LESSON 4 – FRACTION MULTIPLICATION & DIVISION

INTRODUCTION

Now that we have learned how to add and subtract fractions, we will work with the remaining two major mathematical operations: multiplication and division.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Multiply fractions	1, YT2
Solve applications of fraction multiplication	3, YT4, YT14
Divide fractions	5, YT6
Solve applications of fraction division	7, YT8
Use correct *order of operations* when working with fractions	9, 10, YT11
Compute *special cases* with fractions (multiply or divide by 0).	12, 13

KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Multiply Fractions
- Of
- Divide Fractions
- Reciprocal
- Exponent
- Evaluate
- Order of Operations
- PEMDAS
- Multiplication by Zero
- Division by Zero

LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

MINILESSON

MULTIPLYING FRACTIONS

When would we ever need to multiply fractions? Let's go back to the first pizza example in the last lesson. There, we looked at the following example as an addition problem.

Word Description	Mathematical Computations ADDITION
Josh ate slices of size $\frac{1}{6}$ and $\frac{1}{6}$	Josh ate $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}$ or $\frac{1}{3}$ of the pizza

What if we look at this as multiplication instead? How would that look different?

Word Description	Mathematics Computations MULTIPLICATION
Josh ate 2 slices of size $\frac{1}{6}$	Josh ate $2\cdot\frac{1}{6}=\frac{2}{1}\cdot\frac{1}{6}=\frac{2\cdot1}{1\cdot6}=\frac{2}{6}=\frac{1}{3}$ of the pizza

Let's break apart the mathematics step-by-step and see what happens:

$2\cdot\frac{1}{6}$	Original problem
$=\frac{2}{1}\cdot\frac{1}{6}$	Write 2 as a fraction
$=\frac{2\cdot1}{1\cdot6}$	Multiply straight across
$=\frac{2}{6}$	Compute multiplication
$=\frac{1}{3}$	Reduce

Steps to multiply fractions (short list):
1. Convert any whole numbers to fractions.
2. Multiply straight across and reduce if possible.

Notice that when multiplying, we do NOT obtain a common denominator. Let's apply these rules to some examples and see how more complicated problems work.

Example 1: Multiply each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{1}{4}\cdot\frac{3}{2}=$

b. $\frac{5}{8}\cdot 4=$

c. $2\frac{1}{5}\cdot 3\frac{1}{9}=$

d. $\frac{6}{12}\cdot\frac{14}{24}=$

e. $\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{3}{4}=$

f. $1\frac{1}{4}\cdot 3\frac{2}{3}\cdot\frac{3}{5}=$

Let's modify our fraction multiplication rules to include the new ideas in the examples.

Steps to multiply fractions (full list):
1. Convert any whole numbers to fractions.
2. Convert any mixed numbers to improper fractions
3. Multiply straight across.
4. Reduce along the way if possible.
5. Present final, reduced answer at the end.

NOTE: We do not obtain a common denominator when multiplying fractions!

YOU TRY

2. Multiply each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{4}{5}\cdot 10=$

b. $2\frac{1}{2}\cdot 1\frac{3}{4}=$

APPLICATIONS OF FRACTION MULTIPLICATION

Example 3: Matt was training for a marathon and had a 20-mile run listed on his training calendar. If he only completed $\frac{3}{4}$ of the run, how far did he go?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

In mathematics, the word "of" often implies multiplication as shown in the examples above.

YOU TRY

4. Darcy's iPod has 3525 songs on it. If $\frac{1}{3}$ of those songs are categorized as Hip Hop/Rap, how many Hip Hop/Rap songs are on Darcy's iPod?

GIVEN:

GOAL:

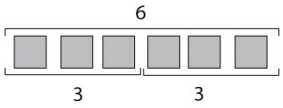
MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

DIVIDING FRACTIONS

Before we discuss division with fractions, let's take a step back and talk just about division in general. What does the operation of division do? Take a look at the problem below.

Problem	Explanation	Result	Check
$6 \div 3$	 There are exactly 2 pieces of size 3 inside 6.	$6 \div 3 = 2$	$2 \cdot 3 = 6$

Are division and multiplication somehow related? Could we turn $6 \div 3$ into a multiplication problem? Let's see how that would be done.

Problem	Multiplication Steps	Result	Check
$6 \div 3$	$6 \div 3 = 6 \cdot \frac{1}{3} = \frac{6}{3} = 2$	$6 \div 3 = 2$	$2 \cdot 3 = 6$

What did we do? We multiplied 6 by the *reciprocal* of 3. That is, we multiplied 6 by $\frac{1}{3}$ and we achieved the same result as $6 \div 3$. Division, then, can be converted to multiplication by using a *reciprocal.*

Let's see if this process makes logical sense when we divide by a fraction.

> Suppose you want to share a candy bar with 3 friends. You know each of your friends would get $\frac{1}{3}$ of the bar. You want to be sure so you ask, "how many pieces of size $\frac{1}{3}$ are there in one candy bar?"

