height, $h$ (in feet), of a golf ball is a function of the time, $t$ (in seconds), it has been in flight. A golfer strikes the golf ball with an initial upward velocity of 96 feet per second. The maximum height of the ball is 144 feet. The height of the ball above the ground is given by the function $h(t)=-16 t^{2}+96 t$.
a. Use the TABLE feature on your graphing calculator to complete the table below.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $h(t)$ |  |  |  |  |  |  |  |

b. Determine $h(3)$. Write a sentence explaining the meaning of your answer.
c. For what values of $t$ is $h(t)=0$ ? Explain the meaning of your answers.
d. Determine the practical domain $\qquad$
e. Determine the practical range $\qquad$
f. Use your graphing calculator to generate a graph of $h(t)$. Use the practical domain and range to determine a "good" viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.


$$
\begin{aligned}
& \mathrm{X} \min = \\
& \mathrm{Xmax}= \\
& \mathrm{Ymin}= \\
& \mathrm{Ymax}=
\end{aligned}
$$

17. Darby signs a 48 -month lease agreement for a new Chevrolet Camaro 2LT convertible. The function $T(n)=3491.88+580.85 n$ gives the total amount paid $n$ months after signing.
a. Using complete sentences, interpret $T(12)=10462.08$ in the context of the story.
b. Determine the practical domain of $T(n)$. Include units.
c. Determine the practical range of $T(n)$. Include units.
18. A candy company has a machine that produces candy canes. The table below is a partial list of the relationship between the number of minutes the machine is operating and the number of candy canes produced by the machine during that time period.

| Minutes $\boldsymbol{t}$ | 3 | 5 | 8 | 12 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Candy Canes $\quad \boldsymbol{C}(\boldsymbol{t})$ | 12 | 20 | 32 | 48 | 60 |

a. Include units. $C(12)=$
b. In a complete sentence and including all appropriate units, explain the meaning of your answer in part a.
19. The function $D(t)$ is shown below.

a. Determine $D(0)$ and interpret its meaning in a complete sentence.
b. For what value of $t$ is $D(t)=10$ ? Write a sentence explaining the meaning of your answer.
c. Determine the practical domain of $D(t)$.
d. Determine the practical range of $D(t)$.

## Extension

20. In a relation, we say that the output depends on the input. If the relation is a function, then we say that the output is a function of the input. For each of the following, identify the input variable and the output variable, and then determine if the relation is a function.
a. Is the outside temperature in Tempe, AZ a function of the time of day?

Input Variable: $\qquad$
Output Variable: $\qquad$
Function? Yes No
b. Is your letter grade a function of your numerical grade in the class?

Input Variable: $\qquad$
Output Variable: $\qquad$
Function? Yes No
c. Is your numerical grade a function of your letter grade?

Input Variable: $\qquad$
Output Variable: $\qquad$
Function? Yes No
$\qquad$
$\qquad$

## Lesson 9 Assessment

1. Paul is planning to sell bottled water at the local Lollapalooza. He buys 2 crates of water (2000 bottles) for $\$ 360$ and plans on selling the bottles for $\$ 1.50$ each. Paul's profit, $P$ in dollars, from selling $b$ bottles of water is given by the formula $P(b)=1.5 b-360$.
a. Determine $P(500)$ and interpret its meaning in a complete sentence.
b. For what value of $b$ is $P(b)=0$ ? Write a sentence explaining the meaning of your answer.
c. Determine the practical domain. Include units.
d. Determine the practical range. Include units. $\qquad$
2. The graph of $f(x)$ is given below.

a) Domain: $\qquad$ $\leq x \leq$ $\qquad$
b) Range $\qquad$ $\leq f(x) \leq$ $\qquad$
c) $f(0)=$ $\qquad$
d) $f(x)=0$ when $x=$ $\qquad$
3. Consider the following table of values. Fill in the blanks below, and identify the corresponding ordered pairs.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $g(x)$ | 1 | 4 | 8 | 6 | 5 | 0 | 2 |

$$
g(1)=
$$

$g(x)=1$ when $x=$ $\qquad$

## Lesson 10: Linear Functions, Part 1

Linear functions are such a part of our everyday life that usually we don't even realize it. Many of the characteristics of linear functions are taken as common sense. In this lesson, we take a closer look at those characteristics and investigate how to make use of them. We also investigate the special case of a linear equation that is not a linear function, the vertical line.

## Mini-Lesson

Section 10.1: Linear Functions
Section 10.2: Graphing Linear Functions
Section 10.3: Interpreting the Slope of a Linear Function
Section 10.4: The Equation of a Linear Function

Lesson 10 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$

## Mini-Lesson 10

## Section 10.1: Linear Functions

LINEAR FUNCTIONS

SLOPE $=\frac{\text { Change in OUTPUT }}{\text { Change in INPUT }}$

## Slope

Example 1: Determine the slope for each of the following:
a. $(-2,3)$ and $(4,-1)$

b. $(-3,-1)$ and $(4,2)$

c. $(3,2)$ and $(-1,2)$

d. $(2,-3)$ and $(2,1)$


1. Determine the slope of the line between the given points. $(-4,-1)$ and $(5,-6)$

## Section 10.2: Graphing Linear Functions

## USING THE SLOPE TO GRAPH A LINEAR FUNCTION

SLOPE $=\frac{\text { Change in OUTPUT }}{\text { Change in INPUT }}$

Example 1: Draw an accurate graph for each of the following
a. $(-2,-3)$ slope $\frac{1}{2}$
b. $(0,-1)$ slope $-\frac{2}{3}$


c. $(2,1)$ slope 3
d. $(1,-4)$ slope 0


e. $(5,2)$ undefined slope


## You Try

2. Sketch the graph of a linear function that passes through the point $(-3,4)$ with slope $=-\frac{3}{2}$.


Your line must extend accurately from edge to edge of the graph shown

Give the coordinates of two additional
points
on the line.

