

Scottsdale Community College

Intermediate Algebra

Student Workbook

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Lesson 1a – Introduction to Functions

Throughout this class, we will be looking at various Algebraic Functions and the characteristics of each. Before we begin, we need to review the concept of what a Function is and look at the rules that a Function must follow. We also need to investigate the different ways that we can represent a Function. It is important that we go beyond simple manipulation and evaluation of these Functions by examining their characteristics analyzing their behavior. Looking at the Functions modeled as Graphs, Tables and Sets of Ordered Pairs is critical to accomplishing that goal.

Lesson Objectives:

1. Define FUNCTION
2. Determine if data sets, graphs, statements, or sets of ordered pairs define functions
3. Use proper function notation
4. Identify INPUT and OUTPUT variables and quantities
5. Identify DOMAIN and RANGE or PRACTICAL DOMAIN and PRACTICAL RANGE of a function
6. Evaluate functions
7. Use a function to find INPUT given OUTPUT or find OUTPUT given INPUT

Lesson 1a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 1a

Intermediate Algebra is a study of functions and their characteristics. In this class, we will study LINEAR, EXPONENTIAL, LOGARITHMIC, QUADRATIC, RATIONAL, & RADICAL functions. Before we learn the specifics of these functions, we need to review/learn the language and notation of FUNCTIONS.

What is a Function?

The concept of “function” is one that is very important in mathematics. The use of this term is very specific and describes a particular relationship between two quantities: an input quantity and an output quantity. Specifically, a relationship between two quantities can be defined as function if it is the case that “each input value is associated with only one output value”.

Why Do We Care About Functions?

Imagine that you are a nurse working the emergency room of a hospital. A very sick person arrives. You know just the medicine needed but you are unsure the exact dose. First, you determine the patient’s weight (200 pounds). Then you look at the table to the right and see the given dosage information:

Weight in lbs	ml of Medicine
200	10
200	100

You are immediately confused and very concerned. How much medicine do you give? 10 ml or 100 ml? One amount could be too much and the other not enough. How do you choose the correct amount? What you have here is a situation that does NOT define a function (and would not occur in real life). In this case, for the input value 200 lbs, there are two choices for the output value. If you have a function, you will not have to choose between output values for a given input. In the real case of patients and medicine, the dosage charts are based upon functions.

A More Formal Definition of Function

A FUNCTION is a rule that assigns a single, unique output value to each input value.

Problem 1 | WORKED EXAMPLE – DO THE DATA REPRESENT A FUNCTION?

1a) Does the data set $\{(2, 4), (3, 5), (3, 6)\}$ represent a function?

This data set DOES NOT represent a function because the input “3” is associated with two different outputs, (3, 5) and (3, 6).

1b) Does the data set $\{(3, 5), (2, 5), (1, 5)\}$ represent a function?

This data set DOES represent a function. Repeated outputs are ok (so the repeated 5’s are not a problem). Each input is still associated with a single output.

Problem 2	WORKED EXAMPLE – INVESTIGATING FUNCTIONAL RELATIONSHIPS
------------------	--

Let's investigate the functional relationship between the two quantities, "numerical grade" and "letter grade". First, let Numerical Grade be the input quantity and Letter Grade be the output quantity. Below is a sample data set that is representative of the situation.

Numerical grade	Letter Grade
95	A
92	A
85	B
73	C

The numbers above are made up to work with this situation. Other numbers could be used. We are assuming a standard 90, 80, 70, etc... grading scale. Hopefully you can see from this data that no matter what numerical value we have for input, there is only one resulting letter grade. Notice that the repeated outputs "A" are not a problem since the inputs are different. You can uniquely predict the output for any numerical grade input.

So, from this information we can say that Letter Grade (output) is a function of Numerical Grade (input).

Now let's switch the data set above.

Letter Grade	Numerical Grade
A	95
A	92
B	85
C	73

Can you see there is a problem here? If you say that you have an A in a class, can you predict your numerical grade uniquely? No. There are a whole host of numerical scores that could come from having an A. The same is true for all the other letter grades as well. Therefore, Numerical Grade (output) is NOT a function of Letter Grade (input).

Summary:

- Letter Grade IS a function of Numerical Grade but
- Numerical Grade is NOT a function of Letter Grade

Additional Terminology

In the language of functions, the phrase INDEPENDENT VARIABLE means input and the phrase DEPENDENT VARIABLE means output. The dependent variable (output) "depends on" or is a "function of" the independent variable (input).

Problem 3 | MEDIA EXAMPLE – DO THE DATA REPRESENT A FUNCTION?

The table below gives the height, in feet, of a golf ball t seconds after being hit.

$t =$ Time (in seconds)	H = Height (in feet)
0	0
1	80
2	128
3	144
4	128
5	80
6	0

a) Identify the input quantity (include units). _____

Identify the input variable. _____

Identify the output quantity (include units). _____

Identify the output variable. _____

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (3, 144).

d) Is height of the golf ball a function of time? Why or why not?

e) Is time a function of the height of the golf ball? Why or why not?

Problem 4 | MEDIA EXAMPLE – DOES THE STATEMENT DESCRIBE A FUNCTION?

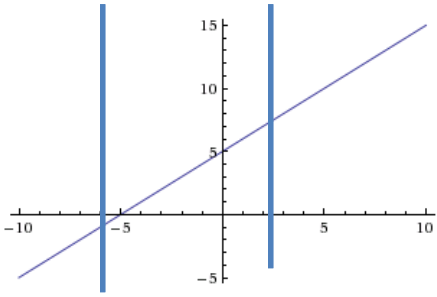
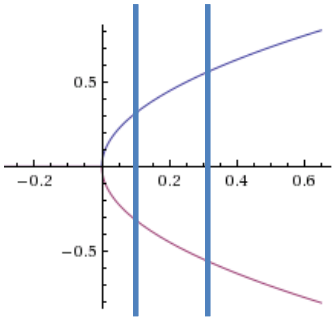
Explain your choice for each of the following. Remember when the word “function” is used, it is in a purely MATHEMATICAL sense, not in an everyday sense.

a) Is the number of children a person has a function of their income?

b) Is your weekly pay a function of the number of hours you work each week? (Assume you work at an hourly rate job with no tips).

Problem 5 **WORKED EXAMPLE – DETERMINE FUNCTIONAL RELATIONSHIPS USING MULTIPLE REPRESENTATIONS**

The table below shows 3 different representations for two relationships. Determine which relationship defines a function.

	Relationship 1	Relationship 2																																																
Data Set	<table border="1"> <tr> <td>x</td> <td>-4</td> <td>-2</td> <td>0</td> <td>3</td> <td>5</td> </tr> <tr> <td>y</td> <td>1</td> <td>3</td> <td>5</td> <td>8</td> <td>10</td> </tr> </table> <p style="text-align: center;">OR</p> <table border="1"> <tr> <td>x</td> <td>y</td> </tr> <tr> <td>-4</td> <td>1</td> </tr> <tr> <td>-2</td> <td>3</td> </tr> <tr> <td>0</td> <td>5</td> </tr> <tr> <td>3</td> <td>8</td> </tr> <tr> <td>5</td> <td>10</td> </tr> </table> <p>All input values are associated with one, unique output value.</p>	x	-4	-2	0	3	5	y	1	3	5	8	10	x	y	-4	1	-2	3	0	5	3	8	5	10	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>1</td> <td>4</td> <td>4</td> </tr> <tr> <td>y</td> <td>0</td> <td>1</td> <td>-1</td> <td>2</td> <td>-2</td> </tr> </table> <p style="text-align: center;">OR</p> <table border="1"> <tr> <td>x</td> <td>y</td> </tr> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>-1</td> </tr> <tr> <td>4</td> <td>2</td> </tr> <tr> <td>4</td> <td>-2</td> </tr> </table> <p>Two of the listed input values (1 & 4) are associated with more than one output value.</p>	x	0	1	1	4	4	y	0	1	-1	2	-2	x	y	0	0	1	1	1	-1	4	2	4	-2
x	-4	-2	0	3	5																																													
y	1	3	5	8	10																																													
x	y																																																	
-4	1																																																	
-2	3																																																	
0	5																																																	
3	8																																																	
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x	0	1	1	4	4																																													
y	0	1	-1	2	-2																																													
x	y																																																	
0	0																																																	
1	1																																																	
1	-1																																																	
4	2																																																	
4	-2																																																	
Set of Ordered Pairs	<p>$\{(-4, 1), (-2, 3), (0, 5), (3, 8), (5, 10)\}$</p> <p>No input value is repeated in an ordered pair.</p>	<p>$\{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2)\}$</p> <p>Two of the listed input values (1 & 4) are associated with more than one output value.</p>																																																
Graph	 <p>No vertical line intersects the graph in more than one point. We say the graph PASSES the VERTICAL LINE TEST.</p>	 <p>Vertical lines intersect the graph at more than one point meaning inputs are repeated with different outputs. We say that the graph FAILS the VERTICAL LINE TEST.</p>																																																
Function?	YES	NO																																																

Function Notation: FUNCTION NOTATION is used to indicate a functional relationship between two quantities as follows:

$$y = f(x)$$

means that
y is a function of x

read $y = f(x)$ as “y equals f of x”

Problem 6	MEDIA EXAMPLE – UNDERSTANDING APPLICATIONS OF FUNCTIONS
------------------	--

Suppose that the cost to fill your 15-gallon gas tank is determined by the function $C(g) = 3.29g$ where C is the output (cost in \$) and g is the input (gallons of gas).

- Graph this function using your calculator. [Go to $Y=$ and type $3.29x$ into the $Y1$ slot. Then, press $WINDOW$ and enter $xmin = 0$, $xmax = 20$, $ymin = 0$, and $ymax = 70$ then press $GRAPH$]. Show a good graph in the space below.
- Use the Table feature of your graph and identify the first and last ordered pairs that are on the graph (based on the information above). [2^{nd} >Graph will take you to the table]. Include both ordered pairs and function notation.
- What is the INPUT quantity (including units) for this function? Name the smallest and largest possible input quantity then use this information to identify the PRACTICAL DOMAIN.
- What is the OUTPUT quantity (including units) for this function? Name the smallest and largest possible output quantity then use this information to identify the PRACTICAL RANGE.

Function Domain: The DOMAIN of a function is the set of all possible values for the input quantity.

Practical Domain: The PRACTICAL DOMAIN of a function is the set of all possible input values that are realistic for a given problem.

Function Range: The RANGE of a function is the set of all possible values for the output quantity

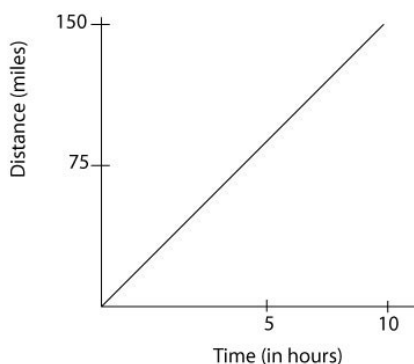
Practical Range: The PRACTICAL RANGE of a function is the set of all possible output values that are realistic for a given problem.

Problem 7 | WORKED EXAMPLE – PRACTICAL DOMAIN AND RANGE

Let the function $D(t) = 3t$ represent the distance you would travel, hiking, after t hours assuming a constant rate of 3 miles per hour. Assume you can hike no more than 5 hours. Find the practical domain and practical range for this function.

Let the function $M(t) = 15t$ represent the distance you would travel bicycling t hours. Assume you can bike no more than 10 hours. Find the practical domain and practical range for this function.

BEGIN by drawing an accurate graph of the situation. Try and determine the smallest and largest input values then do the same thing for the output values.



PRACTICAL DOMAIN:
 In this situation, the input values you can use are related to biking and the input is TIME. You are told you can bike no more than 10 hours. You also cannot bike a negative number of hours but you CAN bike 0 hours.
 Therefore, the Practical Domain is $0 \leq t \leq 10$ which means “all the values of t between and including 0 and 10”.

PRACTICAL RANGE
 In this situation, the outputs represent distances traveled depending on how long you bike. Looking at the endpoints for Practical Domain, you can find you Practical Range as follows:

$$M(0) \leq M(t) \leq M(10)$$
 thus $0 \leq M(t) \leq 150$ is your Practical Range which means you can bike a minimum of 0 miles and a maximum of 150 miles in this situation.

Problem 8 | YOU TRY – APPLICATIONS OF FUNCTIONS

A local towing company charges \$3.25 per mile driven plus a base fee of \$30.00. They tow a maximum of 25 miles.

- a) Let C represent the total cost of any tow and x represent miles driven. Using correct and formal function notation, write a function that represents total cost as a function of miles driven.
- b) Identify the practical domain of this function by filling in the blanks below.

Minimum miles towed $\leq x \leq$ Maximum miles towed

Practical Domain: _____ $\leq x \leq$ _____

- c) Identify the practical range of this function by filling in the blanks below.

Minimum Cost $\leq C(x) \leq$ Maximum Cost

Practical Range: _____ $\leq C(x) \leq$ _____

- d) Enter the equation for C into the $Y=$ part of your calculator. Then use the TABLE feature to complete the table below:

x	0	5	15	20
$C(x)$				

- e) Use the TABLE to determine the value of $C(15)$. Circle the appropriate column in the table.

$C(15) =$ _____

- f) Use the TABLE to determine x when $C(x) = 30$. Circle the appropriate column.

When $C(x) = 30$, $x =$ _____

- g) Use your FUNCTION from part a) to determine algebraically the value of x when $C(x) = 30$. Set up the equation, $C(x) = 30$ then solve for the value of x .
- h) Write a complete sentence to explain the meaning of $C(0) = 30$ in words.

Function Evaluation: To evaluate a function at a particular value of the input variable, replace each occurrence of the input variable with the given value and compute the result. Use of () around your input value, especially if the input is negative, can help achieve correct results.

Problem 9 | WORKED EXAMPLE – FUNCTION EVALUATION

If $f(x) = 5x^2 - 1$, find $f(2)$, $f(-1)$, $f(x + 1)$ and $f(-x)$.

$$\begin{aligned} f(2) &= 5(2)^2 - 1 \\ &= 5(4) - 1 \\ &= 20 - 1 \\ &= 19 \end{aligned}$$

$$\begin{aligned} f(-1) &= 5(-1)^2 - 1 \\ &= 5(1) - 1 \\ &= 5 - 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} f(x+1) &= 5(x+1)^2 - 1 \\ &= 5(x+1)(x+1) - 1 \\ &= 5(x^2 + x + x + 1) - 1 \\ &= 5(x^2 + 2x + 1) - 1 \\ &= 5x^2 + 10x + 5 - 1 \\ &= 5x^2 + 10x + 4 \end{aligned}$$

$$\begin{aligned} f(-x) &= 5(-x)^2 - 1 \\ &= 5x^2 - 1 \end{aligned}$$

[Key to this problem is the use of () when substituting $-x$ for x]

[Key to this problem is that $(x+1)^2 = (x+1)(x+1)$ then FOIL]

Problem 10 | MEDIA EXAMPLE – FUNCTION EVALUATION

For each of the functions below, evaluate $f(2)$, $f(-1)$, $f(1+x)$, and $f(-x)$:

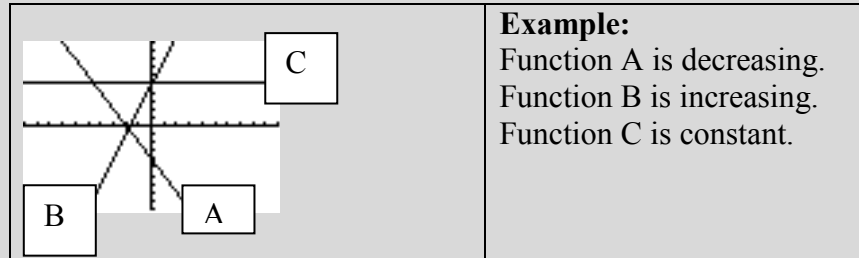
a) $f(x) = 2x - 5$

b) $f(x) = -4x^2 + 5x + 12$

Graphs of Functions:

We read graphs just like we read a book...from left to right. A function is:

- INCREASING if the outputs get larger,
- DECREASING if the outputs get smaller,
- CONSTANT if the outputs do not change.

**Problem 11 | MEDIA EXAMPLE – DOMAIN AND RANGE**

Graph the following function on your graphing calculator restricting the input window to $X_{\min} = -5$ and $X_{\max} = 5$ and draw an accurate sketch here [Go to $Y=$ and type in the equation. Then go to Window and enter -5 for X_{\min} and 5 for X_{\max} . Leave Y_{\min} at -10 and Y_{\max} at 10]. Indicate the domain and range given the window above.

a) $y = x - 1$

Domain:

Range:

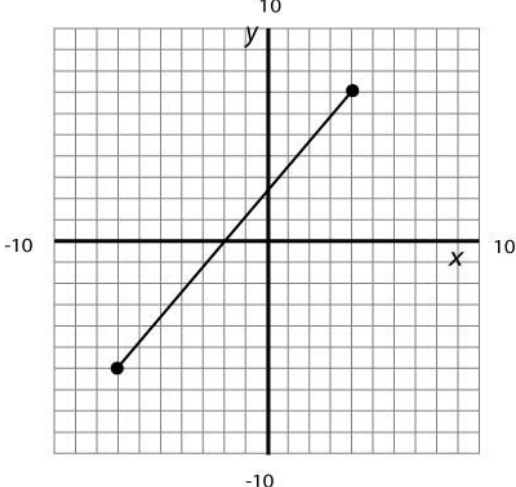
b) If the input and output are not restricted as above, indicate the domain and range for this function.

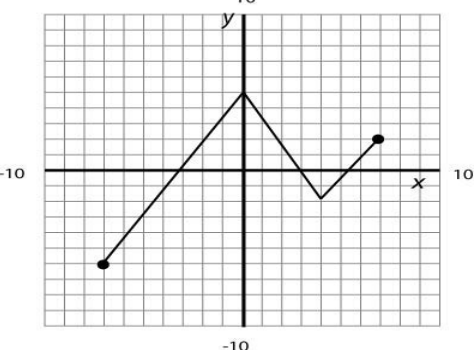
Domain:

Range:

Problem 12 | YOU TRY – DOMAIN, RANGE, FUNCTION EVALUATION

Find the domain and range for the graphs below then fill in the other blanks. Use proper INEQUALITY notation for your domain/range responses (as in Worked Example 3/Media Problem 5).