Problem	Multiplication Steps	Result	Check
$1 \div \frac{1}{3}$	$1 \div \frac{1}{3} = 1 \cdot 3 = 3$	$1 \div \frac{1}{3} = 3$	$3 \cdot \frac{1}{3} = \frac{3}{3} = 1$

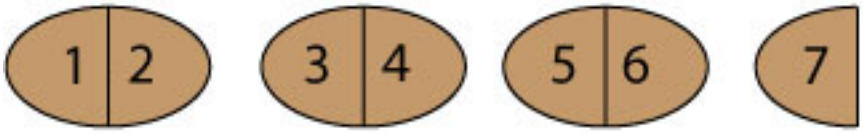
We changed division by $\frac{1}{3}$ to multiplication by 3 (the reciprocal of $\frac{1}{3}$) giving us a result of 3. There are indeed 3 pieces of size $\frac{1}{3}$ inside one full candy bar. Our division process works for fractions as well.

Let's look at one more example just to be sure you have the idea.

> You and some friends sit down to dinner and discover there are $3\frac{1}{2}$ rolls left from your meal the night before. You decided to split all the rolls into $\frac{1}{2}$–size pieces (except for the one that already is) and then divvy them up between you. How many $\frac{1}{2}$–size pieces will there be?

We start with 3 full-size rolls and half of a roll as seen below.

Then, we break the rolls in half and count the total number of $\frac{1}{2}$–size pieces.



From the diagram, we can see easily that are there are 7 pieces of size $\frac{1}{2}$. So, depending on how many friends you have eating with you that night, you can pass out the pieces and maybe keep some extra for yourself. :-0)

What would the mathematics look like for this problem?

Problem	Multiplication Steps	Result	Check
$3\frac{1}{2} \div \frac{1}{2}$	$3\frac{1}{2} \div \frac{1}{2} = \frac{7}{2} \div \frac{1}{2}$ Convert $3\frac{1}{2}$ to an improper fraction. $= \frac{7}{\mathbf{2}} \cdot \frac{\mathbf{2}}{1}$ Multiply by reciprocal of $\frac{1}{2}$. $= \frac{7}{1}$ Remove the common factor 2. $= 7$ Simplify to get final result.	$3\frac{1}{2} \div \frac{1}{2} = 7$	$7 \cdot \frac{1}{2} = \frac{7}{3} = 3\frac{1}{2}$

Steps to divide fractions (full list):
1. Convert any whole numbers to fractions (over 1).
2. Convert any mixed numbers to improper fractions.
3. Change DIVISION to MULTIPLICATION TIMES THE RECIPROCAL of the SECOND fraction.
4. Multiply straight across.
4. Reduce along the way if possible (only after switching to multiplication).
5. Present final, reduced answer at the end.

NOTE: We do not need to obtain a common denominator when dividing fractions!

Example 5: Divide each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a) $2 \div \frac{1}{4} =$

b) $2 \div \frac{2}{5} =$

c) $\frac{7}{2} \div \frac{3}{4} =$

d) $\frac{8}{12} \div 4 =$

d) $3\frac{1}{2} \div 5\frac{3}{8} =$

e) $3 \div 2 =$

YOU TRY

6. Divide each of the following. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{2}{3} \div 11 =$

b. $\frac{1}{5} \div 7 =$

c. $3\frac{1}{4} \div \frac{1}{2} =$

APPLICATIONS OF FRACTION DIVISION

Example 7: If part of a recipe for Albondigas Soup calls for 3 small potatoes, $1\frac{1}{2}$ cups of salsa, and 2 pounds of ground beef, how much of each of these ingredients would be needed to make half of the recipe?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

YOU TRY

Sally was cutting a large tree into log sections that would fit into her fireplace. If her fireplace would take a log that was $1\frac{1}{4}$ feet long and her tree was 100 feet long, how many sections of $1\frac{1}{4}$ feet length would she cut out of the tree?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE:

EXPONENTS/ORDER OF OPERATIONS

Remember again our order of operations from Lesson 1? We will use the same order when working with fraction expressions that involve multiple operations and exponents.

P	Simplify items inside Parenthesis ( ), brackets [ ] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M D	Perform Multiplication and Division next (as they appear from **Left to Right**)
A	Perform Addition and Subtraction on what is left. (as they appear from **Left to Right)**

Example 9: Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\left(\frac{3}{4}\right)^3 =$

b. $\frac{3}{5}\left(\frac{2}{3}\right)^2 =$

Example 10: Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{3}{4}+\frac{4}{5}\div\frac{2}{3}$

b. $\left(2-\frac{1}{3}\right)^2 \div \left(\frac{1}{4}+\frac{1}{6}\right)$

YOU TRY

11. Evaluate. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\left(\frac{3}{7}\right)^3 =$

b. $\frac{4}{5}\div\left(\frac{2}{3}\right)^2 =$

SPECIAL CASES

Example 12: What happens when you multiply by 0?

$$\left(\frac{3}{4}-\frac{1}{3}\right)\cdot\left(\frac{2}{4}-\frac{1}{2}\right)$$

Example 13: What happens when you divide by 0?

$$\frac{2}{3}\div\frac{0}{1}$$

YOU TRY

14. Bill earns $10 for every hour he works each week up to 40 hours. Any additional hours are considered overtime and he earns "time and a half" wages. If he worked 56 hours one week, what were his total earnings?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL RESULT AS A COMPLETE SENTENCE: