Lesson Objectives:
1. Use proper FUNCTION NOTATION when reading, writing and working with functions and when performing OPERATIONS with functions.
2. Use a function to find INPUT given OUTPUT or find OUTPUT given INPUT
3. ADD and SUBTRACT functions by combining like terms
4. MULTIPLY or DIVIDE functions using rules of exponents
5. Perform OPERATIONS with functions using tables, graphs, or symbolic notation
6. Work with APPLICTIONS of function OPERATIONS (Cost, Revenue, Profit functions)

Function Notation: FUNCTION NOTATION is used to indicate a functional relationship between two quantities as follows:

\[ y = f(x) \]

means that
\[ y \text{ is a function of } x \]

Read \( y = f(x) \) as “\( y \) equals \( f \) of \( x \)”

Problem 1 YOU TRY – READING FUNCTIONS

Complete the table below. Learning to read functions using correct language and to identify input/output quantities is very important for our work in this class. The first one is done for you.

<table>
<thead>
<tr>
<th>Function</th>
<th>Function in words</th>
<th>Function in input/output</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3x + 7 )</td>
<td>( f ) of ( x ) equals three times ( x ) plus seven</td>
<td>Output = 3(input) + 7</td>
<td>( x )</td>
<td>( f(x) )</td>
</tr>
<tr>
<td>( g(x) = 4x + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = \frac{1}{2}x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = \frac{2x}{3} + 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h(x) = x^2 + 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Let’s continue our review of functions and function notation, as understanding these topics is CRITICAL for your work in the rest of this course. Functions describe specific relationships between inputs and outputs. Work with the table below as a reminder of how to find inputs and outputs for a given function. If you can learn to think about functions in terms of “inputs” and “outputs”, then the remainder of our topics will be much easier to understand.

### Problem 2 | YOU TRY — FINDING INPUTS, FINDING OUTPUTS

Complete the table below. The first one is done for you.

<table>
<thead>
<tr>
<th>Function</th>
<th>(Input, Output)</th>
<th>Given Input, Find Output</th>
<th>Given Output, Find Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x + 7$</td>
<td>$(x, f(x))$</td>
<td>Find $f(x)$ when $x = 2$.</td>
<td>Find $x$ when $f(x) = 8$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Looking for $(2, ___)$</td>
<td>Looking for $(___, 8)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>To find $f(2)$</td>
<td>To find $SOLVE 8 = x + 7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f(2) = (2) + 7 = 9$</td>
<td>$8 - 7 = x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2, 9)$</td>
<td>$1 = x$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(1, 8)$</td>
</tr>
<tr>
<td>$g(t) = 3t - 4$</td>
<td>$(t, g(t))$</td>
<td>Find $g(t)$ when $t = 4$.</td>
<td>Find $t$ when $g(t) = -1$.</td>
</tr>
<tr>
<td>$f(x) = x - 5$</td>
<td></td>
<td>Find $f(x)$ when $x = -4$.</td>
<td>Find $x$ when $f(x) = 10$.</td>
</tr>
<tr>
<td>$g(x) = 2x + 4$</td>
<td></td>
<td>Find $g(x)$ when $x = 3$.</td>
<td>Find $x$ when $g(x) = 5$.</td>
</tr>
</tbody>
</table>
Functions notation can be expanded to include notation for the different ways we can combine functions as described below.

## Basic Mathematical Operations

The basic mathematical operations are: addition, subtraction, multiplication, and division. When working with function notation, these operations will look like this:

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>Function Notation – How to Read</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$f(x) + g(x)$ “f of x plus g of x”</td>
<td>$3x^2 + 4x$</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$f(x) - g(x)$ “f of x minus g of x”</td>
<td>$(2x + 3) - (4x^2 + 1)$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$f(x) \cdot g(x)$ “f of x times g of x”</td>
<td>$(3x - 1)(5x + 7)$</td>
</tr>
<tr>
<td>Division</td>
<td>$\frac{f(x)}{g(x)}$ “f of x divided by g of x”</td>
<td>$\frac{8x^3 - 2x + 1}{4x}$</td>
</tr>
</tbody>
</table>

The examples on the right side of the table may look fairly familiar. That is because you have probably studied these before in a previous class. Many of the problems we will work in this lesson are problems you may already know how to do. You will just need to get used to some new notation.