Function F(x)	
	
Domain:	Range:
$F(0) = \underline{\hspace{2cm}}$	When $F(x) = 0$, $x = \underline{\hspace{2cm}}$

Function G(x)	
	
Domain:	Range:
$G(0) = \underline{\hspace{2cm}}$	When $G(x) = 0$, $x = \underline{\hspace{1cm}}$ or $\underline{\hspace{1cm}}$ or $\underline{\hspace{1cm}}$ (approximate as needed)

When working with FUNCTIONS, there are two main questions we will ask and solve as follows:

- Given a particular INPUT value, what is the corresponding OUTPUT value?
- Given a particular OUTPUT value, what is the corresponding INPUT value?

Problem 13 MEDIA EXAMPLE – WORKING WITH INPUT AND OUTPUT

Given $f(x) = 2x + 5$, determine each of the following. Round to two decimals as appropriate.

GIVEN INPUT FIND OUTPUT	GIVEN OUTPUT FIND INPUT
Find $f(0)$	Find x if $f(x) = 7$
Find $f(-2)$	Find x if $f(x) = -11$

Problem 14 YOU TRY – WORKING WITH INPUT AND OUTPUT

Given $f(x) = -3x - 4$, compute each of the following. Round to two decimals as appropriate.

GIVEN INPUT FIND OUTPUT	GIVEN OUTPUT FIND INPUT
Find $f(2)$	Find x if $f(x) = 7$
Find $f(-3)$	Find x if $f(x) = -12$

Problem 15 | **YOU TRY – APPLICATIONS OF FUNCTIONS**

The value V (in dollars) of a washer/dryer set decreases as a function of time t (in years). The function $V(t) = -100t + 1200$ models this situation.

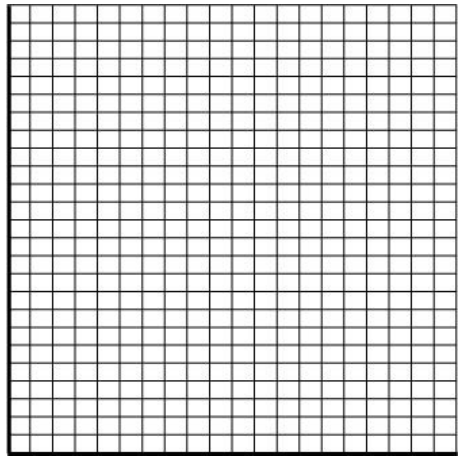
- a) Identify the input quantity (including units) and the input variable.

- b) Identify the output quantity (including units) and the output variable.

- c) Fill in the ordered pair table below.

t	0	6	12
$V(t)$			

- d) Draw a GOOD graph of this function in the space below. Provide labels for your axes. Plot and label the ordered pairs from part c). You may use the graphing feature of your calculator to help you ($Y=$). The recommended window is $xmin = 0$, $xmax = 15$, $ymin = 0$, $ymax = 1500$.



e) A washer/dryer set that is worth \$400 would be how old? *Hint: This is a GIVEN INPUT FIND OUTPUT question. You must show work.*

f) After 2 years, how much would the washer/dryer set be worth? *Hint: This is a GIVEN INPUT FIND OUTPUT question. You must show work.*

g) What is the practical domain for $V(t)$? Use proper notation.

h) What is the practical range for $V(t)$? Use proper notation

Name: _____

Date: _____

Lesson 1a Practice Problems

1. The tables below show 6 different data sets. For each set determine if it meets the criteria for a function. If it fails, explain why and circle the data points that make it invalid.

a)

x	$f(x)$
3	0
1	5
2	8
3	12
4	14

b)

x	$g(x)$
0	0
1	1
2	-1
3	2
4	-2

c)

x	$f(x)$
0	-3
1	-4
2	-5
3	
4	-6

d)

$$R = \{(2, 4), (3, 8), (-2, 6)\}$$

e)

$$T = \{(3, -2), (4, -1), (5, 8), (3, -2)\}$$

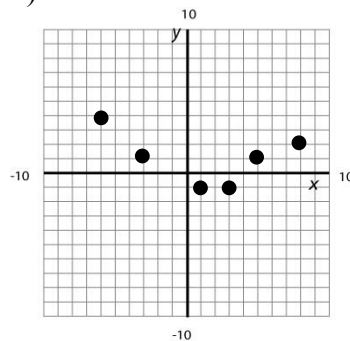
f)

$$T = \{(3, -5), (1, -2), (2, -2), (3, 5)\}$$

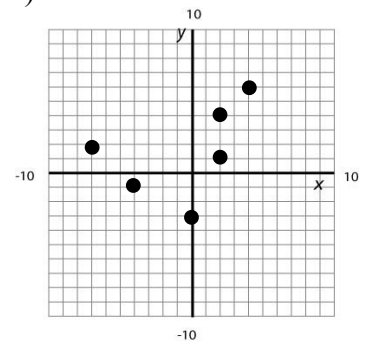
g)

t	0	1	2
$p(t)$	43	45	43

h)



i)



4. Given the function $f(x) = 14 - 2x$, evaluate each of the following:

a) $f(-3) =$

b) $f(4) =$

c) $f\left(\frac{1}{2}x\right) =$

d) $f(-x) =$

5. Find the domain and range for the functions below. Use proper INEQUALITY notation for your domain/range responses

a)

x	$f(x)$
-10	3
-5	8
0	12
5	15
10	18

b)

x	$g(x)$
-20	-4
-10	14
0	32
10	50
20	68
30	86

c)

$$g = \{ (3, -2), (5, -1), (7, 8), (9, -2), (11, 4), (13, -2) \}$$

d)

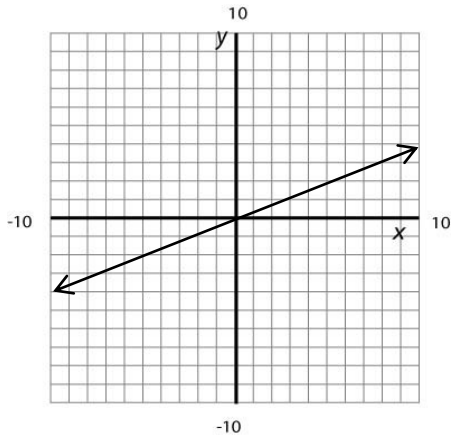
$$f = \{ (-2, -5), (-1, -5), (0, -5), (1, -5) \}$$

e)

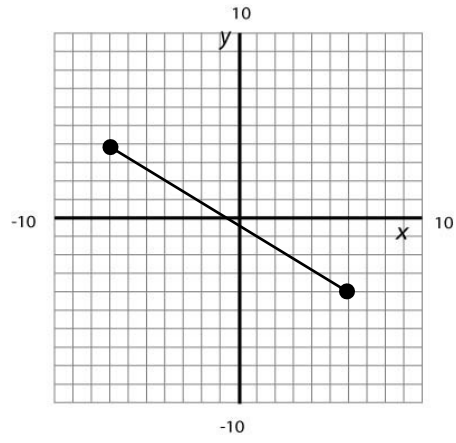
h	8	9	10	11	12	1	2	3	4
$T(h)$	54	62	66	69	72	73	74	73	72

6. Find the domain and range for the functions below. Use proper INEQUALITY notation for your domain/range responses.

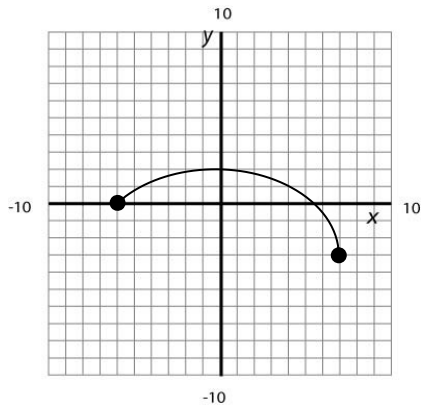
a)



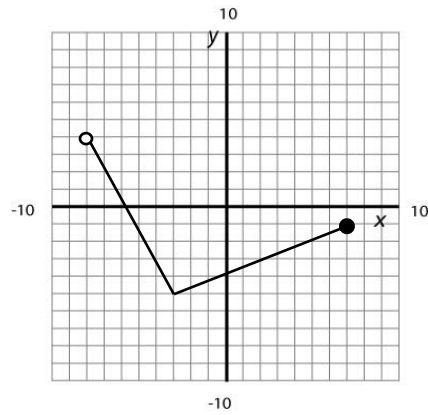
b)



c)



d)



7. Given $f(x) = 3x - 6$, determine each of the following. Round to two decimals as appropriate.

a) Find $f(2) =$

b) Find x if $f(x) = 3$

c) Find $f(-4) =$

d) Find x if $f(x) = -12$

8. Given $f(x) = \frac{3}{2}x - \frac{1}{2}$, determine each of the following. Round to two decimals as appropriate.

a) Find $f(4)$

b) Find x if $f(x) = 2$

c) Find $f(-1)$

d) Find x if $f(x) = \frac{1}{2}$

9. A local window washing company charges \$0.50 per window plus a base fee of \$20.00 per appointment. They can wash a maximum of 200 windows per appointment.
- a) Let C represent the total cost of an appointment and w represent the number of windows washed. Using correct and formal function notation, write a function that represents total cost as a function of windows washed.

- b) Identify the practical domain of this function by filling in the blanks below.

Minimum windows washed $\leq w \leq$ Maximum windows washed

Practical Domain: _____ $\leq w \leq$ _____

- c) Identify the practical range of this function by filling in the blanks below.

Minimum Cost $\leq C(w) \leq$ Maximum Cost

Practical Range: _____ $\leq C(w) \leq$ _____

- d) Enter the equation for C into the $Y=$ part of your calculator. Then use the TABLE feature to complete the table below:

w	0	50	150	200
$C(w)$				

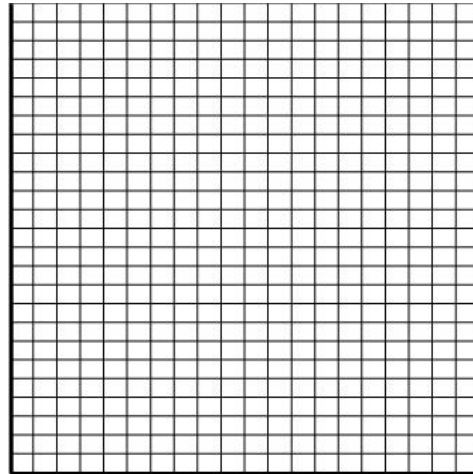
- e) Use the TABLE to determine the value of $C(50)$. Circle the appropriate column in the table. Write a sentence explaining the meaning of your answer.
- f) Use the TABLE to determine w when $C(w) = 45$. Circle the appropriate column. Write a sentence explaining the meaning of your answer.
- g) Use your FUNCTION from part a) to determine the value of w when $C(w) = 45$. Set up the equation, $C(w) = 45$ then solve for the value of w .

10. Suppose the number of pizzas you can make in an 8 hour day is determined by the function $P(t) = 12t$ where P is the output (Pizzas made) and t is the input (Time in hours).
- Graph this function using your calculator. [Go to Y= and type $12x$ into the Y1 slot. Then, press WINDOW and enter $x_{\min} = 0$, $x_{\max} = 8$, $y_{\min} = 0$, and $y_{\max} = 96$ then press GRAPH]. Show a good graph in the space below.
 - Use the Table feature of your graph and identify the first and last ordered pairs that are on the graph (based on the information above). [2^{nd} >Graph will take you to the table]. Include both ordered pairs and function notation.
 - What is the INPUT quantity (including units) for this function? Name the smallest and largest possible input quantity then use this information to identify the PRACTICAL DOMAIN.
 - What is the OUTPUT quantity (including units) for this function? Name the smallest and largest possible output quantity then use this information to identify the PRACTICAL RANGE.

11. The life expectancy for males in the United States can be modeled by the function $L(x) = 0.27x + 48.3$, where L is the life expectancy and x is the number of years since 1900.

- a) Which letter, L or x is used for input?
- b) What does the INPUT represent? Include units.
- c) Which letter, L or x , is used for output?
- d) What does the OUTPUT represent? Include units.
- e) Draw a neat, labeled and accurate sketch of this graph in the space below.

X	L(x)
0	
20	
40	
60	
80	
100	
120	



- f) What is the practical domain of $L(x)$? Use proper inequality notation.
- g) What is the practical range of $L(x)$? Use proper inequality notation.
- h) What is the life expectancy of a man born in Iowa in 1950?
- i) If a man is expected to live to the age of 60, approximate the year he was born. (Round to one decimal place)?

4. Bill is planning to sell bottled water at the local carnival. He buys 2 crates of water (800 bottles) for \$240 and plans on selling the bottles for \$1.50 each. Bill's profit, P in dollars, from selling b bottles of water is given by the formula $P(b) = 1.5b - 240$.
- a) In a complete sentence, interpret $P(250) = 135$.
- b) Determine $P(100)$ and interpret its meaning in a complete sentence.
- c) For what value of b is $P(b) = 0$? Write a sentence explaining the meaning of your answer.
- d) Determine the practical domain. Use proper notation and include units.
- e) Determine the practical range. Use proper notation and include units.
- f) Use your graphing calculator to generate a graph of $P(b)$. Use the practical domain and range to define the appropriate viewing window. Sketch the graph from your calculator screen, and write down the viewing window you used.



Xmin= _____

Xmax= _____

Ymin= _____

Ymax= _____

Lesson 1b – Linear Equations

In the first lesson we looked at the concepts and rules of a Function. The first Function that we are going to investigate is the Linear Function. This is a good place to start because with Linear Functions, change is constant and no exponents are involved. Before we begin working with Linear Functions, though, we need to review the characteristics of Linear Equations and operations on Linear Equations.

Lesson Objectives:

1. Identify LINEAR EQUATIONS
2. Determine slope and y-intercept for a LINEAR EQUATION
3. Determine the x-intercept for a LINEAR EQUATION
4. Draw graphs of LINEAR EQUATIONS
5. Graph Horizontal and Vertical lines
6. Write LINEAR EQUATIONS

Lesson 1b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 1b

The topic of linear equations should be at least slightly familiar to students starting Intermediate Algebra. The basics are covered here with reminders of important ideas and concepts that will be heavily utilized in the next lesson.

What is a Linear Equation?

A LINEAR EQUATION is an equation that can be written in the form:

$$y = mx + b$$

with slope, m , and y-intercept $(0, b)$

This is the SLOPE-INTERCEPT form for the equation of a line.

Slope

The slope of a line is denoted by the letter m . Given any two points, (x_1, y_1) , (x_2, y_2) , on a line, the slope is determined by computing the following ratio:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Slope is also referred to as the “change in y” over the “change in x” and is a measure of steepness and direction for a given line.

Problem 1 | WORKED EXAMPLE – DETERMINE SLOPE OF A LINEAR EQUATION

Find the slope of the line through the points $(2, -5)$ and $(-3, 4)$.

$$\begin{aligned} m &= \frac{4 - (-5)}{-3 - (2)} \\ &= \frac{4 + 5}{-5} \\ &= \frac{9}{-5} \\ &= -\frac{9}{5} \end{aligned}$$

Find the slope of the line through the points $(-2, -4)$ and $(4, 8)$

$$\begin{aligned} m &= \frac{8 - (-4)}{4 - (-2)} \\ &= \frac{8 + 4}{4 + 2} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

Note: The slope is negative indicating the line decreases from left to right. If slope is positive, then the line increases from left to right.

y-intercept

Also called the **VERTICAL INTERCEPT**, this is the special ordered pair with coordinates (0, b). 0 is the value of x (input) and the resulting output, b, is the y-coordinate of the y-intercept. The y-intercept is often used to help when graphing a linear equation and/or to determine the initial output value in an application setting.

Problem 2	WORKED EXAMPLE – DETERMINE Y-INTERCEPT FOR A LINEAR EQUATION
------------------	---

There are 3 main methods for finding the y-intercept (also called the vertical intercept) of a linear equation.

Method 1: Read the value of b from $y=mx + b$ form.

Method 2: Solve for y when $x = 0$

Method 3: Solve the equation for y and then read the value b

Example 1: Find the y-intercept for the equation $y = -2x + 6$.

This equation is written in the form $y = mx + b$. Therefore, (using Method 1) the y-intercept is (0, 6).

Example 2: Find the y-intercept for the equation $4x + 2y = 6$

If the equation is NOT in $y = mx + b$ form. Using Method 2, set x to 0 and solve for y.

$$\begin{aligned}
 4x + 2y &= 6 && \text{Original Equation} \\
 4(0) + 2y &= 6 && \text{Set x to 0} \\
 \frac{2y}{2} &= \frac{6}{2} && \text{Divide both sides by 2} \\
 y &= 3 && \text{y-intercept is (0,3)}
 \end{aligned}$$

Example 3: Find the y-intercept for the equation $4x + 2y = 6$

In this example, we use method 3 to rewrite the equation in $y=mx + b$ form.

$$\begin{aligned}
 4x + 2y &= 6 && \text{Original Equation} \\
 -4x + 4x + 2y &= 6 - 4x && \text{Subtract 4x from both sides} \\
 \frac{2y}{2} &= \frac{-4x + 6}{2} && \text{Divide both sides by 2} \\
 y &= -2x + 3 && \text{y-intercept is (0,3)}
 \end{aligned}$$

Problem 3	MEDIA EXAMPLE – DETERMINE SLOPE/Y-INTERCEPT FOR LINEAR EQUATION
------------------	--

Complete the problems below. *Note: Method 1 and Method 3 will be used to find the y-intercept for all the problems below because we are also asked to determine the slope. Practice finding the y-intercept using Method 2 as well.*

Equation

 $y = mx + b$ form

Slope

Y-intercept

a) $y = -2x + 5$

b) $y = 2 - x$

c) $y = \frac{3}{4}x + 2$

d) $2x - y = 4$

e) $3x + 2y = 6$

f) $8x = 4y$

x-intercept

Also called the **HORIZONTAL INTERCEPT**, this is the special ordered pair with coordinates $(a, 0)$. 0 is the value of y (output) and the resulting input, a , is the x -coordinate of the x -intercept. The x -intercept is often used to help when graphing a linear equation and/or to determine the final input value in an application.

Problem 4 | WORKED EXAMPLE – FIND THE X-INTERCEPT FOR A LINEAR EQUATION

Find the x -intercept (also called the horizontal intercept) for the equation $y = -2x + 6$.

Method: Replace the value of y with 0 then solve for the value of x .

$$\begin{array}{rcl} 0 & = & -2x + 6 \\ -6 & & -6 & \text{Subtract 6 from both sides} \\ -6 & = & -2x \\ \frac{-6}{-2} & \frac{-2x}{-2} & & \text{Divide both sides by -2} \\ 3 & = & x & \text{The x-intercept is (3, 0).} \end{array}$$

Problem 5 | MEDIA EXAMPLE – FIND X-INTERCEPT FOR A LINEAR EQUATION

For each of the following problems, determine the x -intercept as an ordered pair using the methods above.

