

## LESSON 11 – GEOMETRY II: VOLUME & TRIANGLES

### INTRODUCTION

We continue our study of *geometry* by working with *volume*, a measure of three dimensions. In addition, we will spend a little more time working with triangles through the concepts of *similar triangles* and also the *Pythagorean theorem*.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Lesson Objective	Related Examples
Compute the <i>volume</i> of different shapes	1, 2, 3, YT4
Solve application problems involving <i>volume</i> .	5
Determine the lengths of missing sides in <i>similar triangles</i>	6, 7, YT8
Solve application problems involving <i>similar triangles</i> .	9
Find <i>square roots</i> and determine if roots are <i>perfect squares</i>	10, YT11
Use the <i>Pythagorean theorem</i> to find the length of a missing side in a <i>right triangle</i> .	12, 13, YT14
Solve application problems involving the <i>Pythagorean theorem</i> .	15

### KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Cube
- Unit Cube
- Volume
- Rectangular Solid
- Can/Cylinder
- Sphere
- Angle
- Similar Triangle
- Square Root
- Perfect Square
- Right Triangle
- Hypotenuse
- Pythagorean Theorem

### LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

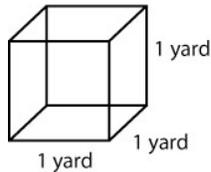
Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

## MINILESSON

## VOLUME

Let's revisit our friend Wally from Lesson 10 and use another aspect of his yard to introduce the concept of *volume*. Wally is a swimmer and wants to install a lap pool in his backyard. Because he has some extra space, he is going to build a pool that is 25 yards long, 2 yards wide, and 2 yards deep. How many cubic yards of water must be used to fill the pool (assuming right to the top).

Much as we did with *area* (counting unit squares), with *volume* we will be counting unit cubes. What is the volume of a unit cube? Let's look at the shape below:



$$\begin{aligned}\text{Volume} &= 1 \text{ yd} \times 1 \text{ yd} \times 1 \text{ yd} \\ &= 1 \text{ cubic yard}\end{aligned}$$

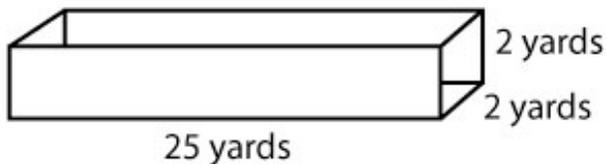
## Finding the Volume of a Cube

The shape at left is a *cube* (all sides are equal length). In particular, because all sides are of length 1, this cube is called a *unit cube*.

We know the area of the base from our previous work (1 yd x 1 yd or 1 square yard). We are going to take that area and extend it vertically through a height of 1 yard so our volume becomes

$$\text{Volume} = 1 \text{ yd} \times 1 \text{ yd} \times 1 \text{ yd} = 1 \text{ cubic yard.}$$

How does this help Wally? Well, if he can count the number of unit cubes in his pool, he can determine the volume of water needed to fill the pool.



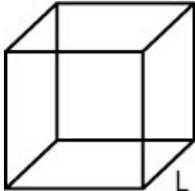
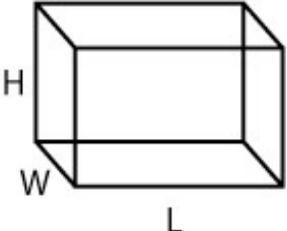
$$\begin{aligned}\text{Volume} &= \text{Length} \times \text{Width} \times \text{Height} \\ V &= LWH\end{aligned}$$

## Filling Wally's Pool

If we fill the pool with unit cubes, we can fill 25 unit cubes along the length, 2 along the width and 2 along the height. That would give us  $25 \times 2 \times 2 = 100$  unit cubes or:

$$\begin{aligned}\text{Volume} &= 25 \text{ yd} \times 2 \text{ yd} \times 2 \text{ yd} \\ \text{Volume} &= 100 \text{ cubic yd}\end{aligned}$$

Explicit formulas for the types of *rectangular solids* used on the previous page are:

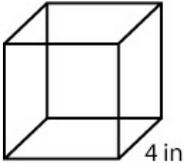
Shape	Volume
Cube of side length $L$ 	$V = L \times L \times L$ $V = L^3$
Box with sides of length $L, W, H$ 	$V = L \times W \times H$

Notes on *Volume*:

- *Volume* is a three-dimensional measurement that represents the amount of space inside a closed three-dimensional shape.
- To find *volume*, count the number of unit cubes inside a given shape.
- If there are units, include units in your final result. Units will always be three-dimensional (i.e. cubic feet, cubic yards, cubic miles, etc...)