We will start with the operations of addition and subtraction.

### Problem 3 WORKED EXAMPLE – ADDING AND SUBTRACTING FUNCTIONS

Given $f(x) = 2x^2 + 3x - 5$ and $g(x) = -x^2 + 5x + 1$, find a) $f(x) + g(x)$ and b) $f(1) - g(1)$

a) $f(x) + g(x) = (2x^2 + 3x - 5) + (-x^2 + 5x + 1) = 2x^2 + 3x - 5 - x^2 + 5x + 1 = x^2 + 8x - 4$

b) $f(1) - g(1) = [2(1)^2 + 3(1) - 5] - [-(1)^2 + 5(1) + 1] = (2 + 3 - 5) - (-1 + 5 + 1) = 0 - 5 = -5$

### Problem 4 MEDIA EXAMPLE – ADDING AND SUBTRACTING FUNCTIONS

Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = x^2 + 2x + 5$, find:

a) $f(x) + g(x)$

b) $f(x) - g(x)$
Problem 5  YOU TRY – ADDING AND SUBTRACTING FUNCTIONS

Given \( f(x) = x + 4 \) and \( g(x) = x^2 + 1 \), determine each of the following. Show complete work.

a) \( f(2) + g(2) = \)

b) \( f(x) - g(x) = \)

Function Multiplication and the Multiplication Property of Exponents

When multiplying functions, you will often need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

MULTIPLICATION PROPERTY OF EXPONENTS
Let \( m \) and \( n \) be rational numbers.
To multiply powers of the same base, keep the base and add the exponents: \( a^m \cdot a^n = a^{m+n} \)

Problem 6  WORKED EXAMPLE – FUNCTION MULTIPLICATION

a) If \( f(x) = 3x + 2 \) and \( g(x) = 2x - 5 \), determine \( f(x) \cdot g(x) \).

\[
f(x) \cdot g(x) = (3x + 2)(2x - 5) \quad \text{NOW USE FOIL}
= (3x \cdot 2x) + (3x \cdot -5) + (2 \cdot 2x) + (2 \cdot -5)
= \text{FIRST} + \text{OUTER} + \text{INNER} + \text{LAST}
\]

(Note: To compute \( 3x \cdot 2x \) remember to first multiply \( 3 \cdot 2 = 6 \) and then \( x \cdot x = x^2 \) from the rule above)

\[
= 6x^2 - 15x + 4x - 10 \quad \text{[Combine like terms]}
= 6x^2 - 11x - 10 \quad \text{[Final Result]}
\]

b) If \( f(x) = 5 \) and \( g(x) = (x+3)^0 \), determine \( f(x) \cdot g(x) \).

\[
f(x) \cdot g(x) = 5(x+3)^0
= 5 \cdot 1 \quad \text{[remember that } a^0 = 1 \text{ if } a \neq 0] 
= 5 \quad \text{[Final Result]}
\]

c) If \( f(x) = -8x^4 \) and \( g(x) = 5x^3 \), determine \( f(x) \cdot g(x) \).

\[
f(x) \cdot g(x) = (-8x^4) \cdot (5x^3)
= (-8) \cdot (5) \cdot (x^4) \cdot (x^3) \quad \text{[Reorder the multiplication]}
= (-40) \cdot (x^7) \quad \text{[Simplify using Multiplication Property of Exponents]}
= -40x^7 \quad \text{[Final Result]}
\]
If \( f(x) = 3x + 2 \) and \( g(x) = 2x^2 + 3x + 1 \), determine \( f(x) \cdot g(x) \).

