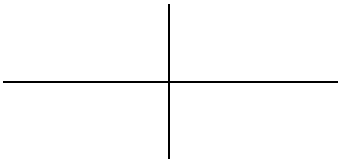# Intermediate Algebra – Final Exam Review

*Note to students: The final exam for MAT120, MAT 121 and MAT122 will consist of 35 multiple-choice questions and a few open-ended questions. The exam itself will cover seven categories: General function understanding, linear functions, quadratic functions, exponential functions, logarithmic functions, rational functions, and radical functions. Sample questions for each topic are given below and are not to be considered identical to those found on the exam.*
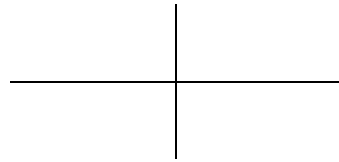
## GENERAL FUNCTION UNDERSTANDING

1.  Draw graphs that meet the given criteria.
    a) A graph of a <u>decreasing</u> function
    b) A graph that does <u>NOT</u> represent a function

2.  Given $f(x) = x^2 - 2x - 15$, find $f(10)$ and $f(-10)$. Write your answers in function notation and as ordered pairs.

3.  Given $f(x) = 8x + 11$, find $x$, such that $f(x) = 3$. Write your answer in function notation and as an ordered pair.
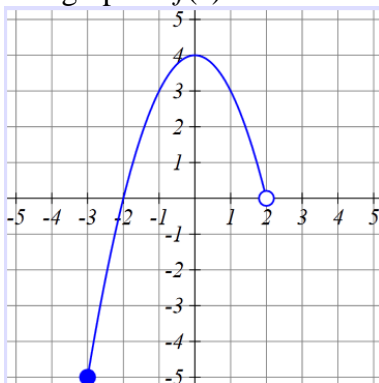
4.  The function $k(x)$ is given by the table below.

    | $x$    | -1 | 2  | 3 | 8 | 17 |
    |--------|----|----|---|---|----|
    | $k(x)$ | 3  | 17 | 5 | 0 | 62 |

    a.  Find $k(3)$. Write your answer in function notation and as an ordered pair.
    b.  Find $x$ such that $k(x) = 17$. Write your answer in function notation and as an ordered pair.
    c.  Determine the domain and range of $k(x)$.

5.  The graph of $f(x)$ is shown below.

    

    a.  Find $f(0)$.

    b.  Find $x$ such that $f(x) = 3$

    c.  Determine the domain and range of the function $f$. Use both inequality notation and interval notation.

6. Given $f(x) = x^2 - 2x - 15$ and $g(x) = x - 5$, determine each of the following:

  a. $f(2) + g(2)$      d. $g(f(2))$      g. $f(g(x))$

  b. $f(x) - g(x)$      e. $g(g(2))$      h. $g(f(x))$

  c. $f(x) \cdot g(x)$      f. $f(g(2))$      i. $g(g(x))$

7. A local towing company charges $3.50 for each mile plus a reservation fee of $12. They tow a maximum of 30 miles.

  a. Let C represent the total cost from the towing company and $x$ represent the number of miles towed. **Using function notation**, write a formula representing the total cost to tow the car $x$ miles.

  b. Interpret the meaning of the statement $C(20) = 82$

  c. Determine $C(8)$. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

  d. Determine $x$ when $C(x) = 100$. Write your answer as an ordered pair and interpret its meaning in a complete sentence.

  e. How much will it cost to tow the car 15 miles? Write your answer in a complete sentence.

  f. Determine the practical domain of $C(x)$. Use inequality notation and **include units.**

  g. Determine the practical range $C(x)$. Use inequality notation and **include units**.

## LINEAR FUNCTIONS

8. Find the equation of the line passing through $(-3,2)$ and $(5,7)$. Leave your answer in $y = mx + b$ form.

9. Find the equation of the horizontal line passing through the point $(3,1)$

10. Find the equation of the vertical line passing through the point $(3,1)$

11. The function $V(m) = 118.4 - 2.31m$ represents the value of an investment (in thousands of dollars) after $m$ months.

  a. Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

  b. Determine the horizontal intercept. Write it as an ordered pair and interpret its practical meaning in a complete sentence.

  c. Identify the slope and write a sentence explaining its meaning.

12. Consider the table shown below.

| Years since 1980 | 0 | 5 | 10 | 15 | 21 | 25 | 26 |
|---|---|---|---|---|---|---|---|
| Total Sales (in millions of dollars) | 1.19 | 1.40 | 1.85 | 1.88 | 2.01 | 2.12 | 2.38 |

  a. Determine the average rate of change between 1985 and 2005. Write a sentence explaining the meaning of your answer.

  b. Does the data represent a linear function? Explain.

  c. Let $S(t)$ represent the total sales of this company $t$ years after 1980. Determine a regression equation for this data set. Use function notation, and round to four decimal places as needed.

  d. Use your graphing calculator to generate a scatterplot of the data *and* regression line on the same screen. Use an appropriate viewing window.

  e. Use the regression equation to determine sales in 2008. Round your answer to the nearest hundredth.

  f. Use the regression equation to determine the year in which sales should reach $3,000,000.

## EXPONENTIAL FUNCTIONS

**13.** Use differences or ratios to determine if each data set is linear or exponential. Write the formula for each.

| $x$ | $p(x)$ |
|---|---|
| 0 | 52 |
| 1 | 41 |
| 2 | 30 |
| 3 | 19 |

| $x$ | $g(x)$ |
|---|---|
| 0 | 128 |
| 1 | 64 |
| 2 | 32 |
| 3 | 16 |

| $x$ | $h(x)$ |
|---|---|
| 0 | 1000 |
| 1 | 1100 |
| 2 | 1210 |
| 3 | 1331 |

$p(x)=$ _____

$g(x) =$ _____

$h(x) =$ _____

**14.** When a new charter school opened in 2005, there were 500 students enrolled. **Using function notation**, $N(t)$, write a formula representing the number of students attending this charter school $t$ years after 2005, assuming that the student population
     a. Increases by 20 students per year.
     b. Increases by 2% per year.
     c. Decreases by 30 students per year.
     d. Decreases by 30% per year.
     e. Increases by 100% each year
     f. Remains constant (does not change).

**15.** Use your calculator to solve for $x$. Round to the nearest hundredth.
     a. $36(1.02)^x = 500$
                            b. $12(0.88)^x = 97$

**16.** The function $f(x) = 300(1.15)^x$ represents the population of a city (in thousands) $x$ years since 2000.
     a. Is the population growing or declining? At what percent rate?
     b. What was the population of this city in 2000.
     c. Predict the population of this city in 2018.
     d. In what year will the population of this city reach 600,000?

**17.** Let $D(x) = 142(1.12)^x$. Write "DNE" if your answer does not exist.
     a. Determine the vertical intercept of $D(x)$.
     b. Determine the horizontal intercept of $D(x)$.
     c. Determine the domain of $D(x)$. Use both inequality notation and interval notation.
     d. Determine the range of $D(x)$. Use both inequality notation and interval notation.
     e. Determine the equation of the horizontal asymptote of $D(x)$.

**18.** You purchased a vehicle for $30,000. Assuming the value of the car decreases at 8% per year.
     a. Write an equation that represents the value, $V(t)$, of the car $t$ years from now.
     b. How many years does it take for the value to decay to $20,000?
     c. Write and solve the equation you would use to find the number of years until the car is worth half of its original value?

**19.** The table below shows the population of a town from 1930 to 2001

| Years since 1930, $t$ | 0 | 10 | 50 | 60 | 71 |
|---|---|---|---|---|---|
| Population, $P(t)$ | 25908 | 25622 | 19057 | 17533 | 16328 |

    a. Fit an exponential regression equation for this data set. Round "a" to the nearest whole number and "b" to 4 decimal places.

    b. Is the population growing or declining? At what percent rate?

    c. What was the population of this town in 1985?

    d. In what year did the population reach 20000?

## LOGARITHMIC FUNCTIONS

**20.** Evaluate the following without a calculator. Write "DNE" if the answer does not exist

$$\log_5(1) \qquad \log_5(5) \qquad \log_5(125) \qquad \log_5(0) \qquad \log_5\left(\frac{1}{5}\right) \qquad \log_5\left(\frac{1}{25}\right) \qquad \log_5\left(\sqrt{5}\right)$$

**21.** Write $4^3 = 64$ in logarithmic form.

**22.** Write the following in exponential form: $\qquad \log_6(36) = 2 \qquad\qquad \log(1000) = 3$

**23.** Use the change of base formula to compute $\log_5 640$. Round to the nearest hundredth.

**24.** Evaluate $30 - 5\log_2 8$ both WITH and WITHOUT your calculator.

**25.** Consider $f(x) = \log_2(x)$. Write "DNE" if your answer does not exist.

    a. Determine the vertical intercept of $f(x)$

    b. Determine the horizontal intercept of $f(x)$.

    c. Determine the domain of $f(x)$. Use both inequality notation and interval notation.

    d. Determine the range of $f(x)$. Use both inequality notation and interval notation.

    e. Determine the equation of the vertical asymptote of $f(x)$.

    f. Determine $f(25)$.

    g. Determine $x$ when $f(x) = 3$.

**26.** Solve $3 + \log_3(120 - x) = 5$ for $x$. Write your answer in exact form AND as a decimal rounded to the nearest thousandth.

**27.** Solve for $x$: $\quad 190 = 95(1.15)^x$. Write your answer in exact form AND as a decimal rounded to the nearest thousandth.

**28.** The function $m(d) = 8.8 + 5.2\log d$ gives the magnitude of a star that can be detected by a telescope of minimum diameter $d$ (in inches). Round answers to the nearest hundredth.

    a. Determine the magnitude of a star that can be detected by a telescope with diameter 120 inches?

    b. What diameter must a lens be to detect a star of magnitude 17.3?

