LESSON 7 – RATIOS, RATES, & PROPORTIONS

INTRODUCTION

In Lessons 2, 3, & 4, we spent time building a base of knowledge about fractions and operations with fractions. In this lesson, we will expand on that knowledge and spend time learning about and solving problems with a special kind of fraction called a *ratio.*

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Write and simplify *ratios*	1, 2, YT5a
Write and simplify *rates*	3, 4, YT5b
Compute *unit rates*	6, 7, 8, YT9
Solve proportions using *cross-products*	10, 11, 12, YT13
Solve applications using *proportional reasoning*	14, YT15

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Ratio
- Rate
- Unit Rate
- Proportion
- Cross Product
- Cross Product Method
- Proportional Reasoning

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

RATIOS & RATES

Write a *ratio* to compare two different quantities. Units are important and are always included if they are present to begin with. The examples below demonstrate the different notations you may see when writing ratios.

Example 1: Write "8 feet to16 feet" as a ratio in simplest form.

Example 2: Write "6:18" as a ratio in simplest form.

If the quantities you are comparing have different units, then your ratio is known as a *rate.* Units are especially important here and should absolutely be included.

Example 3: Write "12 miles in 10 hours" as a ratio in simplest form.

Example 4: In a small bag of mixed nuts, 15 were peanuts, 20 were almonds, and 5 were Brazil nuts. Write the ratio of peanuts to almonds in simplest form.

Note: With ratios, the units will cancel out. With rates, the units will not cancel out.

YOU TRY

5. Use the information to write a ratio in simplest form. Indicate if the ratio is also a rate.

a. 5 feet:10 feet

b. 12 geese to 15 ducks

UNIT RATES

A *unit rate* is a special kind of rate in which the denominator of the ratio is equal to 1. This kind of rate allows for easier comparison of different rates as seen in the example below. As with rates, units are essential and must be included.

Example 6: Which is faster, “12 miles in 10 hours” or “10 miles in 8 hours”? Use unit rates to compare.

Example 7: Determine which bag of Cheetos is the better buy.

Bag A: $4.99 for 20.50 oz   Bag B: $4.29 for 12.50 oz

Example 8: Write each of the following as a unit rate:

a. There are 5280 feet in a mile

b. There are 60 seconds per each minute

c. Gasoline costs $3.49 a gallon

YOU TRY

9. Amazon.com recently advertised the following choices for ibuprofen tablets (200mg). Use unit rates to determine which is the better buy.

Option 1: 360 pills for $15.45   Option 2: 300 pills for $12.98

PROPORTIONS & PROPORTIONAL REASONING

In Example 3, we were given the rate, "12 miles in 10 hours" which we simplified to "6 miles in 5 hours". Let's see how we might write that as a formal mathematical statement of equality:

$$\frac{12\ \text{miles}}{10\ \text{hours}} = \frac{6\ \text{miles}}{5\ \text{hours}}$$

The statement above is called a *proportion* because it sets two rates (or ratios) equal to each other. Because the above rates are equivalent, the equality statement is true.

Suppose, however, that the following problem was posed:

"If George walks 6 miles in 5 hours, how far would he walk in 10 hours?"

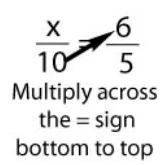
We will use the concept of *variable* from Lesson 6 to set up the following *proportion:*

$$\frac{x\ \text{miles}}{10\ \text{hours}} = \frac{6\ \text{miles}}{5\ \text{hours}}$$

The distance George walks in 10 hours is our unknown value and is represented by the *variable x*. Technically, in this problem, we know that our solution for x is 12. But how would we determine that? First, because our ratios of units are the same (miles/hours) we can simplify our statement this way:

$$\frac{x}{10} = \frac{6}{5}$$

Then, we can use one *cross-product* to rewrite as follows:



$$x = \frac{6 \cdot 10}{5}$$

And finally, we can write our final solution as $x = \frac{60}{5} = 12$.

The final answer to our original question, "if George walks 6 miles in 5 hours, how far would he walk in 10 hours" is that George could walk 12 miles in 10 hours. We solved this problem using *proportional reasoning,* one of the most-used problem solving techniques in mathematics.

The following examples will illustrate additional ways to work with and solve proportions using the *cross-product method*.

Example 10: Use the cross-product method to determine the value for t in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{3}{4}=\frac{t}{40}$

b. $\frac{t}{2}=\frac{3}{5}$

Example 11: Use the cross-product method to determine the value for x in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{6}{12}=\frac{18}{x}$

b. $\frac{2.3}{x}=\frac{4.1}{5.6}$

Example 12: Use the cross-product method to determine the value for r in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{r}{5}=3$

b. $\frac{1/2}{4}=\frac{8}{r}$

YOU TRY

13. Solve the proportions showing all possible steps. Round your answer to the nearest hundredth as needed.

a. $\frac{x}{12} = \frac{3}{6}$

b. $\frac{6}{5} = \frac{10}{p}$

APPLICATIONS OF PROPORTION

Example 14: Ten gallons of water leak from a hose in 20 hours. At this rate, how much water will leak in 10 days? Practice circling the GIVENS and underlining the GOALS to start your problem-solving process.

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

15. Mary earned $112.50 last week working 12 hours at her part-time job. If she works 15 hours this week and is paid the same rate, how much will she earn? Use proportional reasoning to determine your result. Round to the nearest cent.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE: