Lesson 6: Literal Equations, Inequalities, and Absolute Value

Topics and Objectives:

- Literal Equations
- Solving Literal Equations
- Inequalities
  - Symbols
  - Verify that a given value is a solution to an inequality
  - The Solution Set of a Linear Inequality
  - Number Line
  - Interval Notation
  - Translate a statement into an inequality
- Solving Linear Inequalities
- Solving Inequalities – Applications
- Compound Inequalities
  - Number line
  - Interval notation
  - Verify that a given value is a solution to an inequality
  - Translate a statement into a compound inequality
- Absolute Value

Literal Equations

LITERAL EQUATIONS

What are literal equations? _____

What does it mean to "solve" a literal equation? _____

Example 1: Solve for b in each of the following equations.

$2b = 8$ $\qquad$ $ab = c$

$5 + b = 9$ $\qquad$ $a + b = c$

$2b + 1 = 13$ $\qquad$ $ab + c = d$

SOLVING LITERAL EQUATIONS

Example 2: Solve the following equation for c: $4abc = 32$

Example 3: Solve the following equation for B: $A = B + C + \mathrm{D}$

Example 4: Solve the following equation for x: $y = mx + b$

Example 5: Solve the following equation for y: $3x + 4y = 20$

Example 6: Solve the following equation for y: $x - y = 5$

Example 7: Solve the following equation for C: $F = \frac{9}{5}C + 32$

You Try

1. Solve the following equation for y: $3xyz = 9$

2. Solve the following equation for y: $5x - y = 2$

Inequalities

DEFINITION: An <u>algebraic inequality</u> is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign.

Symbol	In words	Examples
$<$		
$\leq$		
$>$		
$\geq$		
$\neq$		

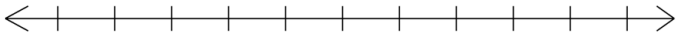
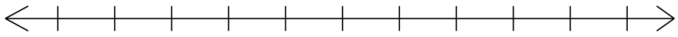
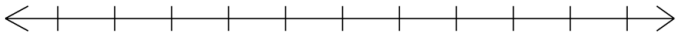
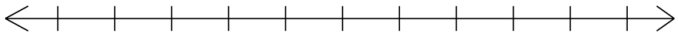
Verify that a given value is a solution to the inequality

DEFINITION: A <u>solution</u> to an inequality is a value that makes the inequality true.

Example 1: Determine whether the number 4 is a solution to the following inequalities.

$x > 1$ $x < 1$ $x \leq 9$ $x > 4$ $x \geq 4$

THE SOLUTION SET OF A LINEAR INEQUALITY

Inequality	Graph	Interval Notation
$x > 2$	$-\infty$  ∞	
$x \geq 2$	$-\infty$  ∞	
$x < 2$	$-\infty$  ∞	
$x \leq 2$	$-\infty$  ∞	

Translate a statement into an inequality

Example 2: Write an inequality to represent the following situation. Clearly indicate what the variable represents.

a. In order to go on the ride, a child must be more than 48 inches tall.

b. Jordan can spend at most $10 on lunch.

You Try

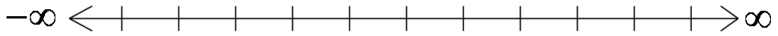
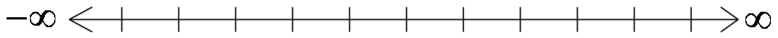
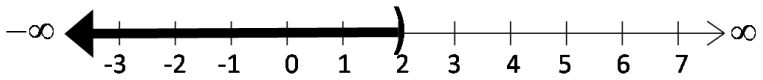
3. Which of the following values are in the solution set for $n > 5$?

$n = -3$ $n = 0$ $n = 4.99$ $n = 5$ $n = 12$

4. Translate the statement into an inequality.

Children age 2 and under are free at Disneyland

5. Complete the table below:

Inequality	Graph	Interval Notation
$x \geq -3$		
		$(-\infty, 11]$
		

Solving Linear Inequalities

STEPS FOR SOLVING A LINEAR INEQUALITY

1. Simplify each side of the inequality. Remove parenthesis if necessary. Collect like terms.
2. Add or subtract terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side.
3. Simplify each side of the inequality by combining like terms.
4. Multiply or divide on both sides to isolate the variable. CAUTION*!!! If you multiply or divide both sides of an inequality by a* ***negative number****, you have to reverse the inequality sign.*
5. Check by substituting the solution (*endpoint and a value from the solution set*) into the original inequality.

Solve the inequality, check your answer, and graph the solution on a number line.

Example 1: Solve the inequality, check your answer, and graph the solution on a number line.

$3x > x + 6$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: ________________

Example 2: Solve the inequality and graph the solution on a number line.

$3 - 5a \leq 2(a + 5)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: ________________

Example 3: Solve the inequality and graph the solution on a number line.

$-5(x + 2) \geq -3(x + 4)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _____

You Try

Solve the inequality, check your answer, and graph the solution on a number line.