# Section 10.3: Interpreting the Slope of a Linear Function 

SLOPE $=\frac{\text { Change in OUTPUT }}{\text { Change in INPUT }} \quad$ Units of the slope:

Example 1: This graph shows the amount of water in a tub over a ten-minute time period.

a. Identify the vertical intercept and interpret its meaning.
b. Identify the horizontal intercept and interpret its meaning.
c. Determine the slope, and interpret its meaning.

## You Try

3. The graph below shows Sally's distance from home over a 30 minute time period.

a. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning.
b. Identify the horizontal intercept. Write it as an ordered pair and interpret its meaning.
c. Determine the slope, and interpret its meaning.

## Section 10.4: The Equation of a Linear Function

## Slope - Intercept Form

$$
\text { SLOPE-INTERCEPT FORM: } y=m x+b \quad y=b+m x \quad f(x)=m x+b
$$

Example 1: Fill in the table below.

| Equation | Slope | I, D, H, V | Vertical Intercept |
| :---: | :---: | :---: | :---: |
| $y=3 x+5$ |  |  |  |
| $y=8-x$ |  |  |  |
| $y=2 x$ |  |  |  |
| $y=-8$ |  |  |  |

Example 2: Determine the horizontal intercepts of each of the following.
a. $y=3 x+5$
b. $y=8-x$
c. $y=2 x$
d. $y=-8$

Example 3: The equation of a vertical line.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$

Two additional points on the line:

## You Try

4. Fill in the table below. Write intercepts as ordered pairs.
$\mathrm{I}=$ Increasing, $\mathrm{D}=$ Decreasing, $\mathrm{H}=$ Horizontal (Constant), V = Vertical

| Equation | Slope | I, D, H, V | Vertical Intercept |
| :---: | :---: | :---: | :---: |
| $y=x-11$ |  |  |  |
| $G(x)=-2 x$ |  |  |  |
| $x=5$ |  |  |  |

5. Draw and accurate graph of the function $y=\frac{3}{4} x-5$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$

Two additional points on the line:
$\qquad$

## Lesson 10 Practice Problems

## Skills Practice

1. Determine the slope of the line between each of the following pairs of points. Show all steps, and reduce your answer to lowest terms.
a. $(4,-5)$ and $(-2,3)$
b. $(-3,2)$ and $(1,8)$
c. $(5,-9)$ and $(5,2)$
d. $(2,-1)$ and $(-2,3)$
e. $(4,3)$ and $(12,-3)$
f. $(2,-4)$ and $(7,-4)$
2. Determine the slope of each of the lines shown below. Reduce your answers to lowest terms.


Slope $=$ $\qquad$
c.


Slope $=$ $\qquad$
e.


Slope $=$ $\qquad$
b.


Slope $=$ $\qquad$
d.


Slope $=$ $\qquad$
f.


Slope $=$ $\qquad$
3. Draw an accurate graph for each of the following by:
i. Plotting the point
ii. Using the slope to find at least two additional points
a. $(1,-2)$ with slope $=\frac{1}{4}$
b. $(5,-2)$ with slope $=-\frac{3}{2}$


c. $(3,0)$ with slope $=5$

e. $(4,-1)$ with undefined slope

d. $(4,-5)$ with slope $=-3$

f. $(-3,5)$ with slope $=0$

4. Complete the table below.

| Equation | Slope | I, D, H, V | Vertical Intercept |
| :---: | :--- | :--- | :--- |
| $y=x-2$ |  |  |  |
| $f(a)=6-4 a$ |  |  |  |
| $P(n)=3 n$ |  |  |  |
| $y=4$ |  |  |  |
| $x=7$ |  |  |  |
| $y=\frac{3}{5} x-4$ |  |  |  |

5. Determine the horizontal intercepts for each of the following.
a. $y=x-2$
b. $f(a)=6-4 a$
c. $P(n)=3 n$
d. $y=4$
e. $x=7$
f. $y=\frac{3}{5} x-4$
6. Draw an accurate graph of the function $f(x)=4 x+5$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
7. Draw an accurate graph of the function $y=\frac{2}{5} x-3$


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
8. Draw an accurate graph of the function $g(x)=3-x$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
9. Draw an accurate graph of the function $y=-2 x$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$

Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
11. Draw an accurate graph of the function $C(x)=\frac{1}{5} x$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
12. Draw an accurate graph of the function $y=x$.

13. Draw an accurate graph of the function $y=3-5 x$.


Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$

Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$

## Applications

14. The function $P(n)=455 n-1820$ represents a computer manufacturer's profit when $n$ computers are sold.
a. Identify the slope, and interpret its meaning in a complete sentence.
b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
c. Determine the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
15. John is a door-to-door vacuum salesman. His weekly salary is given by the linear function $\mathrm{S}(v)=200+50 v$, where $v$ is the number of vacuums sold.
a. Identify the slope, and interpret its meaning in a complete sentence.
b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
16. The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.

a. In a complete sentence, interpret the ordered pair (2, 140).
b. Identify the vertical intercept and interpret its meaning.
c. Determine the slope, and interpret its meaning.
d. At this rate, how far away from home will you be after 7 hours?
e. At this rate, how long will it take for you to be 680 miles from your home?
17. The function $V(n)=221.4+4.25 n$ gives the value, $V$ (in thousands of dollars) of an investment after $n$ years.
a. Identify the slope, and interpret its meaning in a complete sentence.
b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
c. Determine the horizontal intercept. Write it as an ordered pair and discuss its meaning.

## Extension

18. Determine the slope of each of the lines shown below.



Slope $=$ $\qquad$


Slope $=$ $\qquad$
19. Consider the linear equations shown below.
$y=\frac{2}{3} x-5$
$y=\frac{2}{3} x-1$
$y=\frac{2}{3} x+3$
$y=\frac{2}{3} x+7$
a. What do you notice about the equations of the lines given above?
b. Graph all of the lines on the graph below.

c. How are these lines geometrically related?
d. What can you conclude from your answers in part a and part c ?
$\qquad$
$\qquad$

## Lesson 10 Assessment

1. Determine the slope of the line between the points $(2,-1)$ and $(-2,3)$. Show all steps, and reduce your answer to lowest terms.
2. Draw an accurate graph of the function $g(x)=\frac{3}{8} x-2$. Identify the slope, and determine the exact coordinates of the horizontal and vertical intercepts. These points must be clearly shown one your graph.