For each of the following, determine \( f(x) \cdot g(x) \).

a) \( f(x) = 3x - 2 \) and \( g(x) = 3x + 2 \)

b) \( f(x) = 2x^2 + 1 \) and \( g(x) = x^3 - 4x + 5 \)

c) \( f(x) = 4x - 1 \) and \( g(x) = (3x)^0 \)

d) \( f(x) = 4x^3 \) and \( g(x) = -6x \)
Function Division and the Division Property of Exponents

When dividing functions, you will also need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

DIVISION PROPERTY OF EXPONENTS
Let m, n be rational numbers. To divide powers of the same base, keep the base and subtract the exponents.

\[
\frac{a^m}{a^n} = a^{m-n} \quad \text{where } a \neq 0
\]

Problem 9 | WORKED EXAMPLE — FUNCTION DIVISION

For each of the following, determine \( \frac{f(x)}{g(x)} \). Use only positive exponents in your final answer.

a) \( f(x) = 15x^{15} \) and \( g(x) = 3x^9 \)
\[
\frac{f(x)}{g(x)} = \frac{15x^{15}}{3x^9} = 5x^{15-9} = 5x^6
\]

b) \( f(x) = -4x^5 \) and \( g(x) = 2x^8 \)
\[
\frac{f(x)}{g(x)} = \frac{-4x^5}{2x^8} = -2x^{5-8} = -2x^{-3}
\]

This is not our final answer, however, as we need to use only positive exponents in our final result. Recall the rule for negative exponents as follows: If \( a \neq 0 \) and \( n \) is a rational number, then

\[
a^{-n} = \frac{1}{a^n}
\]

Let’s use this rule to rewrite as follows:

\[-2x^{-3} = \frac{-2}{x^3}\]

Notice that the -2 on top did not change or impact the exponent in any way.
Problem 10 | MEDIA EXAMPLE – FUNCTION DIVISION

For each of the following, determine \( \frac{f(x)}{g(x)} \). Use only positive exponents in your final answer.

a) \( f(x) = 10x^4 + 3x^2 \) and \( g(x) = 2x^2 \)

b) \( f(x) = -12x^5 + 8x^2 + 5 \) and \( g(x) = 4x^2 \)

Problem 11 | YOU TRY – FUNCTION DIVISION

For each of the following, determine \( \frac{f(x)}{g(x)} \). Use only positive exponents in your final answer.

a) \( f(x) = 25x^5 - 4x^7 \) and \( g(x) = -5x^4 \)

b) \( f(x) = 20x^6 - 16x^3 + 8 \) and \( g(x) = -4x^3 \)
Functions can be presented in multiple ways including: equations, data sets, graphs, and applications. If you understand function notation, then the process for working with functions is the same no matter how the information is presented.

**Problem 12**  **MEDIA EXAMPLE – WORKING WITH FUNCTIONS IN TABLE FORM**

Functions \( f(x) \) and \( g(x) \) are defined in the tables below. Find \( a – h \) below using the tables.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>8</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>9</th>
<th>11</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-5</td>
</tr>
</tbody>
</table>

a. \( f(1) = \)

b. \( g(9) = \)

c. \( f(0) + g(0) = \)

d. \( g(5) – f(8) = \)

e. \( f(0) \cdot g(3) \)

**Problem 13**  **YOU TRY – WORKING WITH FUNCTIONS IN TABLE FORM**

Given the following two tables, complete the third table. Show work in the table cell for each column. The first one is done for you.

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>4</td>
<td>3</td>
<td>-2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>6</td>
<td>-3</td>
<td>4</td>
<td>-2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) + g(x) )</td>
<td>Find ( f(0) + g(0) = 4 + 6 = 10 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If you remember that graphs are just infinite sets of ordered pairs and if you do a little work ahead of time (as in the example below) then the graphing problems are a lot easier to work with.

**Problem 14 | YOU TRY – WORKING WITH FUNCTIONS IN GRAPH FORM**

Use the graph to determine each of the following. Assume integer answers. The graph of $g$ is the graph in bold. Before you address the questions below, fill in the ordered pairs in the table at the right. A few of them have been done for you.