Equation	Show Work	x-intercept	Equation	Show Work	x-intercept
----------	-----------	-------------	----------	-----------	-------------

a) $y = -2x + 5$

b) $y = 2 - x$

c) $y = \frac{3}{4}x + 2$

d) $2x - y = 4$

e) $3x + 2y = 6$

f) $8x = 4y$

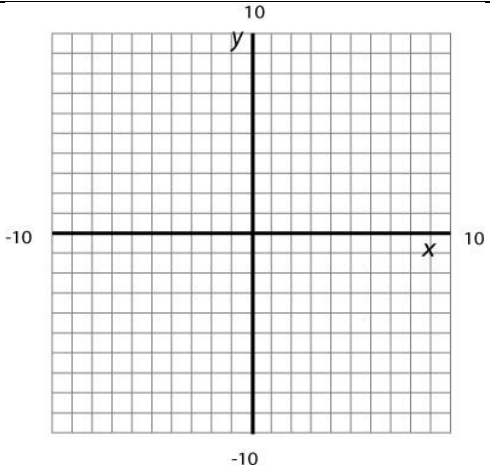
Problem 6 | YOU TRY – DRAW GRAPHS OF LINEAR EQUATIONS

Use the equation $y = -\frac{3}{2}x + 9$ for all parts of this problem. Label all plotted points.

- a) Use the x-intercept and y-intercept to draw the graph of the line. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw your line.

x-intercept : (__ , __)

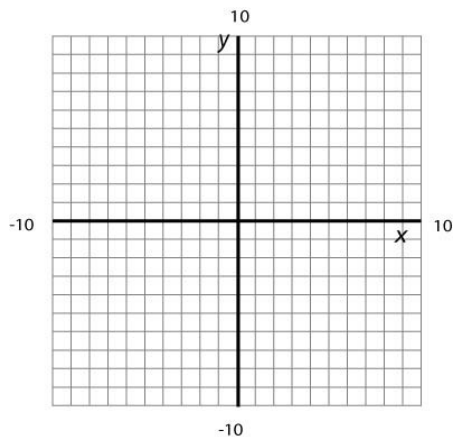
y-intercept: (__ , __)

Work to find x-intercept		Work to find y-intercept
--------------------------	---	--------------------------

- b) Determine the coordinates of two OTHER ordered pairs and use those to graph the line. PLOT and LABEL the points you use.

Remember from your previous math work that you can create a t-table with ordered pairs other than the intercepts OR you can insert the equation into your Y= list on your TI 83/84 calculator then use the TABLE feature to identify two other ordered pairs. Remember to enter as $y1 = (-3/2)x + 9$.

Ordered pair 1: (__ , __) Ordered pair 2: (__ , __)



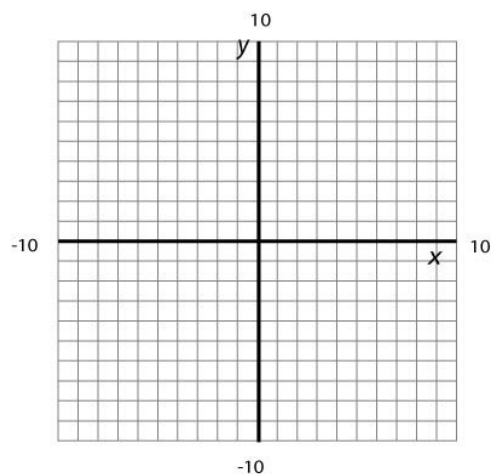
NOTICE: Your graphs for parts a) and b) should look exactly the same.

Special Linear Equations

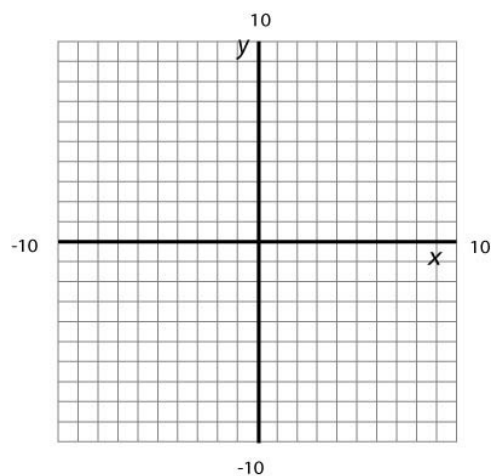
The following media problem will introduce you to two special types of linear equations: horizontal and vertical lines.

Problem 7 | MEDIA EXAMPLE – GRAPHING HORIZONTAL/VERTICAL LINES

a) Use the grid below to graph the equation $y = -2$. Identify the slope and y-intercept.



b) Use the grid below to graph the equations $x = 5$. Identify the slope and y-intercept.



<p>Equations of Vertical Lines</p> <ul style="list-style-type: none"> • equation: $x = a$ • x-intercept: $(a, 0)$ • y-intercept: none • slope: m is undefined 	<p>Equations of Horizontal Lines</p> <ul style="list-style-type: none"> • equation: $y = b$ • x-intercept: none • y-intercept: $(0, b)$ • slope: $m = 0$
---	--

Writing Equations of Lines

Critical to a thorough understanding of linear equations is the ability to write the equation of a line given different pieces of information. The following process will work for almost every situation you are presented with and will be illustrated several times in the media problems to follow.

Step 1: Determine the value of the slope, m .

Step 2: Determine the coordinates of one ordered pair.

Step 3: Plug the values for the ordered pair, and the value for the slope, into $y = mx + b$

Step 4: Solve for b

Step 5: Use the values for m and b to write the resulting equation in $y = mx + b$ form.

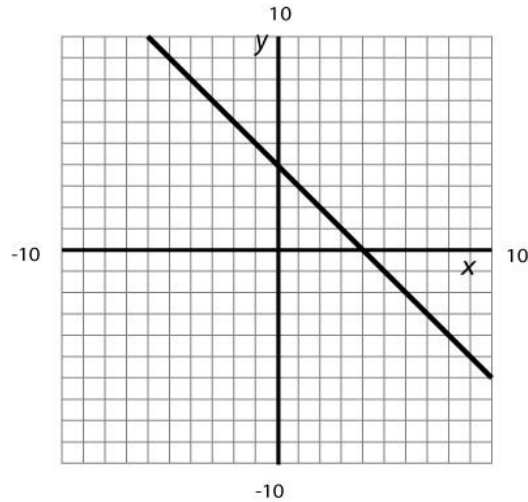
Problem 8 | MEDIA EXAMPLE – WRITING EQUATIONS OF LINES

For each of the following, find the equation of the line that meets the following criteria:

- a) Slope $m = -4$ passing through the point $(0, 3)$.
- b) Passing through the points $(0, -2)$ and $(1, 5)$
- c) Passing through the points $(-2, -3)$ and $(4, -9)$
- d) Parallel to $y = 3x - 7$ and passing through $(2, -5)$
- e) Passing through $(2, 4)$ with an x -intercept of -2 .
- f) Vertical line passing through $(-3, 5)$.
- g) Horizontal line passing through $(-3, 5)$.

Problem 9 | YOU TRY – WRITING LINEAR EQUATIONS FROM GRAPHS

Use the given graph to help answer the questions below. Assume the line intersects grid corners at integer (not decimal) values.



- a) Is the line above increasing, decreasing, or constant? *Read the graph from left to right.*
- b) What is the vertical (y) intercept? *Also, plot and label the y-intercept on the graph.*
- c) What is the horizontal (x) intercept? *Also, plot and label the x-intercept on the graph.*
- d) What is the slope (m)? Work to find slope:
Show work at right to compute.
Hint: Use two points from the graph.
- e) What is the equation of the line in $y=mx + b$ form?
Hint: Use the slope and y-intercept from above.

Problem 10 | **YOU TRY – WRITING EQUATIONS OF LINES**

a) Find the equation of the line passing through the points (1,4) and (3,-2) and write your equation in the form $y = mx + b$. Show complete work in this space.

b) What is the vertical (y) intercept for this equation? Show work or explain your result.

c) What is the horizontal (x) intercept for this equation? Show complete work to find this.

Problem 11 | **YOU TRY – HORIZONTAL AND VERTICAL LINES**

a) Given the ordered pair (2, -3)

- Write the equation of the vertical line through this point. _____
- Identify the slope of the line: _____
- What is the y-intercept? _____
- What is the x-intercept? _____

b) Given the ordered pair (2, -3)

- Write the equation of the horizontal line through this point. _____
- Identify the slope of the line: _____
- What is the y-intercept? _____
- What is the x-intercept? _____

Problem 12 | WORKED EXAMPLE – WRITING EQUATIONS OF LINES

Write an equation of the line to satisfy each set of conditions.

a) A line that contains the points (-3, 5) and (0, 1)

Slope: Use the ordered pairs (-3, 5) and (0, 1) to compute slope.

$$m = \frac{1-5}{0-(-3)} = \frac{-4}{3}$$

y-intercept: Given y-intercept in the ordered pair (0, 1) that was provided then $b = 1$.

Equation: Plug m and b into $y = mx + b$

$$m = \frac{-4}{3}, b = 1$$

$$y = -\frac{4}{3}x + 1$$

b) Line contains points (-4, -3) and (2, 6)

Slope: Use the ordered pairs (-4, -3) and (2, 6) to compute slope.

$$\begin{aligned} m &= \frac{6-(-3)}{2-(-4)} \\ &= \frac{6+3}{2+4} \\ &= \frac{9}{6} \\ &= \frac{3}{2} \end{aligned}$$

y-intercept: Not given y-intercept. Pick one given ordered pair and plug m and that ordered pair into $y = mx + b$. Solve for b .

Using (2, 6)

$$6 = \frac{3}{2}(2) + b$$

$$6 = 3 + b$$

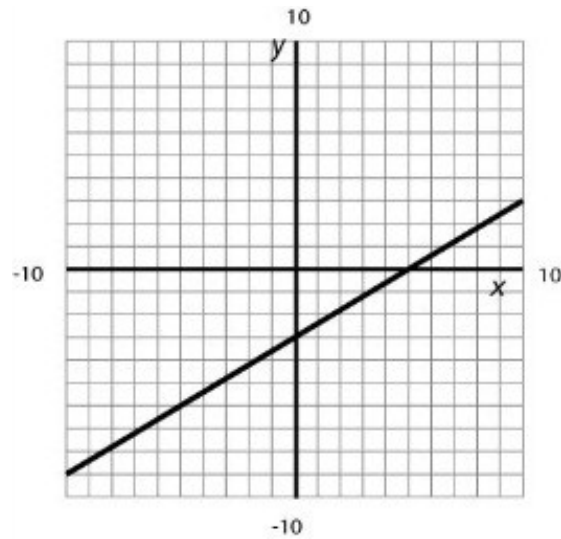
$$3 = b$$

Equation: Plug m and b into $y = mx + b$

$$m = \frac{3}{2}, b = 3$$

$$y = \frac{3}{2}x + 3$$

c) Line has the following graph:



Slope: Identify two ordered pairs from the graph and use them to determine the slope.

(5, 0) and (3, -1)

$$m = \frac{-1 - (0)}{3 - (5)} = \frac{-1}{-2} = \frac{1}{2}$$

y-intercept: Read the y-intercept from the graph. Ordered pair is (0, -3). Therefore $b = -3$.

Equation: Plug m and b into $y = mx + b$

$$m = \frac{1}{2}, b = -3$$

$$y = \frac{1}{2}x - 3$$

Name: _____

Date: _____

Lesson 1b Practice Problems

1. Complete the table below. If the equation is not in $y = mx + b$, show the steps required to convert it to that form. Also show the work required to calculate the X-Int.

Equation	$y = mx + b$ form	Slope	Y-Int	X-Int
----------	-------------------	-------	-------	-------

a) $y = -4x - 8$

b) $y = 3 - 4x$

c) $y = \frac{1}{3}x - 2$

d) $-4x - y = 2$

e) $-6x + 3y = 9$

f) $2x = 4y$

g) $y = 4$

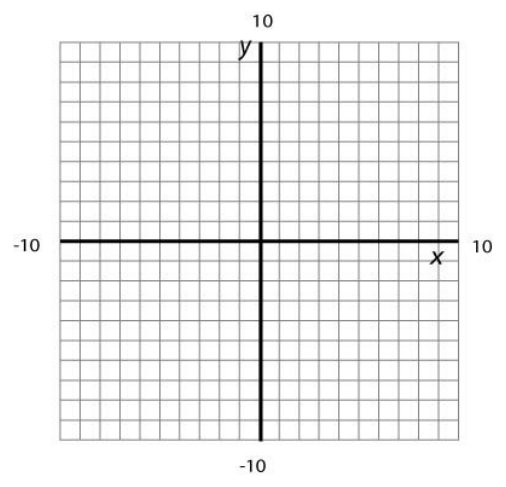
h) $x = -3$

2. Use the equation $y = -x + 2$ for all parts of this problem. Label all plotted points.

- a) Use the x-intercept and y-intercept to draw the graph of the line. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw your line.

x-intercept : (__ , __)

y-intercept: (__ , __)

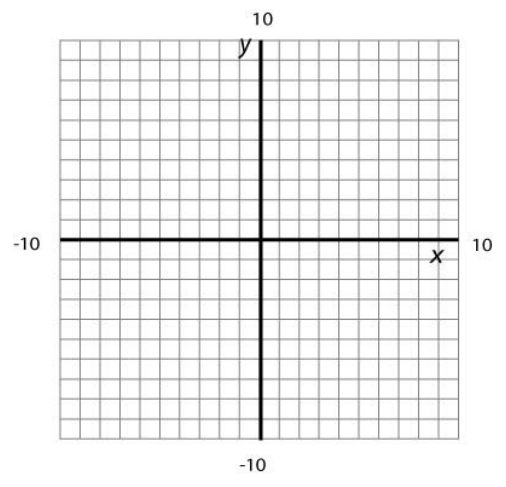
Work to find x-intercept		Work to find y-intercept
--------------------------	---	--------------------------

3. Use the equation $y - 3 = -\frac{1}{3}x$ for all parts of this problem. Label all plotted points.

- a) Use the x-intercept and y-intercept to draw the graph of the line. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw your line.

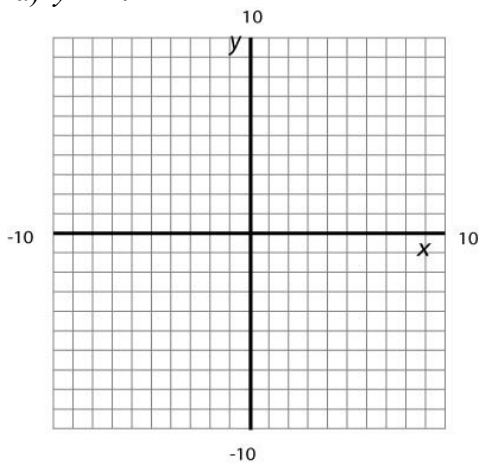
x-intercept: (__ , __)

y-intercept: (__ , __)

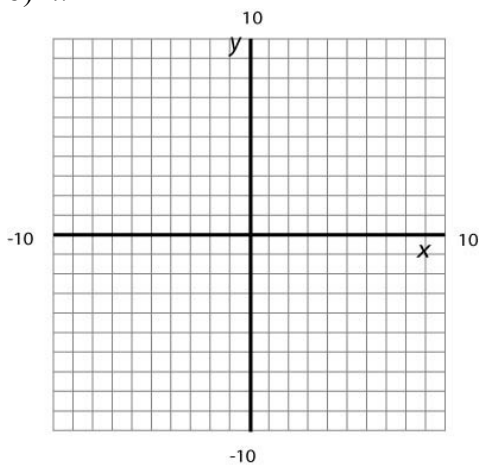
Work to find x-intercept		Work to find y-intercept
--------------------------	--	--------------------------

4. Graph each of the following equations. Identify the slope and the intercepts.

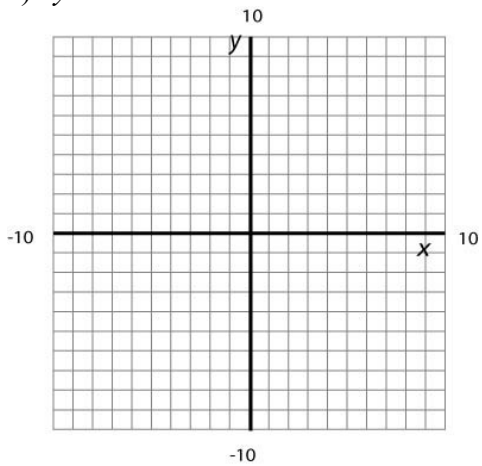
a) $y = -4$



b) $x = 2$



c) $y = -2x$



5. For each of the following, find the equation of the line that meets the following criteria:

a) Slope $m = 2$ passing through the point $(0, -3)$.

b) Passing through the points $(0, -4)$ and $(1, 6)$

c) Passing through the points $(3, 6)$ and $(6, -9)$

d) Parallel to $y = -2x + 5$ and passing through $(2, -5)$

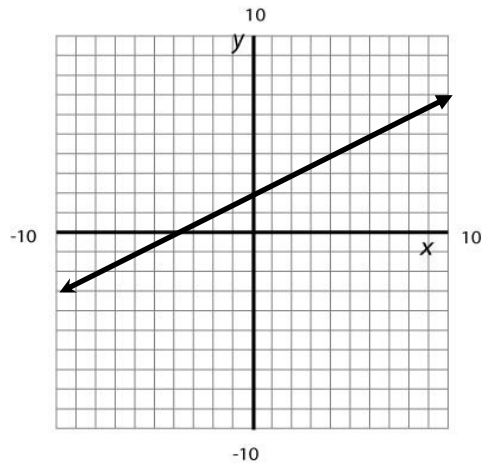
e) Passing through $(2, 6)$ with an x-intercept of -2 .

f) Vertical line passing through $(-5, 3)$.

g) Horizontal line passing through $(2, 7)$.

i) Passing through the points $(0, 0)$ and $(2, 6)$

6. Use the given graph to help answer the questions below.



- a) Is the line above increasing, decreasing, or constant?

- b) What is the vertical (y) intercept? *Also, plot and label the point on the graph.*

- c) What is the horizontal (x) intercept? *Also, plot and label the point on the graph.*

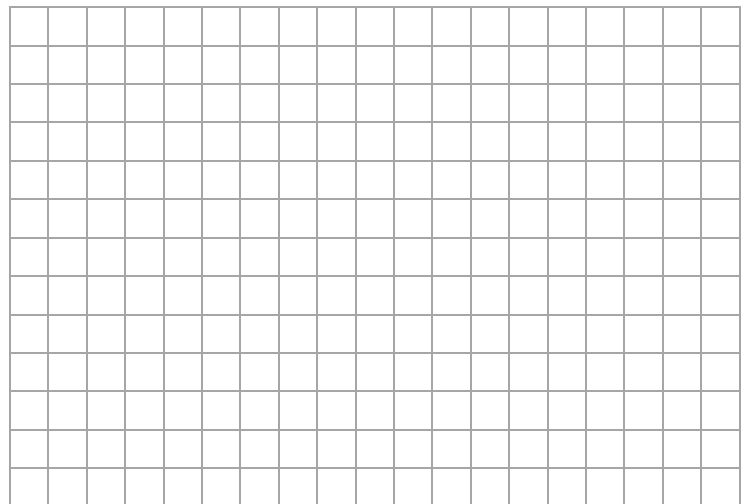
- d) What is the slope (m)? *Show your work.*

- e) What is the equation of the line in $y = mx + b$ form?

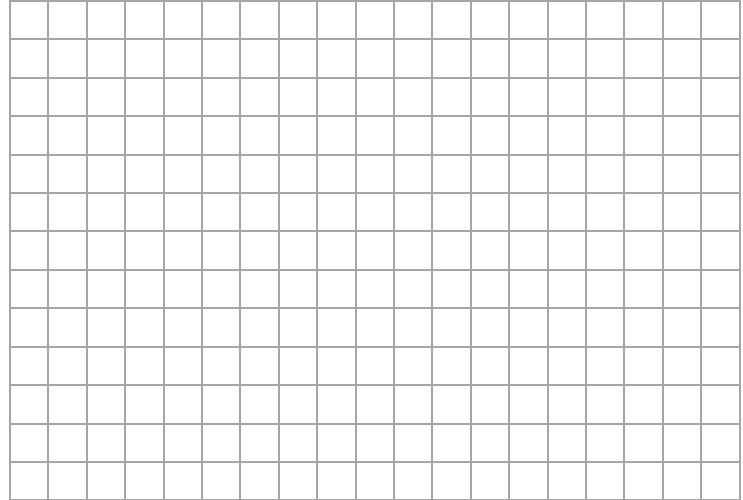
7. A line passes through the points (3,6) and (5,-4).
- a) Determine the equation for the line in the form $y = mx + b$. Show complete work in this space.

 - b) What is the vertical (y) intercept for this equation? Show work or explain your result.

 - c) What is the horizontal (x) intercept for this equation? Show complete work to find this.
8. Find the equation of the line for the following problem. Graph the results.
- a) Jolado decided to go on a diet. On the day he started, he weighed 200 pounds. For the next 8 weeks, he consistently lost 2 pounds a week. At the end of 8 weeks, he decided to make a graph showing his progress. He determined that the y-intercept for his graph would be **(0,200)** and the slope would be **-2** (2 pounds a week).



- b) Jolado needed 200 pounds of roofing nails for his project. He poured one cup filled with nails into a bucket and found that it weighed 2.3 pounds. He then poured 4 more cups of nails into the bucket and found that it weighed 9.5 pounds. He figured if he used the points **(1,2.3)** and **(5,9.5)** he could figure out a formula and calculate how many cups he would need.



- c) Challenge question. The formula you found above doesn't go through the origin. Shouldn't 0 cups of nails weigh 0 pounds? Can you figure out why 0 cups of nails actually weighs MORE than 0 pounds in Jalado's equation?

Name: _____

Date: _____

Lesson 1b Assessment

1. Determine the equation of the line between the points (4, 3) and (12, -3). Write your answer in slope-intercept form $y = mx + b$.

2. The function $P(n) = 455n - 1820$ represents a computer manufacturer's profit when n computers are sold.

a) Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

Ordered Pair: _____

b) Determine the horizontal intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.

Ordered Pair: _____

3. Determine the equation of the vertical line passing through the point (4, 7). _____

4. Determine the equation of the x -axis. _____

5. Fill in the table below. Intercepts must be written as ordered pairs. Write “N” if your answer is undefined.

I = Increasing

D = Decreasing

C = Horizontal (Constant)

V = Vertical

Equation $f(x) = mx+b$	Slope	Vertical Intercept	Horizontal Intercept	I, D, H, V
$f(x) = 2x - 16$				
$f(x) = 8 - 3x$				
$f(x) = -2$				
$f(x) = x$				
	-5	(0, 20)		
		(0, -9)	(-3, 0)	
$x = 6$				

Lesson 2a – Linear Functions and Applications

In lesson 2a, we take a close look at Linear Functions and how real world situations can be modeled using Linear Functions. We study the relationship between Average Rate of Change and Slope and how to interpret these characteristics. We also learn how to create Linear Models for data sets using Linear Regression.

Lesson Objectives:

1. Compute AVERAGE RATE OF CHANGE
2. Explain the meaning of AVERAGE RATE OF CHANGE as it relates to a given situation
3. Interpret AVERAGE RATE OF CHANGE as SLOPE
4. Solve application problems that involve LINEAR FUNCTIONS
5. Use LINEAR REGRESSION to write LINEAR FUNCTIONS that model given data sets
6. Solve application problems that involve using LINEAR REGRESSION

Lesson 2a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 2a

This lesson will combine the concepts of FUNCTIONS and LINEAR EQUATIONS. To write a linear equation as a LINEAR FUNCTION, replace the variable y using FUNCTION NOTATION. For example, in the following linear equation, we replace the variable y with $f(x)$:

$$y = mx + b$$
$$f(x) = mx + b$$

Important Things to Remember about the LINEAR FUNCTION $f(x) = mx + b$

- x represents the INPUT quantity.
- $f(x)$ represents the OUTPUT quantity (where $f(x)$, pronounced, “f of x” really just means “y”).
- The graph of $f(x)$ is a straight line with slope, m , and y -intercept $(0, b)$.
- If $m > 0$, the graph INCREASES from left to right, if $m < 0$, the graph DECREASES from left to right.
- The DOMAIN of a Linear Function is generally ALL REAL NUMBERS unless a context or situation is applied in which case we interpret the PRACTICAL DOMAIN in that context or situation.
- One way to identify the y -intercept is to evaluate $f(0)$. In other words, substitute 0 for input (x) and determine output (y). Remember that y -intercept and vertical intercept are the same thing.
- To find the x -intercept, solve the equation $f(x) = 0$ for x . In other words, set $mx + b = 0$ and solve for the value of x . Then $(x, 0)$ is your x -intercept. Remember that x -intercept and horizontal intercept are the same thing.

Average rate of change (often just referred to as the “rate of change”) of a function over a specified interval is the ratio $\frac{\text{change in output}}{\text{change in input}} = \frac{\text{change in } y}{\text{change in } x}$.

Units for the Average Rate of Change are always $\frac{\text{output units}}{\text{input units}}$.

Problem 2 | MEDIA EXAMPLE – AVERAGE RATE OF CHANGE

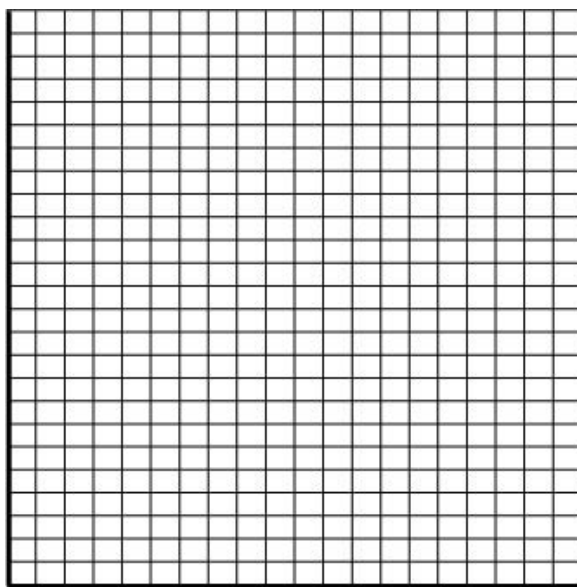
The function $E(t) = 3861 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. Use this function and your graphing calculator to complete the table below.

t , years since 1999	$E(t)$, Surface Elevation of Lake Powell (in feet above sea level)
0	
1	
2	
3	
5	
6	

- Determine the Average Rate of Change of the surface elevation between 1999 and 2000.
- Determine the Average Rate of Change of the surface elevation between 2000 and 2004.
- Determine the Average Rate of Change of the surface elevation between 2001 and 2005.

d) What do you notice about the Average Rates of Change for the function $E(t)$?

e) On the grid below, draw a GOOD graph of $E(t)$ with all appropriate labels.



Because the Average Rate of Change is constant for these depreciation data, we say that a LINEAR FUNCTION models these data best.

Does AVERAGE RATE OF CHANGE look familiar? It should! Another word for “average rate of change” is SLOPE. Given any two points, (x_1, y_1) , (x_2, y_2) , on a line, the slope is determined by computing the following ratio:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

Therefore, AVERAGE RATE OF CHANGE = SLOPE over a given interval.

Problem 3 | MEDIA EXAMPLE – IS THE FUNCTION LINEAR?

For each of the following, determine if the function is linear. If it is linear, give the slope.

a)

x	-4	-1	2	8	12	23	42
y	-110	-74	-38	34	82	214	442

b)

x	-1	2	3	5	8	10	11
y	5	-1	1	11	41	71	89

c)

x	-4	-1	2	3	5	8	9
y	42	27	12	7	-3	-18	-23

Recap – What Have We Learned So Far in this Lesson

- **LINEAR FUNCTIONS** are just **LINEAR EQUATIONS** written using **FUNCTION NOTATION**.
- $f(x) = mx + b$ means the same thing as $y = mx + b$ but the notation is slightly different.
- **AVERAGE RATE of CHANGE** means the same thing as **SLOPE**.
- Data sets that have a **constant average rate of change** (or we can say, constant rate of change), can best be modeled by **LINEAR FUNCTIONS**.

Given Input, Find Output and Given Output, Find Input Questions

When working with LINEAR FUNCTIONS, there are two main questions we will ask and solve as follows:

- Given a particular INPUT value, what is the corresponding OUTPUT value? To address this question, you will EVALUTE the function at the given input by replacing the input variable with the input value and computing the result.
- Given a particular OUTPUT value, what is the corresponding INPUT value? To address this question, you will set the equation equal to the output value and solve for the value of the input.

Problem 4 | YOU TRY – APPLICATIONS OF LINEAR FUNCTIONS

The data below represent your annual salary for the first four years of your current job. The data are exactly linear.

Time, t , in years	0	1	2	3	4
Salary, S , in \$	20,100	20,600	21,100	21,600	22,100

- a) Identify the vertical intercept and average rate of change for the data, then use these to write the linear function model for the data. Use the indicated variables and proper function notation.
- b) Use your model to determine the amount of your salary in year 8. [Note: This is a “Given Input, Find Output” question.] Write your final result as a complete sentence.
- c) Use your model to determine how many years you would need to work to earn a yearly salary of at least \$40,000. Round to the nearest whole year. [Note: This is a “Given Output, Find Input” question.]. Write your final result as a complete sentence.

Once your data columns are clear, enter the input data into L1 (press ENTER after each data value to get to the next row) then right arrow to L2 and enter the output data into L2. Your result should look like this when you are finished (for L1 and L2):

L1	L2	L3	Z
0	196	-----	
1	183		
2	180		
3	177		
4	174		
5	171		

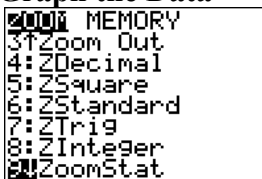
L2(6) =171			

Step 2: Turn on your Stat Plot

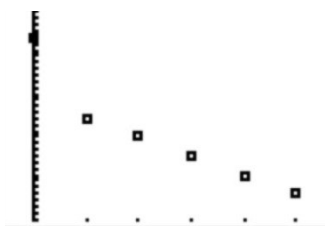


- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be highlighted as at left

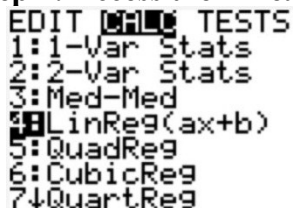
Step 3: Graph the Data



- Press ZOOM
- Scroll down to 9:ZoomStat and press ENTER
- A graph of your data should appear in an appropriate window so that all data points are clearly visible (see below)



Other than the first data point, our data look pretty linear. To determine a linear equation that fits the given data, we could do a variety of things. We could choose the first and last point and use those to write the equation. We could ignore the first point and just use two of the remaining points. Our calculator, however, will give us the best linear equation possible taking into account ALL the given data points. To find this equation, we use a process called LINEAR REGRESSION.

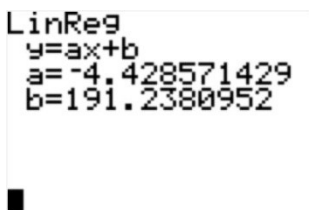
FINDING THE LINEAR REGRESSION EQUATION**Step 1: Access the Linear Regression section of your calculator**


```

EDIT [MODE] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg

```

- Press STAT
- Scroll to the right one place to CALC
- Scroll down to 4:LinReg(ax+b)
- Your screen should look as the one at left

Step 2: Determine the linear regression equation


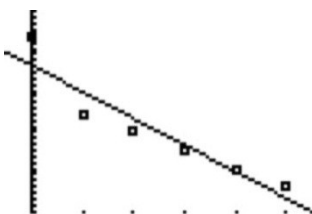
```

LinReg
y=ax+b
a=-4.428571429
b=191.2380952

```

- Press ENTER twice in a row to view the screen at left
- The calculator computes values for slope (a) and y-intercept (b) in what is called the equation of best-fit for your data.
- Identify these values and round to the appropriate places. Let's say 2 decimals in this case. So, $a = -4.43$ and $b = 191.24$
- Now, replace the a and b in $y = ax + b$ with the rounded values to write the actual equation:
 $y = -4.43x + 191.24$
- To write the equation in terms of initial variables, we would write $W = -4.43t + 191.24$

Once we have the equation figured out, it's nice to graph it on top of our data to see how things match up.

GRAPHING THE REGRESSION EQUATION ON TOP OF THE STAT PLOT

- Enter the Regression Equation with rounded values into Y=
- Press GRAPH
- You can see from the graph that the “best fit” line does not hit very many of the given data points. But, it will be the most accurate linear model for the overall data set.

IMPORTANT NOTE: When you are finished graphing your data, TURN OFF YOUR PLOT1. Otherwise, you will encounter an INVALID DIMENSION error when trying to graph other functions. To do this:

- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be UNhighlighted

Problem 7 | YOU TRY – LINEAR REGRESSION

The following table gives the total number of live Christmas trees sold, in millions, in the United States from 2004 to 2011. (Source: Statista.com).

Year	2004	2006	2008	2010	2011
Total Number of Christmas Trees Sold in the U.S. (in millions)	27.10	28.60	28.20	27	30.80

- a) Use your calculator to determine the equation of the *regression line*, $C(t)$ where t represents the number of years since 2004. Refer to the steps outlined in Problem 5 for guidance.

Start by entering new t values for the table below based upon the number of years since 2004. The first few are done for you:

t = number of years since 2004	0	2			
$C(t)$ = Total number of Christmas trees sold in the U.S. (in millions)	27.10	28.60	28.20	27	30.80

Determine the regression equation in $y = ax + b$ form and write it here: _____
Round to three decimals as needed.

Rewrite the regression equation in $C(t) = at + b$ form and write it here: _____
Round to three decimals as needed.

- b) Use the regression equation to determine $C(3)$ and explain its meaning in the context of this problem.
- c) Use the regression equation to predict the number of Christmas trees that will be sold in the year 2013. Write your answer as a complete sentence.
- d) Identify the slope of the regression equation and explain its meaning in the context of this problem.

Problem 8 | YOU TRY – LINEAR REGRESSION

- a) Given the data in the table below, use your calculator to plot a scatterplot of the data and draw a rough but accurate sketch in the space below.

x	0	2	4	6	8	10	12
y	23.76	24.78	25.93	26.24	26.93	27.04	27.93

- b) Use your graphing calculator to determine the equation of the regression line. Round to three decimals as needed.

- c) Using the TABLE, determine the value of y when the input is 6:

Write the specific ordered pair associated with this result:

- d) Using your EQUATION from part b), determine the value of y when the input is 6 (Hint: Plug in 6 for x and compute the y -value):

Write the specific ordered pair associated with this result:

- e) Your y -values for c) and d) should be different. Why is this the case? (refer to the end of Problem 5 for help).

- f) Use the result from part b) to predict the value of x when the output is 28.14. Set up an equation and show your work to solve it here.

Write the specific ordered pair associated with this result: _____

Problem 9 | WORKED EXAMPLE – Multiple Ways to Determine The Equation of a Line

Determine if the data below represent a linear function. If so, use at least two different methods to determine the equation that best fits the given data.

x	1	5	9	13
y	75	275	475	675

Compute a few slopes to determine if the data are linear.

$$\text{Between } (1, 75) \text{ and } (5, 275) \quad m = \frac{275 - 75}{5 - 1} = \frac{200}{4} = 50$$

$$\text{Between } (5, 275) \text{ and } (9, 475) \quad m = \frac{475 - 275}{9 - 5} = \frac{200}{4} = 50$$

$$\text{Between } (9, 475) \text{ and } (13, 675) \quad m = \frac{675 - 475}{13 - 9} = \frac{200}{4} = 50$$

The data appear to be linear with a slope of 50.

Method 1 to determine Linear Equation – Slope Intercept Linear Form ($y = mx + b$):

Use the slope, $m = 50$, and one ordered pair, say $(1, 75)$ to find the y-intercept
 $75 = 50(1) + b$, so $b = 25$.

Thus the equation is given by $y = 50x + 25$.

Method 2 to determine Linear Equation – Linear Regression:

Use the steps for Linear Regression to find the equation. The steps can be used even if the data are exactly linear.

Step 1: Go to STAT>EDIT>1:Edit

Step 2: Clear L1 by scrolling to L1 then press CLEAR then scroll back down one row

Step 3: Enter the values 1, 5, 9, 13 into the rows of L1 (pressing Enter between each one)

Step 4: Right arrow then up arrow to top of L2 and Clear L2 by pressing CLEAR then scroll back down

Step 5: Enter the values 75, 275, 475, 675 into the rows of L2 (pressing Enter between each one)

Step 6: Go to STAT>EDIT>CALC>4:LinReg ($ax + b$) then press ENTER twice

Step 7: Read the values a and b from the screen and use them to write the equation, $y = 50x + 25$

Name: _____

Date: _____

Lesson 2a Practice Problems

1. The data below represent the number of times your friends embarrassing YouTube Video has been viewed per hour since you uploaded it. The data are exactly linear.