Slope: $\qquad$

## Vertical Intercept:

$\qquad$

Horizontal Intercept: $\qquad$
3. Paul is planning to sell bottled water at the local Lollapalooza. Paul's profit, $P$ in dollars, from selling $b$ bottles of water is given by the formula $P(b)=1.75 b-364$.
a. Identify the slope, and interpret its meaning in a complete sentence.
b. Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

Ordered pair: $\qquad$
c. Determine the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

Ordered pair: $\qquad$

## Lesson 11: Linear Functions, Part 2

Lesson 11 continues the study of linear functions. In this lesson, we look at how to write linear equations in slope-intercept and general form and applications where these may be used. We also look at how to identify parallel and perpendicular lines. Finally, we will examine linear inequalities in two variables and graph the solution sets for these inequalities on the Cartesian plane.

## Mini-Lesson

Section 11.1: Writing Linear Equations in Slope-Intercept Form
Section 11.2: Parallel and Perpendicular Lines
Section 11.3: Applications - Slope-Intercept Form
Section 11.4: General Form $a x+b y=c$
Section 11.5: Applications - General Form
Section 11.6: Linear Inequalities in Two Variables
Section 11.7: Graphing Linear Inequalities in Two Variables

Lesson 11 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :---: | :---: | :---: |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$
$\qquad$

## Mini-Lesson 11

## Section 11.1: Writing the Equation of a Line in Slope-Intercept Form

Slope-Intercept Form $y=m x+b$

Example 1: Give the equation of the line in slope-intercept form
a. With $y$-intercept $(0,2)$ and slope -9
b. Passing through $(2,3)$ with slope -5
c. Passing through $(2,6)$ and $(4,16)$

Example 2: Give the equation of the linear function that would generate the following table of values. Use your calculator to check.

| $x$ | $f(x)$ |
| :---: | :---: |
| -5 | 238 |
| -3 | 174 |
| -1 | 110 |
| 1 | 46 |
| 7 | -146 |
| 12 | -306 |

Example 3: Give the equation of the linear function shown below.


Example 4: Give the equation of the horizontal line passing through the point $(1,3)$.

Example 5: Give the equation of the vertical line passing through the point $(1,3)$.

## You Try

1. Give the equation of the line passing through the points $(1,7)$ and $(3,-9)$.
2. Give the equation of the horizontal line passing through the point $(5,11)$.

## Section 11.2: Parallel and Perpendicular Lines

## Parallel Lines

The slopes of Parallel Lines are $\qquad$

Example 1: Give the equation of the line passing through the point $(8,3)$ that is parallel to the line $y=-2 x+3$.

## Perpendicular Lines

The slopes of perpendicular lines are $\qquad$
If Line 1 and Line 2 are perpendicular to each other, then

| Slope of line 1 | Slope of line 2 |
| :---: | :---: |
| $\frac{2}{3}$ |  |
| 5 |  |
| -8 |  |
| $-\frac{4}{5}$ |  |

Example 2: Give the equation of the line passing through the point $(8,3)$ that is perpendicular to the line $y=-2 x+3$.

## You Try

3. Give the equation of the line passing through the point $(-3,1)$ that is parallel to the line $y=8 x-5$.
4. Give the equation of the line passing through the point $(-3,1)$ that is perpendicular to the line $y=8 x-5$.

# Section 11.3: Applications - Slope-Intercept Form 

Example 1: You have just bought a new Sony 55" 3D television set for $\$ 2300$. The TV's value decreases at a rate of $\$ 250$ per year. Construct a linear function to represent this situation.

[^3]
## You Try

5. A new Chevrolet Corvette costs $\$ 56,980$. The car's value depreciates to $\$ 31,870$ after four years.
a. Determine a linear equation to represent this situation. Clearly indicate what each variable represents.
b. Identify the slope and explain its meaning.
c. Identify the vertical intercept, and explain its meaning.
d. Determine the horizontal intercept, and explain its meaning.

## Section 11.4: General Form: $a x+b y=c$

Example 1: Consider the linear equation $3 x-5 y=30$.
a. Write this equation in slope-intercept form.
b. Identify the slope.
c. Determine the vertical intercept.
d. Determine the horizontal intercept.

Example 2: Draw an accurate graph of the function $3 x+2 y=16$.


Slope-Intercept Form:

Slope: $\qquad$
Vertical Intercept:
Horizontal Intercept: $\qquad$

Two additional points on the line:

## You Try

6. Draw an accurate graph of the function $4 x-y=7$


Slope-Intercept Form:

Slope:
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
Two additional points line:

## Section 11.5: Applications - General Form

Example 1: Movie tickets cost $\$ 7$ for adults (matinee), $\$ 5.50$ for children. A total of $\$ 668$ was collected in ticket sales for the Saturday matinee.
a. Write an equation representing the total amount of money collected.
b. If 42 adult tickets were purchased for this matinee, how many children were there?

You Try
7. Tickets to a 3D movie cost $\$ 12.50$ for adults and $\$ 8.50$ for children. A total of $\$ 932$ was collected in ticket sales for the 7:15PM show.
a. Write an equation representing the total amount of money collected.
b. If 17 children's tickets were purchased, how many adults were there?

## Section 11.6: Linear Inequalities in Two Variables

The Solution Set
Example 1: Graph the equation $y=2 x-3$


Example 2: Which of the ordered pairs below satisfy the equation $y=2 x-3$ ?
$(5,3)$
$(2,1)$
$(0,0)$

Example 3: Which of the ordered pairs below satisfy the inequality $y \leq 2 x-3$ ?
$(5,3)$
$(2,1)$
$(0,0)$

Example 4: Graph the linear inequality $y \leq 2 x-3$.


Example 5: Which of the ordered pairs below satisfy the inequality? $y<2 x-3$
$(5,3)$
$(2,1)$
$(0,0)$

Example 6: Graph the linear inequality $y<2 x-3$.