## QUADRATIC FUNCTIONS

**29.** Write in factored form: $12x^2 + 4x$ $\qquad$ $x^2 + 4x - 5$ $\qquad$ $x^2 - 36$

**30.** Solve $x^2 + 4x = 21$ for $x$ by (1) graphing, (2) factoring and (3) the quadratic formula.

**31.** Given $f(x) = -3x^2 + 6x + 11$, determine each of the following. Round to the nearest hundredth as needed. Write "DNE" if your answer does not exist.
   a. The graph of $f(x)$ opens (circle one)     upward     downward
   b. Vertical intercept
   c. Horizontal intercept(s)
   d. Vertex
   e. Axis of symmetry (as an equation)
   f. Domain (Use both inequality notation and interval notation)
   g. Range (Use both inequality notation and interval notation)

**32.** Solve $8x^2 + 6x = -2$ for $x$ using the quadratic formula. Write your answer in exact form and approximate form (decimal approximation). Round to the nearest hundredth.

**33.** Suppose $h(t) = -16t^2 + 40t + 80$ represents the height of a ball (measured in feet above the ground) thrown from a roof as a function of time (in seconds).
   a. Interpret $h(2) = 96$ in the context of this problem.
   b. Find the value(s) of $t$ such that $h(t) = 24$. Interpret your results in the context of this problem.
   c. Write the equation you would solve to determine when the ball will hit the ground. Solve this equation to an accuracy of two decimal places.
   d. Determine the maximum height of the ball. Explain how you found this.

**34.** Write each of the following as a complex number of the form $a + bi$. Write your answers in both exact form and approximate (decimal) form rounded to the nearest hundredth.

$\sqrt{-9}$ $\qquad$ $\sqrt{-11}$ $\qquad$ $\sqrt{-18}$ $\qquad$ $3 - \sqrt{-16}$ $\qquad$ $\dfrac{4 + \sqrt{-3}}{6}$

**35.** Simplify the following. Write your anwers in the form $a + bi$.

$i^2$ $\qquad$ $-i^2$ $\qquad$ $3i(5 - 2i)$ $\qquad$ $(3 + i) - (2 - 3i)$ $\qquad$ $(3 + i)(2 - 3i)$

## RATIONAL FUNCTIONS

**36.** Determine the domain and vertical asymptote of each of the following:

$f(x) = \dfrac{4x + 1}{5x^2}$ $\qquad$ $f(x) = \dfrac{x^2 + 4x - 6}{x + 3}$ $\qquad$ $f(x) = \dfrac{5x - 1}{2x + 8}$

**37.** Determine the horizontal asymptotes of each of the following:

$f(x) = \dfrac{4x + 1}{5x^2}$ $\qquad$ $f(x) = \dfrac{x^2 + 4x - 6}{x + 3}$ $\qquad$ $f(x) = \dfrac{5x - 1}{2x + 8}$

**38.** Solve for $x$:     $-2 = \dfrac{5x-1}{2x+8}$.

**39.** Use $f(x) = \dfrac{3x-4}{1-x}$ to determine each of the following. Write "DNE" if your answer does not exist.
   a. Determine the vertical intercept of $f(x)$.
   b. Determine the horizontal intercept of $f(x)$.
   c. Determine the domain of $f(x)$. Use both inequality notation and interval notation.
   d. Determine the equation of the vertical asymptote of $f(x)$.
   e. Determine the equation of the horizontal asymptote of $f(x)$.
   f. Determine $f(5)$.
   g. Determine $x$ when $f(x) = 3$.

## RADICAL FUNCTIONS

**40.** Use your calculator to evaluate the following. Write "DNE" if your answer does not exist. Round answer to the nearest hundredth as needed.

$\sqrt{32}$                    $\sqrt[4]{42}$                    $\sqrt[3]{-30}$                    $\sqrt[6]{-36}$

**41.** Determine the domain following radical functions.

$f(x) = \sqrt{6-2x}$                    $f(x) = \sqrt[3]{2x+1}$                    $f(x) = \sqrt[4]{x+7}$

**42.** Use $f(x) = \sqrt{2x+8}$ to determine the following. Write "DNE" if your answer does not exist.
   a. Determine the domain of $f(x)$.
   b. Determine the vertical intercept of $f(x)$.
   c. Determine the horizontal intercept of $f(x)$.
   d. Determine $f(5)$.
   e. Determine $x$ when $f(x) = 3$.

**43.** Solve $\sqrt{2x+1}+3 = 5$ for $x$. Write "DNE" if your answer does not exist. Write your answer in both exact form and approximate (decimal) form rounded to the nearest hundredth.

**44.** Solve $1+\sqrt{7-x} = x$ for $x$. Write your answer(s) in both exact form and approximate (decimal) form rounded to the nearest hundredth.

**45.** Solve $6 + \sqrt[3]{3x} = 1$ for $x$. Write "DNE" if your answer does not exist. Write your answer in both exact form and approximate (decimal) form rounded to the nearest hundredth.