Time, t , in hours	0	1	2	3	4
Views, V , in thousands	0	6200	12400	18600	24800

- a) Identify the vertical intercept and average rate of change for the data, then use these to write the linear function model for the data. Use the indicated variables and proper function notation.
- b) Use your model to determine the number of views in hour 8. Write your final result as a complete sentence.
- c) Use your model to determine how many hours until the number of views reaches 100,000. Round to the nearest whole year. Write your final result as a complete sentence.
2. You adopted an adult cat four years ago. The data below represent your cat's weight for four years she's lived with you. The data are exactly linear.

Time, t , in years	0	1	2	3	4
Weight, W , in pounds	6	7.25	8.5	9.75	11

- a) Identify the vertical intercept and average rate of change for the data, then use these to write the linear function model for the data. Use the indicated variables and proper function notation.

- b) Use your model to determine how much the cat will weigh in year 8.
- c) Use your model to determine how many years it would take for your cat to reach 20 pounds. Round to the nearest whole year.
3. Data below represent how many pushups Tim can do in a minute at the start of a 5-week exercise program and each week thereafter.

Time, t , in weeks	0	1	2	3	4	5
Number of Pushups in a minute	2	6	10	14	18	20

- a) Compute the average rate of change for the first three weeks. Be sure to include units.
- b) Compute the average rate of change for the 5-wk period. Be sure to include units.
- c) What is the meaning of the average rate of change in this situation?
- d) Do the data points in the table define a perfectly linear function? Why or why not?

4. You decided to save up for a vacation to Europe by throwing all your loose change in a large coffee can. After a few months, you discover that the jar is 2 inches full and contains \$124.
- Determine the average rate of change, in \$/inch (Dollars per inch), for the coffee can from when it was empty (0 inches) to when it was 2 inches deep.
 - A month later, you check the can and find the change is 3 inches deep and adds up to \$186. Find the average rate of change, in \$/inch, for the coffee can from 0 inches to 3 inches.
 - What is the MEANING of the average rate of change in this situation?

You do some additional calculations and create a table for your can of change.

d , depth of the change in inches	V , value of the can in dollars
0	0
2	124
3	186
5	310
10	620

- Use the information found so far to write an equation that describes this situation. Use function notation and the variable names from the table.
- You need \$1000 for your vacation. In a complete sentence, state how deep the change has to be to reach your goal. Also, write the results as an ordered pair and in function notation.

5. The following table shows the growth of newspaper subscriptions in Middletown, USA with t equaling the number of years since 1995 ($t = 0$ in 1995) and $S(t)$ representing the total subscriptions in each year.

t (year)	0	2	4	6	8
$S(t)$ (total subscriptions)	450,000	430,000	300,000	250,000	180,000

- a) Use your calculator to determine the equation of the *regression line*.

Determine the regression equation in $y = ax + b$ form and write it here:

Rewrite the regression equation in $S(t) = at + b$ form and write it here:

- b) Write in words the meaning of the mathematical statement $S(2) = 430,000$ as it relates to this problem.
- c) Use the result from part a) to estimate the total subscriptions in 2007 (i.e. when $t = 12$). Show your computations here and your final result. (Hint: This is a “Beyond the Data, given input, find output” question, so compute the value of $S(12)$ and show your work here)
- d) What is the slope of $S(t)$ and what is its meaning in the context of this problem?
- e) Use your last result from part a) to estimate the year in which the circulation will be 100,000. Round to the closest whole year. (Hint: This is a “Given output find input” question so solve the equation $S(t) = 100,000$ for t and round to the nearest year. Add your result to 1995).

6. Your turn. Create a story problem where the data changes linearly and then create a table that has data points for that story.

a) Write the story problem.

b) Create a table for the story problem. Make sure you use Function Notation.

c) Compute the average rate of change for your data. Be sure to include units.

d) What is the meaning of the average rate of change in this situation?

e) Determine the vertical intercept for your data. What is the meaning of this vertical intercept?

f) Use the vertical intercept and the rate of change to write the linear function model for the data. Use proper variable names and proper function notation.

g) Write a **read the data** question given the input. Write your question as a complete sentence and in function notation.

h) Write a **read the data** question given the output. Write your question as a complete sentence and in function notation.

- i) Write a **read between the data (Interpolating the Data)** question given the input. Write your question as a complete sentence and in function notation.

- j) Write a **read between the data (Interpolating the Data)** question given the output. Write your question as a complete sentence and in function notation.

- k) Write a **read beyond the data (Extrapolating the Data)** question given the input. Write your question as a complete sentence and in function notation.

- l) Write a **read beyond the data (Extrapolating the Data)** question given the output. Write your question as a complete sentence and in function notation.

Name: _____

Date: _____

Lesson 2a Assessment

Years Since 1980	Total Sales (in millions of dollars)
0	1.19
5	1.40
10	1.91
15	1.88
21	2.01
25	2.12
26	2.38

1. Let $S(t)$ represent the total sales of this company t years after 1980. Use your calculator to determine regression equation for this data set. Use function notation, and round to four decimal places as needed.

2. Use the TABLE to determine the sales in 2005.

3. Use the EQUATION found in problem 1 to determine sales in 2005. Round your answer to the nearest tenth.

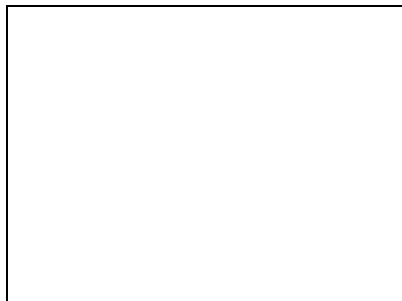
4. Your answers for questions 2 and 3 should be different. Why is this the case?

5. Use the equation found in problem 1 to predict the sales in 2015. Round your answer to the nearest tenth.

6. Interpret the meaning of the statement $S(30) = 2.44$.

7. Use the equation found in problem 1 to determine the year in which sales should reach \$3,000,000.

8. Use your graphing calculator to generate a scatterplot of the data *and* regression line on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.



Xmin= _____

Xmax= _____

Ymin= _____

Ymax= _____

Lesson 2b – Functions and Function Operations

As we continue to work with more complex functions it is important that we are comfortable with Function Notation, operations on Functions and operations involving more than one function. In this lesson, we study using proper Function Notation and then spend time learning how add, subtract, multiply and divide Functions, both algebraically and when the functions are represented with a tables or graphs. Finally, we take a look at a couple of real work examples that involve operations on functions.

Lesson Objectives:

1. Use proper FUNCTION NOTATION when reading, writing and working with functions and when performing OPERATIONS with functions.
2. Use a function to find INPUT given OUTPUT or find OUTPUT given INPUT
3. ADD and SUBTRACT functions by combining like terms
4. MULTIPLY or DIVIDE functions using rules of exponents
5. Perform OPERATIONS with functions using tables, graphs, or symbolic notation
6. Work with APPLICATIONS of function OPERATIONS (Cost, Revenue, Profit functions)

Lesson 2b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 2b

Function Notation: FUNCTION NOTATION is used to indicate a functional relationship between two quantities as follows:

$$y = f(x)$$

means that
y is a function of x

Read $y = f(x)$ as “y equals f of x”

Problem 1 YOU TRY – READING FUNCTIONS
--

Complete the table below. Learning to read functions using correct language and to identify input/output quantities is very important for our work in this class. The first one is done for you.

Function	Function in words	Function in input/output	Input	Output
$f(x) = 3x + 7$	f of x equals three times x plus seven	Output = $3(\text{input}) + 7$	x	f(x)
$g(x) = 4x + 2$				
$f(x) = \frac{1}{2}x$				
$g(x) = \frac{2x}{3} + 1$				
$h(x) = x^2 + 1$				

Let’s continue our review of functions and function notation, as understanding these topics is CRITICAL for your work in the rest of this course. Functions describe specific relationships between inputs and outputs. Work with the table below as a reminder of how to find inputs and outputs for a given function. If you can learn to think about functions in terms of “inputs” and “outputs”, then the remainder of our topics will be much easier to understand.

Problem 2 | YOU TRY – FINDING INPUTS, FINDING OUTPUTS

Complete the table below. The first one is done for you. Write your results as an ordered pair.

Function	(Input, Output)	Given Input, Find Output	Given Output, Find Input
$f(x) = x + 7$	$(x, f(x))$	Find $f(x)$ when $x = 2$. Looking for $(2, \underline{\quad})$ To find the output EVALUATE $f(2)$ $f(x) = x + 7$ $f(2) = (2) + 7$ $f(2) = 9$ Ordered pair: $(2, 9)$	Find x when $f(x) = 8$. Looking for $(\underline{\quad}, 8)$ To find the output, SOLVE for $f(x) = 8$ $f(x) = x + 7$ $8 = x + 7$ $8 - 7 = x + 7 - 7$ $1 = x$ Ordered pair: $(1, 8)$
$g(t) = 3t - 4$	$(t, g(t))$	Find $g(t)$ when $t = 4$.	Find t when $g(t) = -1$.
$f(x) = x - 5$		Find $f(x)$ when $x = -4$.	Find x when $f(x) = 10$.
$g(x) = 2x + 4$		Find $g(x)$ when $x = 3$.	Find x when $g(x) = 5$.

Functions notation can be expanded to include notation for the different ways we can combine functions as described below.

Basic Mathematical Operations

The basic mathematical operations are: addition, subtraction, multiplication, and division. When working with function notation, these operations will look like this:

OPERATION	Function Notation – How to Read	EXAMPLE
Addition	$f(x) + g(x)$ “f of x plus g of x”	$3x^2 + 4x$
Subtraction	$f(x) - g(x)$ “f of x minus g of x”	$(2x + 3) - (4x^2 + 1)$
Multiplication	$f(x) * g(x)$ “f of x times g of x”	$(3x - 1)(5x + 7)$
Division	$\frac{f(x)}{g(x)}$ “f of x divided by g of x” Note the $g(x) \neq 0$	$\frac{8x^3 - 2x + 1}{4x}$

The examples on the right side of the table may look fairly familiar. That is because you have probably studied these before in a previous class. Many of the problems we will work in this lesson are problems you may already know how to do. You will just need to get used to some new notation.

We will start with the operations of addition and subtraction.

Problem 3 | WORKED EXAMPLE – ADDING AND SUBTRACTING FUNCTIONS

Given $f(x) = 2x^2 + 3x - 5$ and $g(x) = -x^2 + 5x + 1$.

a) find $f(x) + g(x)$

$$\begin{aligned} f(x) + g(x) &= (2x^2 + 3x - 5) + (-x^2 + 5x + 1) \\ &= 2x^2 + 3x - 5 - x^2 + 5x + 1 \\ &= 2x^2 - x^2 + 3x + 5x - 5 + 1 \\ f(x) + g(x) &= x^2 + 8x - 4 \end{aligned}$$

b) find $f(x) - g(x)$

$$\begin{aligned} f(x) - g(x) &= (2x^2 + 3x - 5) - (-x^2 + 5x + 1) \\ &= 2x^2 + 3x - 5 + x^2 - 5x - 1 \\ &= 2x^2 + x^2 + 3x - 5x - 5 - 1 \\ f(x) - g(x) &= 3x^2 - 2x - 6 \end{aligned}$$

c) find $f(1) - g(1)$

$$\begin{aligned} f(1) - g(1) &= [2(1)^2 + 3(1) - 5] - [-(1)^2 + 5(1) + 1] \\ &= (2 + 3 - 5) - (-1 + 5 + 1) \\ &= 0 - 5 \\ f(1) - g(1) &= -5 \end{aligned}$$

Problem 4 | MEDIA EXAMPLE – ADDING AND SUBTRACTING FUNCTIONS

Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = x^2 + 2x + 5$:

a) find $f(x) + g(x)$

b) find $f(x) - g(x)$

Problem 5 | YOU TRY – ADDING AND SUBTRACTING FUNCTIONS

Given $f(x) = x + 4$ and $g(x) = x^2 + 1$, determine each of the following. Show complete work.

a) find $f(2) + g(2)$

b) find $f(x) - g(x)$

c) find $f(2) - g(2)$

Function Multiplication and the Multiplication Property of Exponents

When multiplying functions, you will often need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

MULTIPLICATION PROPERTY OF EXPONENTS

Let m and n be rational numbers.

To multiply powers of the same base, keep the base and add the exponents:

$$a^m * a^n = a^{m+n}$$

Problem 6 | WORKED EXAMPLE – FUNCTION MULTIPLICATION

a) Given $f(x) = 3x + 2$ and $g(x) = 2x - 5$, find $f(x) * g(x)$

$$\begin{aligned} f(x) * g(x) &= (3x + 2)(2x - 5) && \text{Now use FOIL} \\ &= (3x)(2x) + (3x)(-5) + (2)(2x) + (2)(-5) && \text{Remember the rules of exp.} \\ &= \text{FIRST} + \text{OUTER} + \text{INNER} + \text{LAST} && (3x)(2x) = (3)(2)(x)(x) \\ & && = 6x^2 \\ &= (6x^2) + (-15x) + (4x) + (-10) && \text{Combine Like Terms} \\ f(x) * g(x) &= 6x^2 - 11x - 10 && \text{Final Result} \end{aligned}$$

b) Given $f(x) = 5$ and $g(x) = (x+3)^0$, find $f(x) * g(x)$

$$\begin{aligned} f(x) * g(x) &= (5)[(x + 3)^0] \\ &= (5)(1) && \text{Remember that } a^0 = 1 \text{ if } a \neq 0 \\ f(x) * g(x) &= 5 && \text{Final Result} \end{aligned}$$

c) Given $f(x) = -8x^4$ and $g(x) = 5x^3$, find $f(x) * g(x)$

$$\begin{aligned} f(x) * g(x) &= (-8x^4)(5x^3) \\ &= (-8)(5)(x^4)(x^3) && \text{Reorder using Commutative Property} \\ &= (-40)(x^{4+3}) && \text{Simplify using the Multiplication Property of Exponents} \\ f(x) * g(x) &= -40x^7 && \text{Final Results} \end{aligned}$$

Problem 7 | MEDIA EXAMPLE – FUNCTION MULTIPLICATION

Given $f(x) = 3x + 2$ and $g(x) = 2x^2 + 3x + 1$, find $f(x) * g(x)$

Problem 8 | YOU TRY – FUNCTION MULTIPLICATION

For each of the following, find $f(x) * g(x)$

a) $f(x) = 3x - 2$ and $g(x) = 3x + 2$

b) $f(x) = 2x^2 + 1$ and $g(x) = x^3 - 4x + 5$

c) $f(x) = 4x - 1$ and $g(x) = (3x)^0$

d) $f(x) = 4x^3$ and $g(x) = -6x$

Function Division and the Division Property of Exponents

When dividing functions, you will also need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

DIVISION PROPERTY OF EXPONENTS

Let m, n be rational numbers. To divide powers of the same base, keep the base and subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n} \text{ where } a \neq 0$$

Problem 9 | WORKED EXAMPLE – FUNCTION DIVISION

For each of the following, find $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 15x^{15}$ and $g(x) = 3x^9$

$$\begin{aligned} \frac{f(x)}{g(x)} &= \frac{15x^{15}}{3x^9} \\ &= 5x^{15-9} \\ &= 5x^6 \end{aligned}$$

b) $f(x) = -4x^5$ and $g(x) = 2x^8$

$$\begin{aligned} \frac{f(x)}{g(x)} &= \frac{-4x^5}{2x^8} \\ &= -2x^{5-8} \\ &= -2x^{-3} \end{aligned}$$

This is not our final answer, however, as we need to use only positive exponents in our final result. Recall the rule for negative exponents as follows: If $a \neq 0$ and n is a rational number, then

$$a^{-n} = \frac{1}{a^n}$$

Let's use this rule to rewrite as follows:

$$-2x^{-3} = \frac{-2}{x^3} \text{ Therefore, our final results are: } \frac{f(x)}{g(x)} = \frac{-2}{x^3}$$

Notice that the -2 on top did not change or impact the exponent in any way.

Problem 10 | MEDIA EXAMPLE – FUNCTION DIVISION

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 10x^4 + 3x^2$ and $g(x) = 2x^2$

b) $f(x) = -12x^5 + 8x^2 + 5$ and $g(x) = 4x^2$

Problem 11 | YOU TRY – FUNCTION DIVISION

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 25x^5 - 4x^7$ and $g(x) = -5x^4$

b) $f(x) = 20x^6 - 16x^3 + 8$ and $g(x) = -4x^3$

Functions can be presented in multiple ways including: equations, data sets, graphs, and applications. If you understand function notation, then the process for working with functions is the same no matter how the information is presented.

Problem 12 **MEDIA EXAMPLE – WORKING WITH FUNCTIONS IN TABLE FORM**

Functions $f(x)$ and $g(x)$ are defined in the tables below. Find a – h below using the tables.

x	-3	-2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	-2	-5

- a) $f(1) =$
- b) $g(9) =$
- c) $f(0) + g(0) =$
- d) $g(5) - f(8) =$
- e) $f(0) * g(3) =$

Problem 13 **YOU TRY – WORKING WITH FUNCTIONS IN TABLE FORM**

Given the following two tables, complete the third table. Show work in the table cell for each column. The first one is done for you.

x	0	1	2	3	4
$f(x)$	4	3	-2	0	1

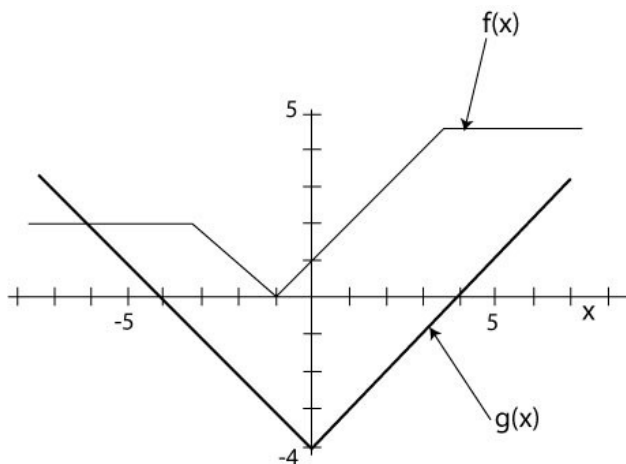
x	0	1	2	3	4
$g(x)$	6	-3	4	-2	2

x	0	1	2	3	4
$f(x) + g(x)$	$= f(0) + g(0)$ $= 4 + 6$ $= 10$	$= f(1) + g(1)$			

If you remember that graphs are just infinite sets of ordered pairs and if you do a little work ahead of time (as in the example below) then the graphing problems are a lot easier to work with.

Problem 14 | YOU TRY – WORKING WITH FUNCTIONS IN GRAPH FORM

Use the graph to determine each of the following. Assume integer answers. The graph of g is the graph in bold.



a) Complete the following ordered pairs from the graphs above. Use the information to help you with the problems below. The first ordered pair for each function has been completed for you.

f : $(-7, 2), (-6, \quad), (-5, \quad), (-4, \quad), (-3, \quad), (-2, \quad), (-1, \quad), (0, \quad), (1, \quad), (2, \quad), (3, \quad), (4, \quad), (5, \quad), (6, \quad), (7, \quad)$

g : $(-7, 3), (-6, \quad), (-5, \quad), (-4, \quad), (-3, \quad), (-2, \quad), (-1, \quad), (0, \quad), (1, \quad), (2, \quad), (3, \quad), (4, \quad), (5, \quad), (6, \quad), (7, \quad)$

a) $g(4) =$

b) $f(2) =$

c) $g(0) =$

d) $f(-6) =$

e) If $f(x) = 0, x =$

f) If $g(x) = 0, x =$ or $x =$

g) If $f(x) = 1, x =$ or $x =$

h) If $g(x) = -4, x =$

i) $f(-1) + g(-1) =$

j) $g(-6) - f(-6) =$

k) $f(1) * g(-2) =$

l) $\frac{g(6)}{f(-1)} =$

One of the classic applications of function operations is the forming of the Profit function, $P(x)$ by subtracting the cost function, $C(x)$, from the revenue function, $R(x)$ as shown below.

PROFIT, REVENUE, COST

Given functions $P(x) = \text{Profit}$, $R(x) = \text{Revenue}$, and $C(x) = \text{Cost}$:

$$P(x) = R(x) - C(x)$$
$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Problem 15 | MEDIA EXAMPLE – COST, REVENUE, PROFIT

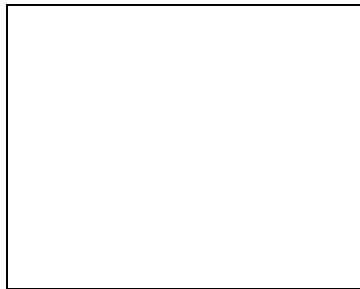
A local courier service estimates its monthly operating costs to be \$1500 plus \$0.85 per delivery. The service generates revenue of \$6 for each delivery. Let x = the number of deliveries in a given month.

- Write a function, $C(x)$, to represent the monthly costs for making x deliveries per month.
- Write a function, $R(x)$, to represent the revenue for making x deliveries per month.
- Write a function, $P(x)$, that represents the monthly profits for making x deliveries per month.
- Determine algebraically the break-even point for the function $P(x)$ and how many deliveries you must make each month to begin making money. Show complete work. Write your final answer as a complete sentence.
- Determine the break-even point graphically by solving the equation $P(x) = 0$. Explain your work and show the graph with appropriate labels. Write your final answer as a complete sentence.

Problem 16 | **YOU TRY – COST, REVENUE, PROFIT**

February is a busy time at Charlie's Chocolate Shoppe! During the week before Valentine's Day, Charlie advertises that his chocolates will be selling for \$1.50 a piece (instead of the usual \$2.00 each). The fixed costs to run the Chocolate Shoppe total \$450 for the week, and he estimates that each chocolate costs about \$0.60 to produce.

- Write a function, $C(x)$, to represent Charlie's total costs for the week if he makes x chocolates.
- Write a function, $R(x)$, to represent the revenue from the sale of x chocolates during the week before Valentine's Day.
- Write a function, $P(x)$, that represents Charlie's profit from selling x chocolates during the week before Valentine's Day. Show complete work to find the function.
- How many chocolates must Charlie sell in order to break even? Show complete work. Write your final answer as a complete sentence.
- In the space below, sketch the graph of $P(x)$ with all appropriate labels. Mark the break even point on the graph.



- Identify the vertical intercept of the graph. Write it as an ordered pair and write a sentence explaining its meaning in this situation.

Name: _____

Date: _____

Lesson 2b Practice Problems

1) Simplify using the properties of exponents. Show all steps for credit. Write your final result using positive exponents.

a) t^3t^4

b) $(x^3y^4)(xy^2z^3)(z^2)$

c) $(-rt^4)(r^2s^3t^2)(-3t^4)$

d) $(t^3)^4$

e) $(-2x^4yz^2)^4$

f) $\frac{3x^2y^4z}{-27x^4y^7z^3}$

g) $\left(\frac{t^3z^6}{-2v^2}\right)^3$

h) $\frac{3x^{-3}}{y^{-2}}$

i) $\left(\frac{3x^{-2}y^{-4}z}{-27x^4y^{-7}z^3}\right)^0$

2. Multiply the following Polynomials. Show all steps. Write your answer in Descending order.

a) $4(3x - 5)$

b) $(2x - 4)(3x - 5)$

c) $(x + 2)(x - 2)$

d) $(x - 5)^2$

e) $3x(4x - 2)$

f) $(x + 5)(2x^2 - x)$

g) $(x - 4)(x^2 + x - 5)$

h) $3(x + 2)(x + 4)$

i) $4(x + 2)^2$

3) Add, subtract and multiply the following functions. Simplify your answers.

a) $f(x) = -4x + 7$ and $g(x) = -3x$

$$f(x) + g(x) =$$

$$f(x) - g(x) =$$

$$f(x) * g(x) =$$

$$f(2) + g(3) =$$

b) $f(x) = -x + 2$ and $g(x) = -3x + 7$

$$f(x) + g(x) =$$

$$f(x) - g(x) =$$

$$f(x) * g(x) =$$

$$f(-3) + g(5) =$$

c) $f(x) = 3x^2 + 4x + 2$ and $g(x) = 6x + 1$

$$f(x) * g(x) =$$

$$f(4) + g(-1) =$$

4. Simplify each of the following functions. Use only positive exponents in your final answer.

a) $f(x) = 32x^4 - 3x^7$ and $g(x) = -6x^4$

b) $f(x) = 48x^9 - 16x^3 + 4$ and $g(x) = -8x^3$

$$\frac{f(x)}{g(x)} =$$

$$\frac{f(x)}{g(x)} =$$

5) Functions $f(x)$ and $g(x)$ are defined in the tables below. Use those tables to evaluate problems the problems below.

x	-3	-2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	-2	-5

a) $f(5) =$

b) $g(5) =$

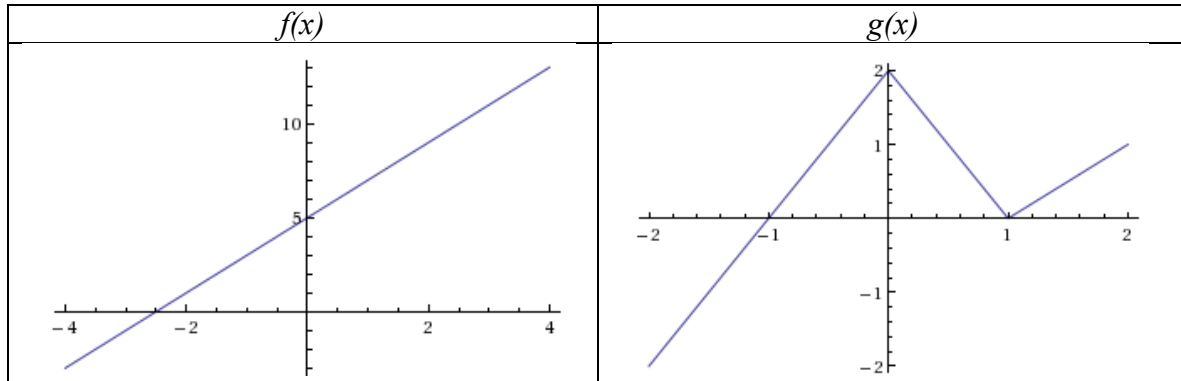
c) $f(5) + g(5) =$

d) $f(0) - g(0) =$

e) $f(8) * g(8) =$

f) $f(4) * g(0) =$

6) Use the graph to determine each of the following. Assume integer answers.



a) $f(0) + g(0) =$

b) $f(0) - g(2) =$

c) $f(-2.5) * g(0) =$

d) $f(-1) * g(-2) =$

7) Functions f and g are defined below. Use those functions to evaluate problems the problems below.

$f = \{(-3,4), (-2,6), (-1,8), (0,6), (1, -2)\}$

$g = \{(-1,8), (0,2), (4,3), (8,4)\}$

a) $f(-2) + g(0) =$

b) $f(1) - g(4) =$

c) $f(0) * g(0) =$

d) $f(-1) * g(8) =$

8. The function $E(n)$ represents Jenelle's budgeted monthly expenses for the first half of the year 2013. In the table, $n = 1$ represents January 2013, $n = 2$ February 2013, and so on.

n	1	2	3	4	5	6
$E(n)$	2263	2480	2890	2263	2352	2550

The function $L(n)$ shown in the table below represents Jenelle's monthly income for the first half of the year 2013. In the table, $n = 1$ represents January 2013, $n = 2$ February 2013, and so on.

n	1	2	3	4	5	6
$L(n)$	2850	2850	2850	2850	2850	2850

At the end of each month, Jenelle puts any extra money into a savings account. The function $S(n)$ represents the amount of money she puts into savings each month. Using the information above, complete the following table for the function $S(n)$.

n	1	2	3	4	5	6
$S(n)$						

Her goal is to save enough money to take a trip to Hawaii in July, 2013. She estimates that the trip will cost \$2000. Will she be able to save up enough money to go to Hawaii in July? If so, how much extra money will she have to spend while she is there? If not, how much more does she need to earn?

9. BoJim and Sariloo are organizing the 20 year reunion for their high school. The high school alumni association has given them \$1000 for the event. They talk to the local caterer and find out the following:
- It will cost \$15 per person plus a \$50 setup fee to provide food for the event.
 - It will cost \$3 per person plus a \$80 setup fee to rent the Meeting Hall at the local Holiday Motel.

To help determine the costs, they come up with the following functions:

- The cost for food is \$50 + \$15 per person. $F(x) = 15x + 50$
- The cost for the Hall is \$80 + \$3 per person $H(x) = 3x + 80$

In addition, they decide to charge each person \$5 to get in the door. This can be modeled by the following function:

- Income for the event is \$1000 from the alumni + \$5 per person. $I(x) = 5x + 1000$

Given this information, answer the following questions. Show how you use the functions to calculate the answers. And give your final answers in complete sentences.

If 400 people attend the event:

- a) How much will it cost for food?
- b) How much will it cost to rent the Meeting Hall?
- c) How much will it cost for food AND to rent the Meeting Hall? Show how you use the functions to calculate this. Hint: $(F+H)(400)$
- d) The final bill for the event is found by subtracting the costs from the income. What would the final bill for the event be?
- e) Challenge question. How many people can attend if the costs have to equal the income?

Name: _____

Date: _____

Lesson 2b Assessment

1. If possible, simplify each of the following by combining like terms or using properties of exponents.

a) $2n^5 + 3n^5 =$ _____

b) $2n^5 * 3n^5 =$ _____

c) $3n^3 + 3n^5 =$ _____

d) $3n^3 * 3n^5 =$ _____

2. The functions A and B are defined by the following tables

x	-3	-2	0	1	4	5	8	10	12
$A(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$B(x)$	1	3	5	10	4	2	0	-2	-5

Determine the values for each of the following.

a) $B(3) =$ _____

b) $A(8) =$ _____

c) $A(0) + B(0) =$ _____

d) $A(8) - B(8) =$ _____

e) $A(4) * B(4) =$ _____

f) $\frac{A(5)}{B(5)} =$ _____

3. Let $p(x) = x^2 + 2x + 3$ and $r(x) = x - 5$. Determine each of the following. Show all work. Box your answers.

a) $p(x) - r(x) =$

b) $p(0) * r(0) =$

c) $p(-2) + r(-2) =$

d) $r(7) - p(7) =$

Lesson 3a – Introduction to Exponential Functions

Exponential Functions play a major role in our lives. Many of the challenges we face involve exponential change and can be modeled by an Exponential Function. Financial considerations are the most obvious, such as the growth of our retirement savings, how much interest we are paying on our home loan or the effects of inflation.

In this lesson, we begin our investigation of Exponential Functions by comparing them to Linear Functions, examining how they are constructed and how they behave. We then learn methods for solving exponential functions given the input and given the output.

Lesson Objectives:

1. State the important characteristics of LINEAR FUNCTIONS
2. Compare LINEAR GROWTH to EXPONENTIAL GROWTH using graphs, data, or equations
3. Write LINEAR and EXPONENTIAL equations/functions from data tables
4. Use COMMON RATIO to identify exponential data
5. Identify EXPONENTIAL FUNCTIONS and state their important characteristics
6. Use EXPONENTIAL FUNCTIONS to find INPUT given OUTPUT or find OUTPUT given INPUT
7. Graph EXPONENTIAL FUNCTIONS in an appropriate window

Lesson 3a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 3a

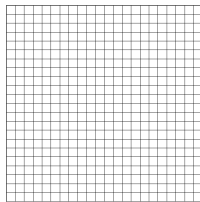
Exponential Functions vs. Linear Functions

So far in this course, the only function type we have studied is LINEAR FUNCTIONS. You spent the first lessons learning the characteristics of LINEAR FUNCTIONS. Let's remind ourselves of those now.

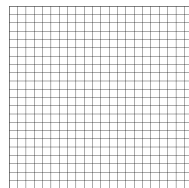
Problem 1 | YOU TRY – CHARACTERISTICS OF LINEAR FUNCTIONS

Given a function, $f(x) = mx + b$, respond to each of the following. Refer back to previous lessons as needed.

- The variable x represents the _____ quantity.
- $f(x)$ represents the _____ quantity. $f(x)$ is just another name for _____.
- The graph of $f(x)$ is a _____ with slope _____ and y-intercept _____.
- On the graphing grid below, draw an INCREASING linear function. In this case, what can you say about the slope of the line? m _____ 0 (Your choices here are $>$ or $<$)



- On the graphing grid below, draw a DECREASING linear function. In this case, what can you say about the slope of the line? m _____ 0 (Your choices here are $>$ or $<$)



- The defining characteristic of a LINEAR FUNCTION is that the RATE OF CHANGE (also called the SLOPE) is _____. We say, then, that LINEAR FUNCTIONS change in an ADDITIVE way.

This next example is long but will illustrate the key difference between EXPONENTIAL FUNCTIONS and LINEAR FUNCTIONS.

Problem 2 | WORKED EXAMPLE – DOLLARS & SENSE

On December 31st around 10 pm, you are sitting quietly in your house watching Dick Clark's New Year's Eve special when there is a knock at the door. Wondering who could possibly be visiting at this hour you head to the front door to find out who it is. Seeing a man dressed in a three-piece suit and tie and holding a briefcase, you cautiously open the door.

The man introduces himself as a lawyer representing the estate of your recently deceased great uncle. Turns out your uncle left you some money in his will, but you have to make a decision. The man in the suit explains that you have three options for how to receive your allotment.

Option A: \$1000 would be deposited on Dec 31st in a bank account bearing your name and each day an additional \$1000 would be deposited (until January 31st).

Option B: One penny would be deposited on Dec 31st in a bank account bearing your name. Each day, the amount would be doubled (until January 31st).

Option C: Take \$30,000 on the spot and be done with it.

Given that you had been to a party earlier that night and your head was a little fuzzy, you wanted some time to think about it. The man agreed to give you until 11:50 pm. Which option would give you the most money after the 31 days???

A table of values for option A and B are provided on the following page. Before you look at the values, though, which option would you select according to your intuition?

Without “doing the math” first, I would instinctively choose the following option (circle your choice):

Option
A

Option
B

Option
C

Option A: \$1000 to start + \$1000 per day		Option B: \$.01 to start then double each day	
Note that t = 0 on Dec. 31st			
Table of input/output values		Table of input/output values	
$t =$ time in # of days	$A(t) =$ \$ in account after t days	$t =$ time in # of days	$B(t) =$ \$ in account after t days
0	1000	0	.01
1	2000	1	.02
2	3000	2	.04
3	4000	3	.08
4	5000	4	.16
5	6000	5	.32
6	7000	6	.64
7	8000	7	1.28
8	9000	8	2.56
9	10,000	9	5.12
10	11,000	10	10.24
11	12,000	11	20.48
12	13,000	12	40.96
13	14,000	13	81.92
14	15,000	14	163.84
15	16,000	15	327.68
16	17,000	16	655.36
17	18,000	17	1,310.72
18	19,000	18	2,621.44
19	20,000	19	5,242.88
20	21,000	20	10,485.76
21	22,000	21	20,971.52
22	23,000	22	41,943.04
23	24,000	23	83,886.08
24	25,000	24	167,772.16
25	26,000	25	335,544.32
26	27,000	26	671,088.64
27	28,000	27	1,342,177.28
28	29,000	28	2,684,354.56
29	30,000	29	5,368,709.12
30	31,000	30	10,737,418.24
31	32,000	31	21,474,836.48

WOWWWWW!!!!!!

What IS that number for Option B? I hope you made that choice... it's 21 million, 4 hundred 74 thousand, 8 hundred 36 dollars and 48 cents. Let's see if we can understand what is going on with these different options.

Problem 3	MEDIA EXAMPLE – COMPARE LINEAR AND EXPONENTIAL GROWTH
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For the example discussed in Problem 2, respond to the following:

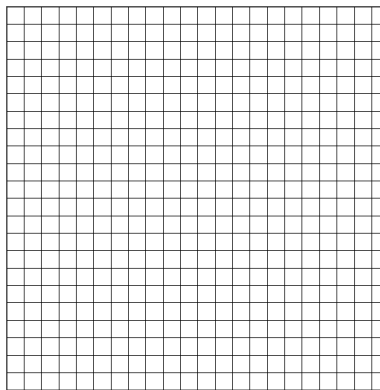
a) Symbolic representation (model) for each situation:

$A(t) =$ _____ Type of function _____

$B(t) =$ _____ Type of function _____

$C(t) =$ _____ Type of function _____

b) Provide a rough but accurate sketch of the graphs for each function on the same grid below:



c) What are the practical domain and range for each function?

	Practical Domain	Practical Range
$A(t):$		
$B(t):$		
$C(t):$		

d) Based on the graphs, which option would give you the most money after 31 days?

e) Let’s see if we can understand WHY option B grows so much faster. Let’s focus just on options A and B. Take a look at the data tables given for each function. Just the later parts of the initial table are provided.