8. Which of the ordered pairs below satisfy the linear inequality $y \geq 4-2 x$ ?
$(1,2)$
$(0,0)$
$(5,0)$
9. Which of the ordered pairs below satisfy the linear inequality $y<4-2 x$ ?

$$
(1,2)
$$

$(0,0)$
$(5,0)$

# Section 11.7: Graphing Linear Inequalities in Two Variables 

## Graphing The Solution Set of a Linear Inequality in Two Variables

Step 1: Rewrite the inequality as an equality statement.
Step 2: Graph the linear equation. This is the boundary of the solution region.
Step 3: Determine if the line should be solid or dotted.

- If the original inequality statement is either $<$ or $>$, draw a dotted line.
- If the original inequality statement is either $\leq$ or $\geq$, draw a solid line.

Step 4: Choose a test point and plug it into the original inequality.

- If the test point satisfies the inequality, shade in the direction of the test point.
- If the test point does not satisfy the inequality, shade in the opposite direction of the test point.

Example 1: Graph the inequality $y<5-3 x$.


Example 2: Graph the inequality $3 x-2 y \geq 6$.


Example 3: Graph the inequality $y \geq 2 x$.

$\square$ You Try
10. Graph the inequality $y>2 x-1$.

$\qquad$

## Lesson 11 Practice Problems

## Skills Practice

1. Determine the equation of the line between each of the following pairs of points.
a. $(4,-5)$ and $(2,3)$
b. $(-3,2)$ and $(1,8)$
c. $(5,-9)$ and $(5,2)$
d. $(2,-1)$ and $(-2,3)$
e. $(4,3)$ and $(12,-3)$
f. $(2,-4)$ and $(7,-4)$
2. Give the equation of the linear function that generates the following table of values. Write your answer in slope-intercept form.

| $x$ | $f(x)$ |
| :---: | :---: |
| -5 | 91 |
| -2 | 67 |
| 1 | 43 |
| 4 | 19 |
| 9 | -21 |

3. Give the equation of the linear function that generates the following table of values. Write your answer in slope-intercept form.

| $t$ | $C(t)$ |
| :---: | :---: |
| 5 | -1250 |
| 15 | -900 |
| 20 | -725 |
| 35 | -200 |
| 45 | 150 |

4. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

5. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

6. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

7. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

8. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

9. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

10. Give the equation of the horizontal line passing through the point $(-6,11)$. $\qquad$
11. Give the equation of the vertical line passing through the point $(4,7)$. $\qquad$
12. Give the equation of the $x$-axis. $\qquad$
13. Give the equation of the $y$-axis. $\qquad$
14. Give the equation of the line passing through the point $(1,-5)$ that is parallel to $y=12-8 x$.
15. Give the equation of the line passing through the point $(6,0)$ that is parallel to $y=9-\frac{3}{2} x$.
16. Give the equation of the line passing through the point $(10,3)$ that is perpendicular to $y=\frac{2}{5} x+1$.
17. Give the equation of the line passing through the point $(-12,-1)$ that is perpendicular to $y=3-4 x$.
18. Draw an accurate graph of the linear equation $2 x+3 y=6$.


## Slope-Intercept Form:

Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
19. Draw an accurate graph of the function $3 x-2 y=4$.


Slope-Intercept Form:

Slope: $\qquad$
Vertical Intercept: $\qquad$
Horizontal Intercept: $\qquad$
20. Graph the solution sets of each of the following linear inequalities.
a. $y>3-x$

b. $y \geq \frac{3}{5} x-1$

c. $4 x-y<3$

d. $x+y \leq-5$

e. $y>\frac{1}{2} x$
f. $y<-4$
g. $x \geq 2$


## Applications

21. A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes.
a. Determine the equation of the linear function that represents this situation. Let $\mathrm{C}(x)$ represent the number of candy canes produced in $x$ minutes. Write your answer in function notation.
b. Determine $\mathrm{C}(10)$. Write a sentence explaining the meaning of your answer.
c. What is the practical meaning of the slope of this linear function? Include units.
d. Determine horizontal intercept of this linear function. Write it as an ordered pair and interpret its meaning.
e. How many candy canes will this machine produce in 1 hour?
22. Your workplace is 20 miles from your house. The graph below shows the distance you are from your house if you leave work and drive in the opposite direction.

a. Determine the equation of the linear function that represents this situation. Clearly indicate what each variable represents.
b. Use the equation from part a to determine how long it would take for you to be 500 miles from your house. Express your answer in hours and minutes.
c. How far from your house would you be after 12 hours?
d. Interpret the meaning of the slope of $\mathrm{C}(x)$.
23. With good credit, and a $\$ 5000$ down payment, you can finance a new 2012 Chevrolet Camaro convertible for 60 months for $\$ 615.17$ per month.
a. Determine the equation of the linear function that represents this situation. Clearly indicate what each variable represents.
b. Use the equation from part a to determine the total payment over the 60 -month time period.
c. A new 2012 Chevrolet Camaro convertible has a base MSRP of $\$ 35,080$. Why is this value lower than your answer in part b?
24. At a concession stand, three hot dogs and five sodas cost $\$ 18.50$.
a. Let $h$ represent the price of each hot dog, and $s$ represent the price of each soda. Write a linear equation in general form to represent this situation.
b. If hot dogs cost $\$ 3.25$ each, how much is each soda?
25. The Science Museum charges $\$ 14$ for adult admission and $\$ 11$ for each child. The museum bill for a school field trip was $\$ 896$.
a. Write a linear equation in general form to represent this situation. Clearly indicate what each variable represents.
b. Nine adults attended the field trip. How many children were there?
26. Bill begins a 50 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 18 miles per hour.
a. Let $b$ represent the amount of time Bill spent bicycling before the chain broke, and $w$ represent the amount of time Bill spent walking. Write a linear equation in general form to represent this situation. (Hint: Distance $=$ rate $\cdot$ time .)
b. Bill had been riding his bike for two hours when the chain broke. Use the equation in part a to determine the amount of time he spent walking.

## Extension

27. The graph below shows the cost and revenue for a widget company. The function $\mathrm{R}(x)$ gives the revenue earned when $x$ widgets are sold. The function $\mathrm{C}(x)$ gives the total cost to produce $x$ widgets.

a. Identify the vertical intercept of $\mathrm{C}(x)$. Write it as an ordered pair, and interpret its meaning.
b. Determine the slope of $\mathrm{C}(x)$. Interpret its meaning.
c. Identify the vertical intercept of $\mathrm{R}(x)$. Write it as an ordered pair, and interpret its meaning.
d. Determine the slope of $\mathrm{R}(x)$. Interpret its meaning.
e. Point $P$ has coordinates $(40,100)$. Discuss the significance of this point in terms of the cost, revenue, and profit for this company.
28. *Refer to your course syllabus*
a. The Final Exam for this class is worth $\qquad$ $\%$ of your course grade.
b. Let $x$ represent the score you make on the Final Exam (as a percent), and $y$ represent your grade in the class (as a percent) just prior to taking the Final exam. Write a linear equation in general form to represent this situation.
Hint: If your final exam is worth $30 \%$ of your course grade, then everything else would be worth $100 \%-30 \%=70 \%$ of your course grade.
c. Suppose you have a $77 \%$ in the class just before taking the final exam. What score do you need to make on the Final Exam to earn a B ( $80 \%$ ) in the class? Assume that your instructor does not round up!
$\qquad$

## Lesson 11 Assessment

1. Determine the equation of the line between the points $(4,3)$ and $(12,-3)$. Your answer must be written in slope-intercept form.
2. Give the equation of the linear function shown below. Write your answer in slope-intercept form.

3. Give the equation of the vertical line passing through the point $(1,8)$. $\qquad$
4. Give the equation of the horizontal line passing through the point $(1,8)$. $\qquad$
5. Graph the solution set of linear inequality $3 x+2 y \leq-16$


## Lesson 12: Systems of Linear Equations

Our final lesson involves the study of systems of linear equations. In this lesson, we examine the relationship between two distinct linear equations. Specifically, we are looking for the point where the two equations are equal. In a graph of the two equations, this would be the point where the two lines intersect.

First we learn how to determine if a given point is a solution to the two equations and then we look at three different methods for finding the solution: Graphing, Substitution, and Elimination.

In addition, we examine special cases in which a system of equations has no solution, or infinitely many possible solutions.

## Mini-Lesson

Section 12.1: Systems of Equations
Section 12.2: The Substitution Method
Section 12.3: The Addition (Elimination) Method
Section 12.4: Applications

Lesson 12 Checklist

| Component | Required? <br> Y or N | Comments | Due | Score |
| :---: | :---: | :--- | :--- | :--- |
| Mini-Lesson |  |  |  |  |
| Online <br> Homework |  |  |  |  |
| Online |  |  |  |  |
| Quiz |  |  |  |  |
| Online |  |  |  |  |
| Test |  |  |  |  |
| Practice |  |  |  |  |
| Problems |  |  |  |  |
| Assessment |  |  |  |  |

$\qquad$

## Mini-Lesson 12

## Section 12.1: Systems of Equations

Two linear equations that relate the same two variables are called a system of equations.

## The Solution to a System of Equations

The SOLUTION to a system of equations is the POINT at which the graphs intersect. This is the single ordered pair that is a solution to both equations.

Types of Solutions to a Linear System of Equations:

1. One unique solution: The lines intersect at exactly one point.
2. No solution: The two lines are parallel and will never intersect.
3. Infinitely many solutions: This occurs when both lines graph as the same line.

Example 1: Verify that the point $(5,4)$ is a solution to the system of equations

$$
\begin{aligned}
& y=2 x-6 \\
& y=x-1
\end{aligned}
$$

## Solving a System of Linear Equations by Graphing

Example 2: Solve the system of equations by graphing. Check your answer.

$$
\begin{aligned}
& y=6-\frac{2}{3} x \\
& y=x+1
\end{aligned}
$$



Example 3: Solve the system of equations by graphing. Check your answer.

$$
\begin{aligned}
& 4 x-3 y=-18 \\
& 2 x+y=-4
\end{aligned}
$$



Example 4: Solve the system of equations by graphing. Check your answer.
$x-3 y=3$ $3 x-9 y=-18$


Example 5: Solve the system of equations by graphing. Check your answer.

$$
\begin{aligned}
& 2 x+y=3 \\
& 6 x+3 y=9
\end{aligned}
$$



## You Try

1. Solve the system of equations by graphing. Check your answer.

$$
\begin{aligned}
& x-y=2 \\
& x+y=6
\end{aligned}
$$



Verify that your solution is correct:

## Section 12.2: The Substitution Method

## Using Substitution to Solve a Linear System of Equations

Step 1: Solve one of the equations of the system for one of the variables.
Step 2: Substitute the expression for the variable obtained in step 1 into the other equation.
Step 3: Solve the equation.
Step 4: Substitute the result back into one of the original equations to find the ordered pair solution.

Step 5: Check your result by substituting your result into either one of the original equations.

Example 1: Solve the system of equations using the Substitution Method.

$$
\begin{aligned}
& 3 x-2 y=16 \\
& 2 x+y=20
\end{aligned}
$$

Example 2: Solve the system of equations using the Substitution Method.
$5 x-4 y=9$
$x-2 y=-3$

Example 3: Solve the system of equations using the Substitution Method.

$$
\begin{aligned}
& 3 x+y=5 \\
& 6 x+2 y=11
\end{aligned}
$$

Example 4: Solve the system of equations using the Substitution Method.

$$
\begin{aligned}
& x-y=-1 \\
& y=x+1
\end{aligned}
$$

2. Solve the system of equations using the Substitution Method. Check your answer.

$$
\begin{aligned}
& x-2 y=-11 \\
& 5 x+2 y=5
\end{aligned}
$$

## Section 12.3: The Addition (Elimination) Method

Consider the following systems of equations.

$$
\begin{aligned}
& x-2 y=-11 \\
& 5 x+2 y=5
\end{aligned}
$$

## Using the Addition (Elimination) Method to Solve a Linear System of Equations

Step 1:"Line up" the variables.
Step 2: Determine which variable you want to eliminate. Make those coefficients opposites.
Step 3: Add straight down (one variable should "drop out")
Step 4: Solve resulting equation
Step 5: Substitute this result into either of the ORIGINAL equations
Step 6: Solve for the variable
Step 7: CHECK!!!!!!! Plug solution into BOTH equations!

Example 1: Solve the system of equations using the Addition (Elimination) Method.
$4 x-3 y=-15$
$x+5 y=2$

Example 2: Solve the system of equations using the Addition (Elimination) Method.

$$
\begin{aligned}
& 3 x-2 y=-12 \\
& 5 x-8 y=8
\end{aligned}
$$

Example 3: Solve the system of equations using the Addition (Elimination) Method.

$$
\begin{aligned}
& 7 x-2 y=41 \\
& 3 x-5 y=1
\end{aligned}
$$

## You Try

3. Solve the system of equations using the Addition (Elimination) Method.

Check your answer.
$2 x+3 y=18$
$x-y=4$

## Section 12.4: Applications

Example 1: Movie tickets cost $\$ 7$ for adults (matinee), $\$ 5.50$ for children. There are 218 seats in the theater. A total of $\$ 1,463$ was collected in ticket sales for the sold-out Saturday matinee. How many adults and how many children were in the theater?
a. Write an equation representing the total number of tickets sold.
b. Write an equation representing the total amount of money collected from the sale of all tickets.
c. Solve this system of linear equations.

## You Try

4. Tickets to a 3D movie cost $\$ 12.50$ for adults and $\$ 8.50$ for children. The theater can seat up to 180 people. A total of $\$ 1,826$ was collected in ticket sales for the sold-out $7: 15 \mathrm{PM}$ show. Determine the number of adult tickets and the number of children's tickets that were sold.
a. Write an equation representing the total number of tickets sold. Clearly indicate what each variable represents.
b. Write an equation representing the total amount of money collected from the sale of all tickets.
c. Solve this system of linear equations.

Number of adult tickets sold: $\qquad$
Number of children's tickets sold: $\qquad$
$\qquad$

## Lesson 12 Practice Problems

## Skills Practice

1. Is the point $(6,1)$ a solution to the system of equations $\begin{aligned} & y=x-5 \\ & y=2 x+4\end{aligned}$ ? YES NO You must show correct work to justify your answer.
2. Is the point $(-2,5)$ a solution to the system of equations

$$
\begin{aligned}
2 x+y & =1 \\
3 x-2 y & =-16
\end{aligned} ? \quad \text { YES } \quad \text { NO }
$$ You must show correct work to justify your answer.

3. Is the point $(5,3)$ a solution to the system of equations $\begin{aligned} & 3 x-2 y=9 \\ & 2 x+5 y=4\end{aligned}$ ? YES NO You must show correct work to justify your answer.
4. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{aligned}
& y=7-x \\
& y=3 x-5
\end{aligned}
$$

Solution: $\qquad$
5. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{aligned}
& x-y=-2 \\
& x+y=4
\end{aligned}
$$

Solution: $\qquad$
6. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{aligned}
& x-2 y=10 \\
& 5 x-y=-4
\end{aligned}
$$

Solution: $\qquad$
7. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{aligned}
& 3 x-y=8 \\
& -3 x+y=1
\end{aligned}
$$

Solution: $\qquad$
8. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{aligned}
& x+2 y=-4 \\
& 2 x+4 y=-8
\end{aligned}
$$

Solution: $\qquad$
9. Solve the system of equations using the substitution method. Show all steps. $5 x+y=2$
$3 x-4 y=15$
Solution: $\qquad$
10. Solve the system of equations using the substitution method. Show all steps.
$2 x+y=8$
$6 x+3 y=24$

Solution: $\qquad$
11. Solve the system of equations using the substitution method. Show all steps. $x-y=9$
$5 x+3 y=21$
Solution: $\qquad$
12. Solve the system of equations using the addition (elimination) method. Show all steps. $-3 x+2 y=12$
$x+y=16$
Solution: $\qquad$
13. Solve the system of equations using the addition (elimination) method. Show all steps. $3 x-2 y=-12$
$12 x-8 y=22$
Solution: $\qquad$
14. Solve the system of equations using the addition (elimination) method. Show all steps.

$$
\begin{aligned}
& 3 x+2 y=-18 \\
& 4 x-3 y=-24
\end{aligned}
$$

Solution: $\qquad$
15. Solve the system of equations using the addition (elimination) method. Show all steps. $5 x+2 y=-10$ $3 x+4 y=8$

Solution: $\qquad$

## Applications

16. Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $\$ 80$ consultation fee plus $\$ 14$ per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $\$ 30$ per hour for the actual work.
a. Write an equation that describes the cost, C, if you hire The Greenhouse for $h$ hours of work.
b. Write a second equation that describes Garden Pros' charge, C, for $h$ hours of work.
c. Solve this system of linear equations. Write your answer as an ordered pair.
d. Interpret the solution in a complete sentence.
e. Your yard needs a lot of work, and you anticipate that the job will take at least 6 hours. Which service do you choose? Why?
17. The graph below shows the cost and revenue for a widget company. The function $\mathrm{R}(x)$ gives the revenue earned when $x$ widgets are sold. The function $\mathrm{C}(x)$ gives the total cost to produce $x$ widgets.

a. Point $P$ has coordinates $(40,100)$. Discuss the significance of this point in terms of the cost, revenue, and profit for this company.
b. What happens if fewer than 40 widgets are sold?
c. What happens if more than 40 widgets are sold?
18. At a concession stand, five hot dogs and five sodas cost $\$ 30$. Two hot dogs and four sodas cost $\$ 15$. Determine the price of each hot dog and each soda.

Price for each soda: $\qquad$
Price for each hot dog: $\qquad$
19. The Science Museum charges $\$ 14$ for adult admission and $\$ 11$ for each child. The total bill for 68 people from a school field trip was $\$ 784$. How many adults and how many children went to the museum?

Number of children $\qquad$
Number of adults $\qquad$
20. Tickets to a 3D movie cost $\$ 12.50$ for adults and $\$ 8.50$ for children. The theater can seat up to 260 people. A total of $\$ 1,734$ was collected in ticket sales for the $7: 15 \mathrm{PM}$ show, in which only $60 \%$ of the tickets were sold. How many adults and how many children were in the theater?

Number of children $\qquad$
Number of adults $\qquad$
21. Emery invested $\$ 10,000$ in two mutual funds. Fund A earned $4 \%$ profit during the first year, while Fund B suffered a $2 \%$ loss. If she received a total of $\$ 130$ profit, how much had she invested in each mutual fund?

Amount invested in Fund A: $\qquad$
Amount invested in Fund B: $\qquad$
22. Bill begins a 100 mile bicycle ride. Unfortunately, his bicycle chain breaks, and he is forced to walk the rest of the way. The whole trip takes 6 hours. If Bill walks at a rate of 4 miles per hour, and rides his bike at a rate of 20 miles per hour, find the amount of time he spent walking. Write your answer in a complete sentence. (Hint: Distance = rate • time.)

## Extension

23. The functions $f(x)$ and $g(x)$ are defined by the following table. At what point is $f(x)=g(x)$ ?

| $\boldsymbol{x}$ | $\mathbf{- 2}$ | $\mathbf{- 1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 5 | -3 | 0 | 4 | 5 | 8 | -2 |


| $\boldsymbol{x}$ | $\mathbf{- 2}$ | $\mathbf{- 1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{g}(\boldsymbol{x})$ | 6 | 3 | 2 | 4 | 8 | 0 | 9 |

Solution (write the ordered pair): $\qquad$
24. Construct a system of linear equations (in slope-intercept form) that has the ordered pair $(3,5)$ as a solution.
25. Construct a system of linear equations (in general form) that has the ordered pair $(2,4)$ as a solution.
$\qquad$
$\qquad$

## Lesson 12 Assessment

1. Solve the system of equations by graphing. Your lines must extend accurately to the edge of the graph. Verify that your solution is correct.


$$
\begin{gathered}
4 x-3 y=-18 \\
3 x+y=-7
\end{gathered}
$$

Solution: $\qquad$
2. Solve the system of equations using the substitution method. Show all steps.

$$
\begin{aligned}
2 x-3 y & =-19 \\
x+2 y & =8
\end{aligned}
$$

Solution: $\qquad$
3. Jamaal invested $\$ 10,000$ in two mutual funds. Fund A earned $6 \%$ profit during the first year, and Fund B earned $2 \%$ profit. If he received a total of $\$ 374$ profit, how much had he invested in each mutual fund?

Amount invested in Fund A: $\qquad$

Amount invested in Fund B: $\qquad$

## Appendix A: You-Try Answers

## Lesson 1

You Try

1. 9
2. 7
3. $\frac{2}{3}$
4. $\frac{21}{5}$
5. $3 \frac{2}{11}$
6. $\frac{3}{5}=\frac{6}{10}=\frac{15}{25}$ Answers will vary.
7. a. $\frac{19}{15}=1 \frac{4}{15}$
b. $\frac{2}{5}$
c. $\frac{9}{10}$
d. $\frac{3}{5}$
e. $\frac{3}{35}$
f. $\frac{25}{32}$
8. $|-5|=5$
$-|-5|=-5$
9. a. -16
b. 8

## Lesson 2

## You Try

1. -39
2. After 3 seconds, the ball is 96 feet high.
3. a. 4
b. -8
c. 2
d. 1
e. $-2, m$
4. a. $-8 x$
b. $3 a^{2}-4 a+2$
5. $-15 x^{2}+6 x-24$
6. $6 x^{2}+6 x+11$
7. $x-2$

Lesson 3
You Try

1. Complete the table.

| Polynomial | Name | Leading <br> Coefficient | Constant Term | Degree |
| :---: | :---: | :---: | :---: | :---: |
| $3 n^{2}-2 n+8$ | Trinomial | 3 | 8 | 2 |
| $4 x$ | Mononomial | 4 | 0 | 1 |
| $x^{2}-7$ | Binomial | 1 | -7 | 2 |

2. $-5 x^{3}+5 x+1$
3. a. $12 x^{8}$
b. $-54 x^{7}$
c. $g^{12}$
d. 2
4. a. $-3 x^{7}-18 x^{5}+15 x^{3}$
b. $15 x^{2}-14 x-8$
c. $4 p^{2}-20 p+25$
5. $x^{2}+7 x+10$
6. $x^{2}+7 x$

## Lesson 4

## You Try

1. a. $\frac{9 a^{20}}{49}=\frac{9}{49} a^{20}$
b. $\frac{2 x^{2} y^{3}}{3}=\frac{2}{3} x^{2} y^{3}$
2. 

a. $7 a^{2}$
b. $n^{3}$
c. $\frac{2 w^{2} x^{3}}{3}=\frac{2}{3} w^{2} x^{3}$
d. $\frac{2}{27 x^{6}}$
3. a. $\frac{11}{3} x-5$
b. $1+\frac{5}{3 x}-\frac{4}{x^{2}}$
4. a. 490,000
b. .0015
5. a. $6.1 \times 10^{-7}$
b. $5.43 \times 10^{9}$

## Lesson 5

You Try

1. Yes
2. Yes
3. a. $x=-52$
b. $n=-\frac{10}{3}$
c. $x=-14$
d. $w=-15$
4. a. $x=18$
b. $w=8$
c. $x=-12$
5. a. $m=-1$
b. $x=2$
6. You can take 10 credits.

## Lesson 6

1. a. $50+36 h=212, h=$ the number of hours worked
b. If the cost is $\$ 212$, then the landscapers worked for 4.5 hours.
2. 2.175
3. If he works 20 hours, he will earn $\$ 375$.
4. $\$ 13.54$
5. $\$ 417.94$
6. His sales would have to be $\$ 10,000$.
7. $y=\frac{3}{x z}$
8. $y=5 x-2$
9. $n=-3,0,4.99$
10. $a \leq 2$
11. 