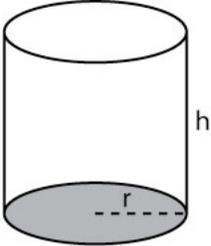


**Example 1:** Find the volume of each shape below.

a. 	b. A box with sides of length 2 ft, 3 ft, $2\frac{1}{2}$ ft.
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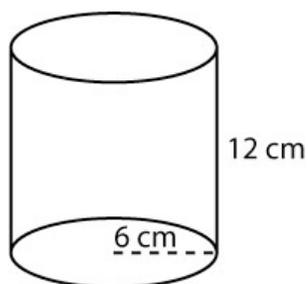
### VOLUME OF A CIRCULAR CYLINDER

Can we use what we know about the area of a circle to formulate the volume of a *can* (also called a *cylinder*)? Take a look at the shape below.

	<p>The base circle is shaded. If we take the area of that circle (<math>A = A = \pi r^2</math>) and extend it up through the height <math>h</math>, then our volume for the can would be:</p> $V = \pi r^2 h$
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**Example 2:** Find the volume of the cylinder shown below.



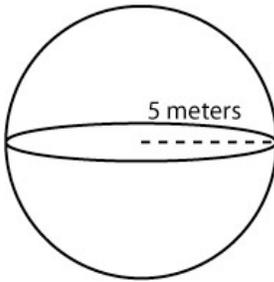
### VOLUME OF OTHER SHAPES

The chart below shows the volumes of some other basic geometric shapes.

Shape	Volume
Sphere	$V = \frac{4}{3} \pi r^3$
Cone	$V = \frac{1}{3} \pi r^2 h$
Pyramid	$V = \frac{1}{3} LWH$



**Example 3:** Find the volume of a sphere with radius 5 meters.



YOU TRY

4. Determine the volume of each of the following. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for  $\pi$  and round answers to tenths as needed.

a. Find the volume of a cube with side 3.25 meters.

b. Find the volume of a box with sides of length 4 feet by  $2\frac{1}{2}$  feet by 6 feet.

c. Find the volume of a can with radius 4.62 cm and height 10 cm.

d. Find the volume of a sphere with diameter 12 yards.

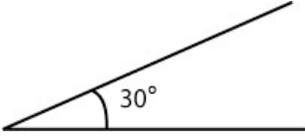
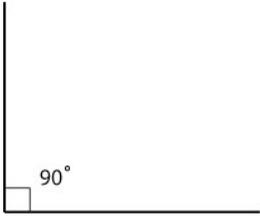
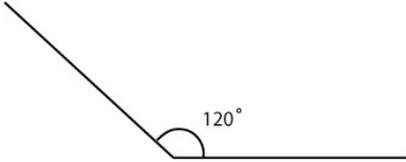
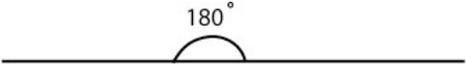
## APPLICATIONS OF VOLUME



**Example 5:** If you drank sodas from 5 cans each of diameter 4 inches and height 5 inches, how many cubic inches of soda did you drink? Use 3.14 for  $\pi$  and round to tenths.

## ANGLES

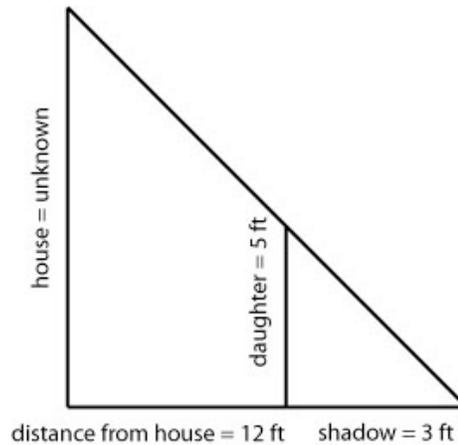
*Angles*, often measured in *degrees*, measure the amount of rotation or “*arc*” between intersecting line segments. We need to have some sense of what an angle is before moving on to the next topic. See some examples and terminology below.