### Ordered Pairs

**Function f:**
- $(-2, 4)$, $(-1, 3)$, $(0, \_)$,
- $(1, \_)$, $(2, \_)$, $(3, \_)$, $(4, \_)$,
- $(5, \_)$, $(6, \_)$, $(7, \_)$, $(8, \_)$

**Function g:**
- $(-2, -1)$, $(-1, 0)$, $(0, \_)$,
- $(1, \_)$, $(2, \_)$, $(3, \_)$, $(4, \_)$,
- $(5, \_)$, $(6, \_)$, $(7, \_)$, $(8, \_)$

\[a) \ g(5) = \]
\[b) \ f(3) = \]
\[c) \ f(0) = \]
\[d) \ g(2) = \]
\[e) \ \text{if } f(x) = 0, x = \]
\[f) \ \text{if } g(x) = 0, x = \quad \text{or } x = \]
\[g) \ \text{if } f(x) = 4, x = \quad \text{or } x = \]
\[h) \ \text{if } g(x) = 4, x = \]
\[i) \ f(1) + g(1) = \]
\[j) \ g(2) - f(2) = \]
\[k) \ f(0) \times g(3) = \]
\[l) \ \frac{g(3)}{f(2)} = \]
One of the classic applications of function operations is the forming of the Profit function, \( P(x) \) by subtracting the cost function, \( C(x) \), from the revenue function, \( R(x) \) as shown below.

<table>
<thead>
<tr>
<th>PROFIT, REVENUE, COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given functions ( P(x) = ) profit, ( R(x) = ) Revenue, and ( C(x) = ) cost, ( P(x) = R(x) - C(x) ) Profit = Revenue - Cost</td>
</tr>
</tbody>
</table>

**Problem 15 | MEDIA EXAMPLE — COST, REVENUE, PROFIT**
A local courier service estimates its monthly operating costs to be $1500 plus $0.85 per delivery. The service generates revenue of $6 for each delivery. Let \( x \) = the number of deliveries in a given month.

a) Write a function, \( C(x) \), to represent the monthly costs for making \( x \) deliveries per month.

b) Write a function, \( R(x) \), to represent the revenue for making \( x \) deliveries per month.

c) Write a function, \( P(x) \), that represents the monthly profits for making \( x \) deliveries per month.

d) Determine algebraically the break-even point for the function \( P(x) \) and how many deliveries you must make each month to begin making money. Show complete work. Write your final answer as a complete sentence.

e) Determine the break-even point graphically by solving the equation \( P(x) = 0 \). Explain your work and show the graph with appropriate labels. Write your final answer as a complete sentence.
Problem 16 | YOU TRY — COST, REVENUE, PROFIT

You have a knitting machine in your home and a business making ski hats. The fixed cost to run your business is $300 per month and the cost to produce each hat averages $12. The hats will sell for $29.95 (these are nice hats!).

a) Write a function, C(x), to represent the cost of making x hats per month.

b) Write a function, R(x), to represent the revenue from the sale of x hats.

c) Write a function, P(x), that represents the profit from selling x hats in a given month. Show complete work to find the function.

d) Determine algebraically the break-even point for the function P(x) and how many hats you must sell each month to begin making money. Show complete work. Write your final answer as a complete sentence.

e) Determine the break-even point graphically by solving the equation P(x) = 0. Explain your work and show the graph with appropriate labels. Write your final answer as a complete sentence.
You Try Problem 1 – follow the given example

You Try Problem 2 –
g(t) = 3t - 4; g(4) = 8; when g(t) = -1, t = 1
f(x) = x - 5; f(-4) = -9; when f(x) = 10, x = 15
g(x) = 2x + 4; g(3) = 10; when g(x) = 5, x = ½

You Try Problem 5 –
a) 11  b) -x² + x + 3

You Try Problem 8 –
a) 9x² - 4  b) 2x⁵ - 7x³ + 10x² - 4x + 5  c) 4x - 1  d) -24x⁴

You Try Problem 11 –
a) -5x + ⁴/₅x³

b) -5x³ + 4 - ⁵/₃x³

You Try Problem 14 – Not all answers provided for this problem
a) 2  c) 2  e) 2  g) 6  i) 3  k) 8

You Try Problem 16 – Not all answers provided for this problem

-1)

c) P(x) = 17.95x - 300
d) e) 17 hats to break even