6. $x \leq 3$, Interval Notation: $(-\infty, 3]$
7. $x>7$, Interval Notation: $(7, \infty)$
8. $3.79 g \leq 20$, where $g=$ the number of gallons pumped

Up to 5.27 gallons of gasoline can be purchased for under $\$ 20$
9. $w=-5$
$w=0$
10. $0<n \leq 8$
11.

| $5<x<11$ | $(5,11)$ |
| :---: | :---: |
| $-3<x \leq 1$ | $(-3,1]$ |
| $-1 \leq x<4$ | $[-1,4)$ |

12. $x=-8,8$

| $\|x\|<3$ | List some values in the solution set: $-2,-1,0,1,2$. Answers will vary. |
| :---: | :---: |
| $\|x\| \geq 3$ | List some values in the solution set: $-5,-4,4,5$. Answers will vary. |

## Lesson 8

## You Try

2. 

a. Elapsed time
b. 62 feet
c. 2.4 seconds
d. After 1 second, the ball was 43 feet above the ground.
4.

| $x$ | $y$ | Ordered Pair |
| :---: | :---: | :---: |
| 2 | 4 | $(2,4)$ |
| 0 | 3 | $(0,3)$ |
| -2 | 2 | $(-2,2)$ |

5. 

| Equation | Vertical Intercept | Horizontal Intercept |
| :---: | :---: | :---: |
| $y=24-6 x$ | $(0,24)$ | $(4,0)$ |
| $5 x-3 y=30$ | $(0,-10)$ | $(6,0)$ |

6. 




## Lesson 9

## You Try

1. a. Yes
b. Yes
c. Yes
2. 

| Ordered Pair | Function Notation |
| :---: | :---: |
| $(8,1)$ | $f(8)=1$ |
| $(0,11)$ | $f(0)=11$ |

3. a. $k(2)=6$
b. $k(x)=1$ when $x=3$
4. It costs $\$ 126$ to produce 580 gallons of ice cream.
5. $r(-2)=14 \quad(-2,14)$
6. $r(-3)=19 \quad(-3,19)$
7. Domain: $\{4,6,8,10\}$ Range: $\{12\}$
8. Domain: $-15 \leq x \leq 20$ Range: $-200 \leq f(x) \leq 400$
9. a. n, number of points earned
b. $0 \leq n \leq 1500$ points
c. $G(n)$, final course grade
d. $0 \leq G(n) \leq 100$ percent

## Lesson 10

## You Try

1. Slope $=-\frac{5}{9}$
2. Additional Points: $(-1,1),(1,-2),(4,-5), \ldots$
3. a. Vertical Intercept $=(0,20)$ Sally was 20 miles from home after 0 minutes.
b. Horizontal Intercept $=(30,0)$ Sally arrives at home after 30 minutes.
c. Slope $=-2 / 3$ mile per minute

Sally's distance from home is decreasing at a rate of $2 / 3$ mile per minute.
4.

| Equation | Slope | I, D, H, V | Vertical Intercept |
| :---: | :---: | :---: | :---: |
| $y=x-11$ | 1 | I | $(0,-11)$ |
| $G(x)=-2 x$ | -2 | D | $(0,0)$ |
| $x=5$ | Undefined | V | None |

5. Draw and accurate graph of the function $y=\frac{3}{4} x-5$.


Slope: $\frac{3}{4}$

Vertical Intercept: $(0,-5)$

Horizontal Intercept: $\left(\frac{20}{3}, 0\right)$

Two additional points on the line:
$(-4,-8),(4,-2),(8,1), \ldots$

## Lesson 11

## You Try

1. $y=-8 x+15$
2. $y=11$
3. $y=8 x+25$
4. $y=-\frac{1}{8} x+\frac{5}{8}$
5. a. $V=-6277.50 n+56980$, where V is the value of the car (in dollars), and $n$ represents the number of years you have owned the car.
b. Slope $=-6277.5$. The value of the car decreases $\$ 6277.59$ a year.
c. Vertical Intercept $=(0,56980)$. When the car was new it was worth $\$ 56980$.
d. Horizontal Intercept $=(9.1,0)$ After a little more than 9 years the car will be worth $\$ 0$.
6. 

Slope-Intercept Form: $\quad y=4 x-7$
Slope: 4
Vertical Intercept: $(0,-7)$
Horizontal Intercept: $\left(\frac{7}{4}, 0\right)$
Two additional points line: $(1,-3),(2,1),(3,5),(4,9)$
7. a. $12.50 \mathrm{~A}+8.50 \mathrm{C}=932, \mathrm{~A}=$ number of adults, $\mathrm{C}=$ number of children
b. There were 63 adult tickets sold.
8. $(1,2)$
9. $(0,0)$
10. $y>2 x-1$


## Lesson 12

$\square$ You Try

1. $x=4, y=2,(4,2)$

2. $(-1,5)$
3. $(6,2)$
4. a. $\mathrm{A}=$ number of adult tickets, $\mathrm{C}=$ number of child tickets $\quad \mathrm{A}+\mathrm{C}=180$
b. $12.5 \mathrm{~A}+8.5 \mathrm{C}=1826$

106 Children's tickets sold, 74 adult tickets sold.


[^0]:    Example 2: You have just bought a new Sony 55" 3D television set for $\$ 1,600$. The value of the television set decreases by $\$ 250$ per year. How long before the television set is worth half of its original value?

[^1]:    Example 3: Tommy's grandma gave him a $\$ 50$ gift card to Toys $R$ Us for his birthday. Sales tax is currently $9 \%$. Determine the price of the most expensive toy Tommy can buy with the $\$ 50$ gift card.

[^2]:    Example 2: Sean owns a business that builds computers. The fixed operating costs for his business are $\$ 2,700$ per week. In addition to fixed operating costs, each computer costs $\$ 600$ to produce. Each computer sells for $\$ 1,500$. Write an inequality that can be used to determine the number of computers Sean needs to sell in order make a profit each week. Solve the inequality, and write your answer in a complete sentence.

[^3]:    Example 2: In 1998, the cost of tuition at a large Midwestern university was $\$ 144$ per credit hour. In 2008, tuition had risen to $\$ 238$ per credit hour. Determine a linear equation to represent the cost, C , of tuition as a function of $x$, the number of years since 1990.