	<ul style="list-style-type: none"> <li>• This angle measures <math>30^\circ</math></li> <li>• This angle measure is less than <math>90^\circ</math></li> <li>• Angles less than <math>90^\circ</math> are called <i>Acute Angles</i></li> </ul>
	<ul style="list-style-type: none"> <li>• This angle measures <math>90^\circ</math></li> <li>• Angles that measure exactly <math>90^\circ</math> are called <i>Right Angles</i></li> <li>• The small box in the angle corner denotes a right angle</li> </ul>
	<ul style="list-style-type: none"> <li>• This angle measures <math>120^\circ</math></li> <li>• This angle measure is more than <math>90^\circ</math></li> <li>• Angles that measure more than <math>90^\circ</math> but less than <math>180^\circ</math> than are called <i>Obtuse Angles</i></li> </ul>
	<ul style="list-style-type: none"> <li>• This angle measures <math>180^\circ</math></li> <li>• Angles that measure exactly <math>180^\circ</math> are called <i>Straight Angles</i></li> </ul>

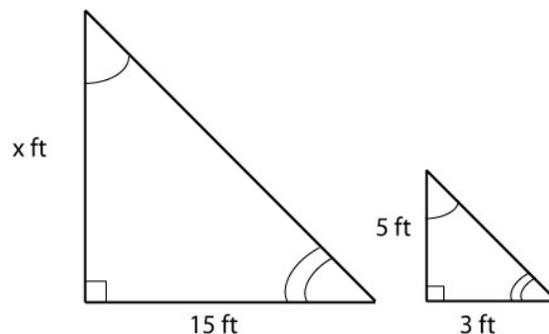
## SIMILAR TRIANGLES

Let's begin our discussion of *similar triangles* with an example.

Mary was out in the yard one day and had her two daughters with her. She was doing some renovations and wanted to know how tall the house was. She noticed a shadow 3 feet long when her daughter was standing 12 feet from the house and used it to set up the drawing below.



We can take that drawing and separate the two triangles as follows allowing us to focus on the numbers and the shapes.



These triangles are what are called *Similar Triangles*. They have the same *angles* and sides in *proportion* to each other. We can use that information to determine the height of the house as seen below.

To determine the height of the house, we set up the following proportion:

$$\frac{x}{15} = \frac{5}{3}$$

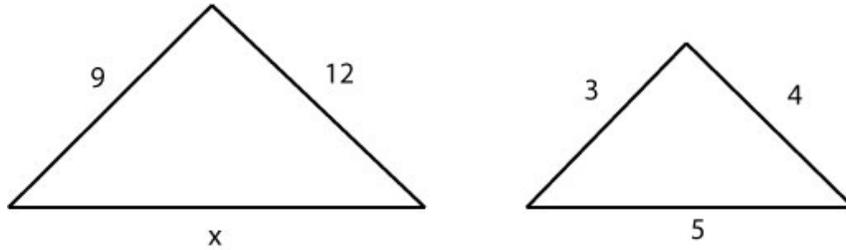
Then, we solve for the unknown  $x$  by using cross products as we have done before:

$$x = \frac{5 \cdot 15}{3} = \frac{75}{3} = 25$$

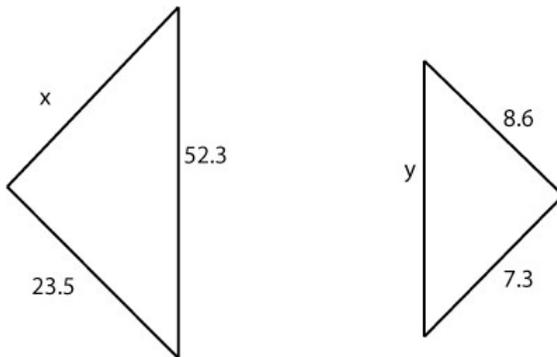
Therefore, we can conclude that the house is 25 feet high.



**Example 6:** Use the Similar Triangles process to determine the length of the missing side. Set up the proportions in as many ways as possible and show the results are all the same.

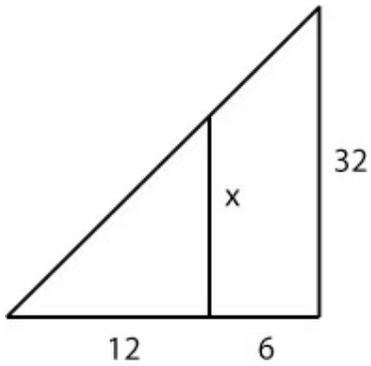


**Example 7:** Use the Similar Triangles process to determine the length of the missing sides. You may need to redraw your triangles to set up the proportions correctly.