$A(t) = 1000t + 1000$		$B(t) = .01(2)^t$	
$t =$ time in # of days	$A(t) =$ \$ in account after t days	$t =$ time in # of days	$B(t) =$ \$ in account after t days
20	21,000	20	10,485.76
21	22,000	21	20,971.52
22	23,000	22	41,943.04
23	24,000	23	83,886.08
24	25,000	24	167,772.16
25	26,000	25	335,544.32
26	27,000	26	671,088.64
27	28,000	27	1,342,177.28
28	29,000	28	2,684,354.56
29	30,000	29	5,368,709.12
30	31,000	30	10,737,418.24
31	32,000	31	21,474,836.48

As t increases from day 20 to 21, describe how the outputs change for each function:

$A(t)$:

$B(t)$:

As t increases from day 23 to 24, describe how the outputs change for each function:

$A(t)$:

$B(t)$:

So, in general, we can say as the inputs increase from one day to the next, then the outputs for each function:

$A(t)$:

$B(t)$:

In other words, $A(t)$ grows _____ and $B(t)$ grows _____. We have just identified the primary difference between **LINEAR FUNCTIONS** and **EXPONENTIAL FUNCTIONS**.

Exponential Functions vs. Linear Functions
 The outputs for **Linear Functions** change by **ADDITION** and the outputs for **Exponential Functions** change by **MULTIPLICATION**.

Problem 4 | WORKED EXAMPLE – ARE THE DATA EXPONENTIAL?

To determine if an exponential function is the best model for a given data set, calculate the ratio $\frac{y_2}{y_1}$ for each of the consecutive points. If this ratio is approximately the same for the entire set, then an exponential function models the data best. For example:

x	1	2	3	4	5
y	1.75	7	28	112	448

For this set of data, $\frac{y_2}{y_1} = \frac{7}{1.75} = \frac{28}{7} = \frac{112}{28} = \frac{448}{112} = 4$

Since $\frac{y_2}{y_1} = 4$ for all consecutive pairs, the data are exponential with a growth factor of 4.

Problem 5 | MEDIA EXAMPLE – LINEAR DATA VS. EXPONENTIAL DATA

Analyze each of the following data sets to determine if the set can be modeled best by a linear function or an exponential function. Write the equation that goes with each data set.

x	-1	-2	0	1	2	3	4
y	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125	625

x	0	1	2	3	4	5	6	7
y	-3.2	-3	-2.8	-2.6	-2.4	-2.2	-2.0	-1.8

Problem 6	YOU TRY – USE COMMON RATIO TO IDENTIFY EXPONENTIAL DATA
------------------	--

a) Given the following table, explain why the data can be best modeled by an exponential function. Use the idea of common ratio in your response.

x	0	1	2	3	4	5	6
f(x)	5.0	7.5	11.2	16.9	25.3	37.9	57.0

b) Determine an exponential model $f(x) = ab^x$ that fits these data. Start by identifying the values of a and b then writing your final result using proper notation.

Exponential Functions are of the form

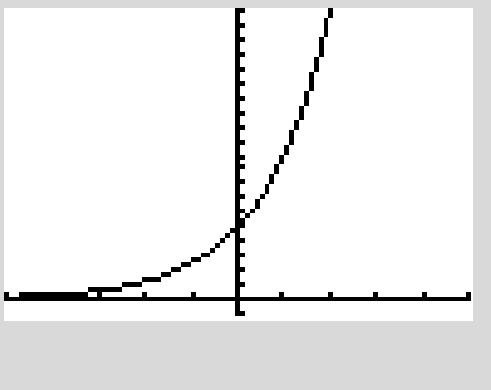

$$f(x) = ab^x$$

where a = the INITIAL VALUE

b = the base ($b > 0$ and $b \neq 1$) also called the GROWTH or DECAY FACTOR

Important Characteristics of the EXPONENTIAL FUNCTION $f(x) = ab^x$

- x represents the INPUT quantity
- $f(x)$ represents the OUTPUT quantity (where $f(x)$ really just means “y”)
- The graph of $f(x)$ is in the shape of the letter “J” with y-intercept $(0, a)$ and base, b (note that b is the same as the COMMON RATIO from previous examples)
- If $b > 1$, the function is an EXPONENTIAL GROWTH function and the graph INCREASES from L to R
- If $0 < b < 1$, the function is an EXPONENTIAL DECAY function and the graph DECREASES from L to R
- One way to identify the y-intercept is to evaluate $f(0)$. In other words, substitute 0 for input (x) and determine output (y). Remember that y-intercept and vertical intercept are the same thing.

<p>Graph of a generic Exponential Growth Function</p> $f(x) = ab^x, b > 1$ <ul style="list-style-type: none"> • Domain: All Real Numbers (all inputs) • Range: $f(x) > 0$ (only outputs bigger than 0) • X-intercept: None (graph does not cross x-axis) • Y-intercept: $(0, a)$ • Horizontal Asymptote: $y = 0$ (guiding line for the left side of the graph is the x-axis or $y = 0$) • Left to right behavior of the function: Y-values INCREASING 	
<p>Graph of a generic Exponential Decay Function</p> $f(x) = ab^x, 0 < b < 1$ <ul style="list-style-type: none"> • Domain: All Real Numbers (all inputs) • Range: $f(x) > 0$ (only outputs bigger than 0) • X-intercept: None (graph does not cross x-axis) • Y-intercept: $(0, a)$ • Horizontal Asymptote: $y = 0$ (guiding line for the right side of the graph is the x-axis or $y = 0$) • Left to right behavior of the function: Y-values DECREASING 	

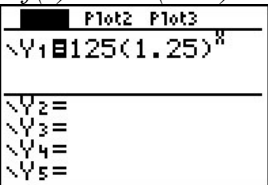
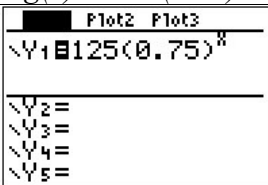
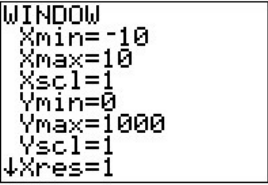
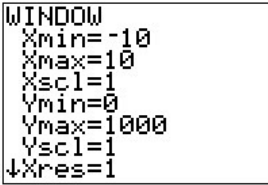
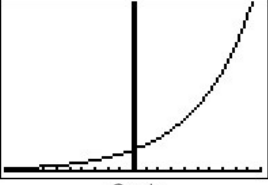

Problem 7	WORKED EXAMPLE – Examples of Exponential Functions
-----------	--

- | | |
|--------------------------|---|
| a) $f(x) = 2(3)^x$ | Initial Value, $a = 2$, y-intercept = $(0, 2)$
Base, $b = 3$.
$f(x)$ is an exponential GROWTH function since $b > 1$. |
| b) $g(x) = 1523(1.05)^x$ | Initial Value, $a = 1523$, y-intercept = $(0, 1523)$
Base, $b = 1.05$.
$g(x)$ is an exponential GROWTH function since $b > 1$. |
| c) $h(x) = 256(0.85)^x$ | Initial Value, $a = 256$, y-intercept = $(0, 256)$
Base, $b = 0.85$.
$h(x)$ is an exponential DECAY function since $b < 1$. |
| d) $k(x) = 32(0.956)^x$ | Initial Value, $a = 32$, y-intercept = $(0, 32)$
Base, $b = 0.956$.
$k(x)$ is an exponential DECAY function since $b < 1$. |

Problem 8 | YOU TRY – CHARACTERISTICS OF EXPONENTIAL FUNCTIONS

Complete the table using the graphs to assist you. You should enter the graphs on your calculator as well using the screens below to help you.

- Functions are entered on the Y= screen
- Press WINDOW to change your values as shown below
- Press GRAPH to show the graphs

$f(x) = 125(1.25)^x$	$g(x) = 125(0.75)^x$
 <p>Equation</p>	 <p>Equation</p>
 <p>Window</p>	 <p>Window</p>
 <p>Graph</p>	 <p>Graph</p>

	$f(x) = 125(1.25)^x$	$g(x) = 125(0.75)^x$
Initial Value (a)?		
Base (b)?		
Domain?		
Range?		
X-intercept?		
Y-intercept?		
Horizontal Asymptote?		
Increasing or Decreasing?		

When working with EXPONENTIAL FUNCTIONS, as with LINEAR FUNCTIONS, there are two main questions we will ask and solve as follows:

- Given a particular INPUT value, what is the corresponding OUTPUT value?
- Given a particular OUTPUT value, what is the corresponding INPUT value?

Problem 9	MEDIA EXAMPLE – GIVEN INPUT FIND OUTPUT
------------------	--

Use the functions $f(x) = 125(1.25)^x$ and $g(x) = 125(0.75)^x$ to respond to the questions below.

- a) Find $f(x)$ when $x = 2$. b) Find $g(x)$ when $x = 2$.
- c) Find $f(x)$ when $x = -4$. d) Find $g(x)$ when $x = -4$.

Problem 10	WORKED EXAMPLE – GIVEN OUTPUT FIND INPUT
-------------------	---

Given $f(x) = 125(1.25)^x$ find x when $f(x) = 300$. Round your respond to two decimal places.

To do this, we need to SOLVE the equation $125(1.25)^x = 300$ using a process called the INTERSECTION METHOD on our graphing calculators.

To solve $125(1.25)^x = 300$

- Press Y= then enter $Y1 = 125(1.25)^x$ and $Y2 = 300$
Note: You could also let $Y1 = 300$ and $Y2 = 125(1.25)^x$

```

Plot1 Plot2 Plot3
Y1=125(1.25)^X
Y2=300
Y3=
Y4=
Y5=
Y6=
Y7=

```

- Press WINDOW then enter the values at right.

```

WINDOW
Xmin=0
Xmax=10
Xscl=1
Ymin=0
Ymax=350
Yscl=1
Xres=1

```

Try to determine why these values were selected. You must see the intersection in your window. Other entries will work. If you graph and do not see both graphs AND where they intersect, you must pick new WINDOW values until you do.

KEEP GOING ON THE NEXT PAGE

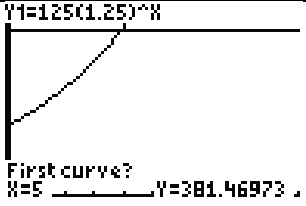
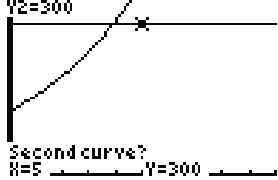
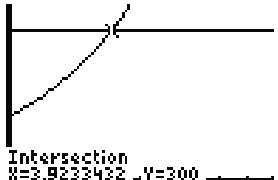
- Press 2nd>CALC
- Scroll to 5: INTERSECT and press ENTER

Notice the question, “First Curve?” The calculator is asking if $Y1 = 125(1.25)^x$ is the first curve in the intersection.

- Press Enter to indicate “Yes”

Notice the question, “Second Curve?” The calculator is asking if $Y2 = 300$ is the second curve in the intersection.

- Press Enter to indicate “Yes”
- Press Enter at the “Guess” question and obtain the screen at right. Your intersection values are given at screen bottom and the intersection is marked with a cursor. Round as indicated in your problem directions or as dictated by the situation.

For this problem, we were asked to find x when $f(x) = 300$. Round to two decimal places. Our response is that, “When $f(x) = 300$, $x = 3.92$ ”. Note that this information corresponds to the ordered pair $(3.92, 300)$ on the graph of $f(x) = 125(1.25)^x$

Problem 11 | YOU TRY – GIVEN OUTPUT FIND INPUT

Given $g(x) = 125(0.75)^x$, find x when $g(x) = 300$ (to two decimal places) using the same process as above.

- a) What equation will you solve to respond to the given question? _____
- b) On your calculator, you will let $Y1 =$ _____ and $Y2 =$ _____
- c) What values are you using for your WINDOW: $xmin=$ ____, $xmax=$ ____, $ymin=$ ____, $ymax=$ ____
- d) What is the value of x that you found when $g(x) = 300$? _____ (using 5: INTERSECT)
- e) To what ordered pair does this information correspond? _____
- f) Include a rough but accurate sketch of the graphs and intersection in the box to the right. Mark and label the intersection.



Problem 12 | **WORKED EXAMPLE – GIVEN OUTPUT FIND INPUT**

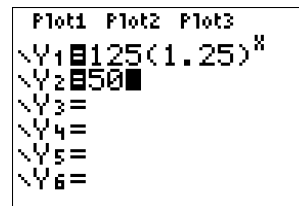
Given $f(x) = 125(1.25)^x$ find x when $f(x) = 50$. Round your response to two decimal places.

To do this, we need to SOLVE the equation $125(1.25)^x = 50$ using a process called the INTERSECTION METHOD on our graphing calculators.

To solve $125(1.25)^x = 50$

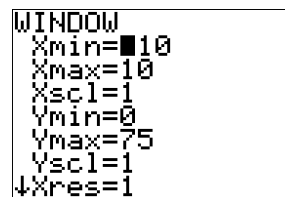
- Press Y= then enter $Y1 = 125(1.25)^x$ and $Y2 = 50$

Note: You could also let $Y1 = 50$ and $Y2 = 125(1.25)^x$



- Press WINDOW then enter the values at right.

Try to determine why these values were selected. You must see the intersection in your window. Other entries will work. If you graph and do not see both graphs AND where they intersect, you must pick new WINDOW values until you do.

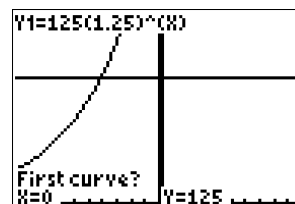


- Press 2nd>CALC
- Scroll to 5: INTERSECT and press ENTER

Notice the question, “First Curve?” The calculator is asking if

$Y1 = 125(1.25)^x$ is the first curve in the intersection.

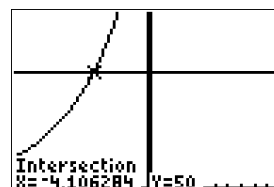
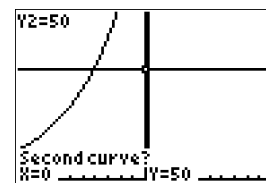
- Press Enter to indicate “Yes”



Notice the question, “Second Curve?” The calculator is asking if

$Y2 = 50$ is the second curve in the intersection.

- Press Enter to indicate “Yes”
- Press Enter at the “Guess” question and obtain the screen at right. Your intersection values are given at screen bottom and the intersection is marked with a cursor. Round as indicated in your problem directions or as dictated by the situation.



For this problem, we were asked to find x when $f(x) = 50$. Round to two decimal places. Our response is that, “When $f(x) = 50$, $x = -4.11$ ”. Note that this information corresponds to the ordered pair $(-4.11, 50)$ on the graph of $f(x) = 125(1.25)^x$

Problem 13 | **YOU TRY – GIVEN OUTPUT FIND INPUT**

Given $g(x) = 125(0.75)^x$, find x when $g(x) = 50$ (to two decimal places) using the same process as above.

- a) What equation will you solve to respond to the given question? _____
- b) On your calculator, you will let $Y1 =$ _____ and $Y2 =$ _____
- c) What values are you using for your WINDOW: $xmin=$ ____, $xmax=$ ____,
 $ymin=$ ____, $ymax=$ ____
- d) What is the value of x that you found when $g(x) = 50$? _____ (using 5:
INTERSECT)
- e) To what ordered pair does this information correspond? _____
- f) Include a rough but accurate sketch of the graphs and intersection on the grid below. Mark and label the intersection.



Writing Exponential Equations/Functions

Given a set of data that can be modeled using an exponential equation, use the steps below to determine the particulars of the equation:

1. Identify the initial value. This is the a part of the exponential equation $y = ab^x$. To find a , look for the starting value of the data set (the output that goes with input 0).
2. Identify the common ratio, b , value. To do this, make a fraction of two consecutive outputs (as long as the inputs are separated by exactly 1). We write this as the fraction $\frac{y_2}{y_1}$ to indicate that we put the second y on top and the first on the bottom. Simplify this fraction and round as the problem indicates to obtain the value of b .
3. Plug in the values of a and b into $y = ab^x$ to write the exponential equation.
4. Replace y with appropriate notation as needed to identify a requested exponential FUNCTION.

Problem 14	MEDIA EXAMPLE – WRITING EXPONENTIAL EQUATIONS/FUNCTIONS
-------------------	--

The growth of the population of a small city is shown in the following table:

Year	Population
2000	12,545
2001	15269
2002	18584

Assume this growth is exponential. Let $t = 0$ represent the year 2000 and let a equal the initial population in 2000. Let b equal the ratio in population between the years 2001 and 2000.

- a) Write the equation of the exponential mathematical model for this situation. Round any decimals to two places. Be sure your final result uses proper function notation.
- b) Using this model, forecast the population in 2008 (to the nearest person).
- c) Also using this model, determine the nearest whole year in which the population will reach 50,000.

Name: _____

Date: _____

Lesson 3a Practice Problems

1. In this problem, we look at the characteristics of Linear and Exponential Functions. Complete the table below.

Function	If Linear, is it Increasing or Decreasing or Constant If Exponential, is it Exponential Growth or Exponential Decay	If Linear, state the slope If Exponential, state the base	Identify the y-intercept as an ordered pair.
$y = 2x + 4$	Linear Increasing	Slope = 2	y-intercept = (0,4)
$f(x) = 3(2)^x$			
$g(x) = -1.5x - 2$			
$p(t) = 100(1.2)^t$			
$f(c) = 1.8c + 32$			
$g(x) = 1000(0.75)^x$			

2. For the following three linear functions, identify the vertical intercept, calculate the slope and then write the equation for the function in slope-intercept form.

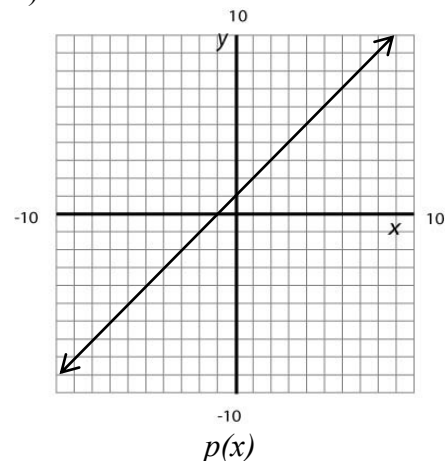
a)

x	f(x)
-1	3
0	2
1	1

b)

$$g = \{(-2,2), (0,3), (2,4)\}$$

c)



3. For the following three exponential functions, identify the initial value (a), calculate the base (b) and then write the equation for the function.