6. $7 - 4x \geq -5$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _____

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7. $6x + 13 < 5(2x - 3)$

Graph:

$-\infty \longleftrightarrow \infty$

Interval Notation: _____

Solving Inequalities – Applications

For each problem, underline the Givens and circle the Goal. Form a Strategy, Solve, and Check. Write your answer in a complete sentence.

Example 1: The cost of tuition at SCC is $76 per credit hour. Write an *inequality* that can be used to determine the number of credit hours a student can take for under $1000. Solve the inequality, and write your answer in a complete sentence.

Example 2: Sean owns a business that builds computers. The fixed operating costs for his business are $2,700 per week. In addition to fixed operating costs, each computer costs $600 to produce. Each computer sells for $1,500. Write an *inequality* that can be used to determine the number of computers Sean needs to sell in order make a profit each week. Solve the inequality, and write your answer in a complete sentence.

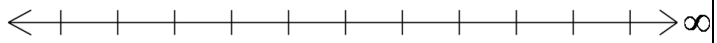
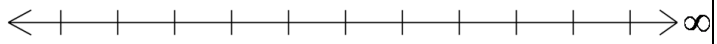
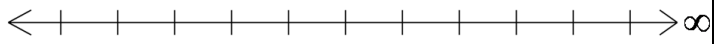
YOU TRY

For each problem, underline the Givens and circle the Goal. Form a strategy, solve, check, and write your answer in a complete sentence.

8. Gasoline costs $3.79 per gallon. Write an *inequality* that can be used to determine how many gallons of fuel can be purchased for under $20. Solve the inequality, and write your answer in a complete sentence.

Compound Inequalities

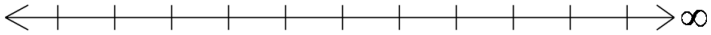
THE SOLUTION SET OF A COMPOUND INEQUALITY

Inequality	Graph	Interval Notation
$-1 < x < 2$	$-\infty$  ∞	
$-1 \le x \le 2$	$-\infty$  ∞	
$-1 \le x < 2$	$-\infty$  ∞	

Verify that a given value is a solution to the inequality

EXAMPLE 1: Which of the following values are in the solution set for $-3 \le n < 5$?

$n = -5$ $n = -3$ $n = 0$ $n = 4.99$ $n = 5$ $n = 12$

$-\infty$  ∞

Translate a statement into an inequality

Example 2: Write a compound inequality to represent the following situation. Clearly indicate what the variable represents.

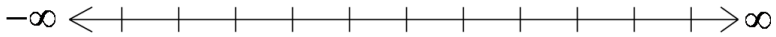
a. A number is greater than or equal to 5 but less than 8.

b. My car's tank can hold a maximum of 20 gallons of gas.

You Try

9. Which of the following values are in the solution set for $-8 < w < 2$?

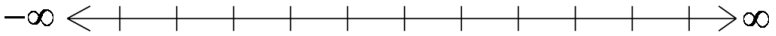
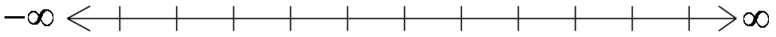
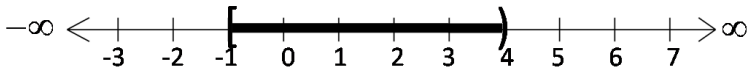
$w = -11$ $w = -8$ $w = -5$ $w = 0$ $w = 2$ $w = 2.1$



10. Translate the statement into a compound inequality.

A number is greater than 0, and less than or equal to 8.

11. Complete the table below:

Inequality	Graph	Interval Notation
$5 < x < 11$		
		(–3, 1]
		

Absolute Value

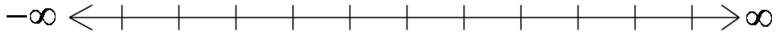
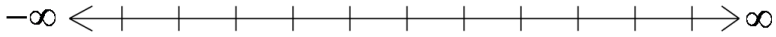
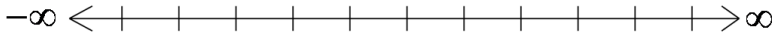
Absolute Value

Example 1: Evaluate the following: $|2|=$ $|-2|=$

Absolute Value Equations

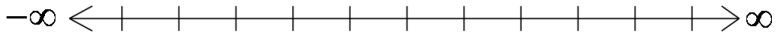
Determine the solution to each of the following equations.

Example 2: $|x| = 2$ $|x| = 3$ $|x| = -4$

Absolute Value Inequalities	
$\lvert x\rvert < 2$	List some values in the solution set: _____ 
$\lvert x\rvert \leq 2$	List some values in the solution set: _____ 
$\lvert x\rvert > 2$	List some values in the solution set: _____ 

YOU TRY

12. Determine the solution to the equation $|x| = 8$

13. Absolute Value Inequalities:

$\lvert x\rvert < 3$	List some values in the solution set: _____
$\lvert x\rvert \geq 3$	List some values in the solution set: _____