## YOU TRY

8. Given the similar triangles below, find the missing lengths. Round to tenths as needed. Feel free to redraw the triangles so you can see the proportional sides.



## APPLICATIONS OF SIMILAR TRIANGLES

 **Example 9:** Mary (from the application that started this topic), decides to use what she knows about the height of the roof to measure the height of her second daughter. If her second daughter casts a shadow that is 1.5 feet long when she is 13.5 feet from the house, what is the height of the second daughter? Draw an accurate diagram and use similar triangles to solve.

Before we get to our last topic in this lesson, the *Pythagorean Theorem*, we need to know a little bit about *square roots*.

### SQUARE ROOTS

- The *square root* of a number is that number which, when multiplied times itself, gives the original number. On your calculator, look for  $\sqrt{\quad}$  to compute square roots.

$$\sqrt{16} = 4 \text{ because } 4 \cdot 4 = 16$$

- A *perfect square* is a number whose square root is a whole number. The square root of a non-perfect square is a decimal value.

$$\sqrt{16} \text{ is a perfect square. } \sqrt{19} \text{ is NOT a perfect square.}$$

- To obtain a decimal value for non-perfect square roots on your calculator, you may need to change the settings under your MODE button. Check your owner's manual for help if needed.

$$\sqrt{19} = 4.36$$



**Example 10:** Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

a.  $\sqrt{169}$

b.  $\sqrt{31}$

c.  $\sqrt{9}$

### YOU TRY

11. Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

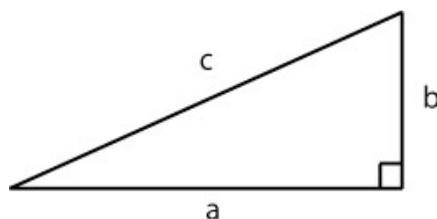
a.  $\sqrt{225}$

b.  $\sqrt{17}$

c.  $\sqrt{324}$

## THE PYTHAGOREAN THEOREM

The mathematician Pythagoras proved the Pythagorean theorem. The theorem states that given any right triangle with sides  $a$ ,  $b$ , and  $c$  as below, the following relationship is always true:  $a^2 + b^2 = c^2$



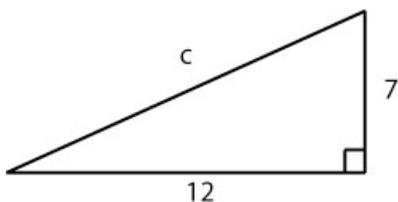
$$a^2 + b^2 = c^2$$

Notes about the Pythagorean theorem:

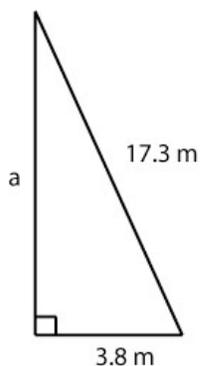
- The triangle must be a **RIGHT** triangle (contains a  $90^\circ$  angle).
- The side  $c$  is called the *Hypotenuse* and **ALWAYS** sits across from the right angle.
- The lengths  $a$  and  $b$  are interchangeable in the theorem but  $c$  cannot be interchanged with  $a$  or  $b$ . In other words, the location of  $c$  is very important and cannot be changed.



**Example 12:** Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to the tenths place.

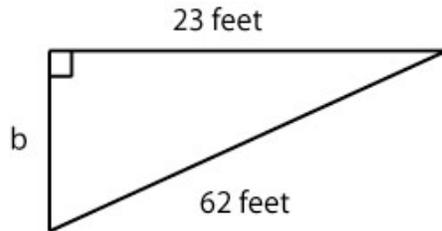


**Example 13:** Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to the tenths place.



## YOU TRY

14. Use the Pythagorean theorem to find the missing sides length for the triangle given below. Round to hundredths.



## APPLICATIONS OF THE PYTHAGOREAN THEOREM



**Example 15:** In NBA Basketball, the width of the free-throw line is 12 feet (reference: <http://www.sportsknowhow.com>). A player stands at one exact corner of the free throw line (Player 1) and wants to throw a pass to his open teammate across the lane and close to the basket (Player 2). If his other teammate (Player 3 – heavily guarded) is directly down the lane from him 16 feet, how far is his pass to the open teammate? Fill in the diagram below and use it to help you solve the problem.