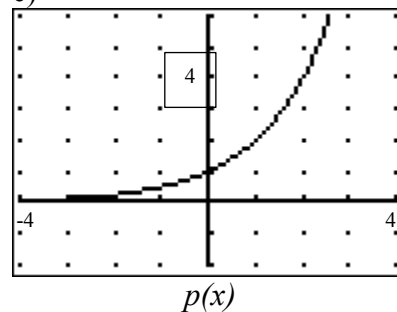
a)

x	$f(x)$
0	4
1	8
2	16

b)

$$g = \{(0, 2), (1, 4.2), (2, 5.88)\}$$

c)



4. Determine if each data set is linear or exponential, and write the formula for each.

a)

x	-2	-1	0	1	2	3	4
$f(x)$.04	.2	1	5	25	125	625

b)

x	-2	-1	0	1	2	3	4
$f(x)$	-1.375	-5	.375	1.25	2.125	3	3.875

c)

x	-2	-1	0	1	2	3	4
$f(x)$	-3	-5.5	-8	-10.5	-13	-15.5	-18

d)

x	-2	-1	0	1	2	3	4
$f(x)$	98.224	99.108	100	100.9	101.81	102.72	103.65

5. In this problem, we look at the characteristics of exponential functions in more depth. For the following three equations, identify each of the characteristics.

	$f(x) = 3.4(1.13)^x$	$g(x) = 42(0.62)^x$	$h(x) = 1000(1.03)^x$
Initial Value (a)?			
Base (b)?			
Domain?			
Range?			
X-intercept?			
Y-intercept?			
Horizontal Asymptote?			
Increasing or Decreasing?			

6. Given $f(x) = 50(1.25)^x$ solve each of the following

a) $f(5) =$

b) $f(50) =$

c) find x when $f(x) = 75$

d) find x when $f(x) = -25$

7. Given $f(x) = 100(0.90)^x$ solve each of the following

a) $f(3) =$

b) $f(30) =$

c) find x when $f(x) = 25$

d) find x when $f(x) = 50$

8. The rabbit population in several counties is shown in the following table.

Assume this growth is exponential. Let $t = 0$ represent the year 2006 and let “a” represent the initial population in 2006. Let “b” represent the ratio in population between the years 2006 and 2007.

	Rabbit Population		
Year	Coconino	Yavapai	Harestew
2006	15000	8000	25000
2007	18000	12800	18750
2008	21600	20480	14063
2009	25920	32768	10547

a) Write the equation of the exponential mathematical model for each situation. Round any decimals to two places. Be sure your final result uses proper function notation. Use $C(t)$ for Coconino, $Y(t)$ for Yavapai and $H(t)$ for Harestew.

b) Using these models, forecast the rabbit population in 2012 (to the nearest rabbit) for each county.

c) Also using these models, predict which will happen first,

- The Rabbit Population in Coconino County reaches 60,000
- The Rabbit Population in Yavapai Country reaches 340,000
- The Rabbit Population in Harestew goes below 5000.

Explain your reasoning.

9. Assume you can invest \$1000 at 5% Simple Interest or 4% Compound Interest (Annual). The equation for Simple Interest is modeled by: $A = P + Prt$. Compound Interest is modeled by $A = P(1+r)^t$. Given that, your two equations are:

$$S(t) = 1000 + 50t$$

$$C(t) = 1000(1.04)^t$$

- a) How much does each investment return after 1 year?
- b) How much does each investment return after 10 years?
- c) How much does each investment return after 20 years?
- d) When would the two investments return the same amount? How much would they return? (Use your calculator and take advantage of the Calc Intersect function)
- e) Which investment would you go with? Why?

10. In 2010, the estimated population of Maricopa County was 3,817,117. By 2011, the population had grown to 3,880,244.
- Assuming that the growth is linear, construct a linear equation that expresses the population, P , of Maricopa County x years since 2010.
 - Assuming that the growth was *exponential*, construct an exponential equation that expresses the population, P , of Maricopa County x years since 2010.
 - Use the equation found in part a to predict the population of Maricopa county in 2015.
 - Use the equation found in part b to predict the population of Maricopa County in 2015.

Name: _____

Date: _____

Lesson 3a Assessment

1. Complete the following table. Use proper notation.

	$f(x) = 24(1.32)^x$	$f(x) = 3324(0.92)^x$	$f(x) = (1.04)^x$
Growth or Decay?			
Vertical Intercept			
Horizontal Intercept			
Domain			
Range			
Horizontal Asymptote (equation)			

2. Determine if each data set is linear or exponential, and write the formula for each.

x	$p(x)$
0	52
1	41
2	30
3	19

x	$g(x)$
0	128
1	64
2	32
3	16

x	$h(x)$
0	1000
1	1100
2	1210
3	1331

$p(x) =$ _____

$g(x) =$ _____

$h(x) =$ _____

3. One 12-oz can of Dr. Pepper contains about 39.4 mg of caffeine. The function $A(x) = 39.4(0.8341)^x$ gives amount of caffeine remaining in the body x hours after drinking a can of Dr. Pepper. Answer in complete sentences.
- a) Determine the half-life of caffeine in the body (i.e. How long will it take the body to metabolize half of the caffeine from one can Dr. Pepper?). Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.
- b) How much caffeine is in the body eight hours after drinking the Dr. Pepper? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.
- c) How much caffeine is in the body one day after drinking the Dr. Pepper? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.
- d) According to this model, how long will it take for *all* of the caffeine to leave the body?

Lesson 3b – More Exponential Functions

Now that we have studied the basics of Exponential Functions, it is time to look at several specific concepts. In this lesson, we study Exponential Growth and Exponential Decay and look at ways to model and measure each. We also learn how to use our calculator to create an Exponential Model by using the Linear Regression tool.

Lesson Objectives:

1. Write EXPONENTIAL GROWTH MODELS
2. Determine DOUBLING TIME
3. Write EXPONENTIAL DECAY MODELS
4. Determine HALVING TIME
5. Use EXPONENTIAL REGRESSION to write exponential growth or decay models

Lesson 3b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 3b

Writing Exponential Models and Doubling Time

In the previous lesson, we worked primarily with data and writing exponential equations from data sets. We will do that again later in this lesson using EXPONENTIAL REGRESSION. First, however, let's see how to write exponential models given just some information about the situation and no actual raw data. We will start with models of EXPONENTIAL GROWTH.

Problem 1	MEDIA EXAMPLE—WRITING EXPONENTIAL GROWTH MODELS/DOUBLING TIME
------------------	--

In 2001, the population of a particular city was 22,395 with an identified growth rate of 6.2% per year. Assume that this growth rate is fairly consistent from year to year.

- Write the EXPONENTIAL GROWTH MODEL for this situation.

- What is the approximate population of the city in 2006? Be sure and round to the nearest person.

- Estimate the number of years (to the nearest whole year) that it will take for the population to double. In what actual year will this take place?



Problem 2	WORKED EXAMPLE-WRITING EXPONENTIAL GROWTH MODELS/DOUBLING TIME
------------------	---

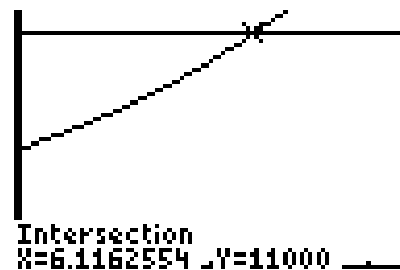
A city has a current population of 5500 people with a growth rate of 12% per year. Write the exponential model for this population and determine the time (to the nearest year) for the population to double.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Initial population = 5500
- Given: Growth rate of 12% per year
- Formula to use: $P(t) = ab^t$
- $a = 5500$ (initial population)
- To find b , convert 12% to a decimal (.12), Then, since the population grows, $b = 1 + .12 = 1.12$ (This value is also called the GROWTH FACTOR).
- Write the model: $P(t) = 5500(1.12)^t$

Second, determine the time for the population to double (DOUBLING TIME)

- Given: $P(t) = 5500(1.12)^t$, initial population = 5500
- Goal: Determine the time for the population to double. Another way to say this is, “find the value of t when the population is twice the initial population” (i.e. find t when $P(t) = 2(5500) = 11000$).
- Mathematically, we want to solve the equation:
 $5500(1.12)^t = 11000$
- Use calculator entering $Y1 = 5500(1.12)^t$ and $Y2 = 11000$.
Use window $X[0..10]$, $Y[0..12000]$ then
 $2^{nd}>Calc>5:Intersect$ to get $t = 6.12$. (See graph below).
Round to get $t = 6$.



Result: The population will double in about 6 years.

Steps to Write an Exponential Growth Model Given the Rate of Growth

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time $t = 0$ and the number will be your number for “ a ”.
- Write the given rate as a decimal and ADD it to 1. This is your value for “ b ” (GROWTH FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Doubling Time

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 2a$
- Solve by graphing and INTERSECTION method

Problem 3	YOU TRY- WRITING EXPONENTIAL GROWTH MODELS/DOUBLING TIME
------------------	---

After graduating from college in 2010, Sara accepts a job that pays \$52,000 per year. At the end of each year, she expects to receive a 3% raise.

- a) Let t represent the number of years Sara works at her new job. Write the exponential growth function, $S(t)$, that models her annual salary given the information above.

Initial Salary (a value): _____

Given growth rate as a decimal: _____

Growth factor (b value): _____

Write the model: $S(t) = \text{InitialValue}(\text{GrowthFactor})^t =$ _____

- b) If Sara’s salary continues to increase at the rate of 3% each year, determine how much will she will make in 2015. Show your work clearly here.

- c) How many years will she have to work before her salary will be double what it was in 2010 (assuming the same growth rate)? Be sure to set up and clearly identify the DOUBLING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

DOUBLING EQUATION: _____



DOUBLING TIME (Rounded to nearest whole year): _____

Problem 4	MEDIA EXAMPLE – WRITING EXPONENTIAL DECAY MODELS/HALVING TIME
------------------	--

The 2000 U.S. Census reported the population of Tulsa, Oklahoma to be 382,872. Since the 2000 Census, Tulsa's population has been decreasing at approximately 2.6% per year.

a) Write an EXPONENTIAL DECAY MODEL, $P(t)$, that predicts the population of Tulsa, OK at any time t .

b) Use the function you wrote for $P(t)$ to predict the population of Tulsa, OK in 2013.

c) In how many years will the population of Tulsa decrease to 300,000 people (round to the nearest whole year)?



d) In how many years will the population of Tulsa decrease to HALF of the initial (2000) population? Round to the nearest whole year.



Problem 5	WORKED EXAMPLE – WRITING EXPONENTIAL DECAY MODELS/HALVING TIME
------------------	---

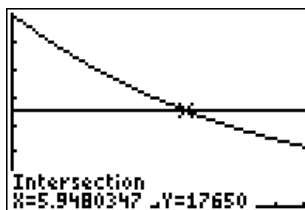
In 2012, Shannon purchased a new Ford Mustang GT convertible for \$35,300. Since then, the value of the car has decreased at a rate of 11% each year.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Purchase price = \$35,300
- Given: Decay rate of 11% per year
- Formula to use: $V(t) = ab^t$
- $a = 35,300$ (initial value)
- To find b , convert 11% to a decimal (0.11), Since the population decays, $b = 1 - .11 = 0.89$ (This value is also called the DECAY FACTOR).
- Write the model: $V(t) = 35300(0.89)^t$

Second, determine the time for the price to halve (HALF-LIFE or HALVING TIME)

- Given: $V(t) = 35300(0.89)^t$, initial price = \$35,300
- Goal: Determine the time for the value of the car to halve. Another way to say this is, “find the value of t when the value is half the initial purchase price” (i.e. find t when $V(t) = 0.5(35,300) = 17,650$).
- Mathematically, we want to solve the equation: $35300(0.89)^t = 17650$
- Use your calculator and enter $Y1 = 35300(0.89)^t$ and $Y2 = 17650$. Use window $X[0..10]$, $Y[0..35300]$ then $2^{nd}>Calc>5:Intersect$ to get $t = 5.95$ (See graph below).



- Result: The value of the car will be worth half the initial purchase price in about 6 years.

Steps to Write an Exponential Decay Model Given the Rate of Decay

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time $t = 0$ and the number will be your number for “ a ”.
- Write the given rate as a decimal and SUBTRACT it from 1. This is “ b ”(DECAY FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Halving Time (also called Half-Life)

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 0.5a$
- Solve by graphing and INTERSECTION method

Problem 6 | **YOU TRY – WRITING EXPONENTIAL DECAY MODELS/HALVING TIME**

In 1970, the population of Buffalo, New York had a population of 462,768 people. Assume the population decreased by 1.4% each year from 1970 to 2000.

- a) Let t represent the number of years since 1970 (i.e. your starting year is 1970 so $t=0$ in this year). Write the exponential decay function, $P(t)$, that models the annual population given the information above.

Initial Population (a value): _____ Given DECAY RATE as a decimal: _____

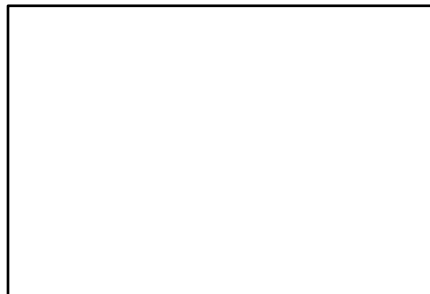
Take 1 – the DECAY RATE decimal to identify your DECAY FACTOR (b value):

Write the model: $P(t) = InitialValue(DecayFactor)^t =$ _____

- b) If the population continues to decrease at the rate above, determine how many people will live in Buffalo in 2015. Show your work clearly here.

- c) How many years will it take for the Buffalo population to decrease to half what it was in 1970 (assuming the same decay rate)? Be sure to set up and clearly identify the HALVING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

HALVING EQUATION: _____



HALVING TIME (Rounded to nearest whole year): _____

Problem 7 | YOU TRY – WRITING EXPONENTIAL GROWTH/DECAY MODELS

Given what we have covered in this lesson so far, you should now be able to quickly write models for exponential growth or decay given a starting value and a rate of growth or decay. Fill in the table below to practice. The first few are done for you.

Given	Initial Value (a value)	Growth or Decay Factor (b value)	Exponential Function $y=ab^t$
Initial population = 12,000 Growth rate = 40%	12,000	40% = 0.40 $b = 1 + .40 = 1.40$	$P(t) = 12,000(1.40)^t$
Initial value = 45 Decay rate = 35%	45	35% = 0.35 $b = 1 - 0.35 = 0.65$	$V(t) = 45(0.65)^t$
Initial population = 145 Growth rate 3%			
Initial value = 314 Decay rate 96.5%			
Initial population = 412 Growth rate 13.24%			
Initial investment = 546 Decay rate 2%			
Initial amount = 3 Decay rate 22%			
Initial population = 3061 Growth rate 205%			

Problem 8 | WORKED EXAMPLE–WRITING EXPONENTIAL MDLS USING EXP REGRESSION

As with LINEAR FUNCTIONS, we can work with a data table and, if appropriate, model that data using EXPONENTIAL REGRESSION. The steps are almost the same as those followed for LINEAR REGRESSION.

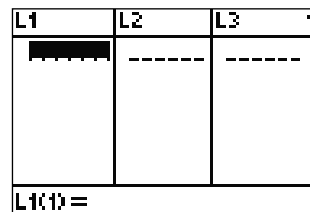
The table below shows the population decay in a given state after t years.

t (years)	Population
5	5,234,456
10	4,892,345
15	4,012,345

Use the Exponential Regression feature of your calculator to generate a mathematical model for this situation. Round “a” to the nearest whole number and “b” to 3 decimals.

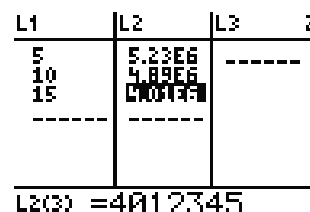
- Press STAT>EDIT>ENTER to show data entry area. The STAT button is on the second row, third column.

Data entry area should be blank to begin. To clear, go column by column. Scroll to column header using the arrow keys then press Clear>Enter. Use the arrow keys to move back and forth.



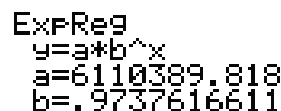
[Note: If you ever accidentally DELETE a column, then go to STAT>5: SetUpEditor>ENTER. When you go back to STAT, your column should be restored.]

- Enter input data into L₁: 5 ENTER 10 ENTER 15 ENTER
- Arrow over to the first entry spot in L₂
- Enter output data into L₂: 5234456 ENTER 4892345 ENTER 4012345 ENTER



[Note: The numbers for L₂ are displayed using scientific notation (the E notation) since they are too long for the column width. Scroll to each number and see its full representation in the bottom of the window. See example highlighted at right.

- Press STAT>CALC>0:ExpReg>ENTER>ENTER to get the screen at right.



Thus, your exponential function (with values rounded as the problem indicates) is $y = 6110390(0.974)^x$. Convert this to the problem’s notation to get $P(t) = 6110390(0.974)^t$.

Problem 9	YOU TRY – WRITING EXPONENTIAL MODELS USING EXP REGRESSION
------------------	--

Determine the exponential regression equation that models the data below:

X	0	1	2	4	6	9	12	16
y	125	75	50	32	22	16	10	5.7

When you write your final equation, round “a” to 1 decimal place and “b” to three decimal places.

- a) Write final exponential regression equation here: _____
 [Note: You may think it strange that your equation does NOT use 125 as the “a” value. That is because of the regression process used by your calculator.]
- b) What is the rate of decay (as a %) for this equation? _____
- c) Using your equation from part a, determine y when $x = 20$ and show your work here. Write the corresponding ordered pair result. [Hint: This is a “Given Input, Find Output” problem]
- d) Using your equation from part a, determine x when $y = 28$ and show your work here. Write the corresponding ordered pair result. [Hint: This is a “Given Output, Find Input” problem]
- e) Using your equation from part a), set up and solve the halving equation. What solution method did you use?

HALVING EQUATION: _____ HALVING TIME: _____

Name: _____

Date: _____

Lesson 3b Practice Problems

1. Convert each of the following percentages to decimals.

a) $12\% =$

b) $112\% =$

c) $7\% =$

d) $7.5\% =$

e) $.5\% =$

f) $200\% =$

2. Complete the following two tables.

Growth Rate as a %	Growth Rate as a decimal	Growth Factor
13%	0.13	1.13
21%		
7%		
	0.20	
	0.05	
		1.25
		1.075

Decay Rate as a %	Decay Rate as a decimal	Decay Factor
12%	0.12	0.88
23%		
3%		
	0.18	
	0.02	
		0.75
		0.98

3. Write the exponential function for each of the following.

a) Initial Value = 1500
Growth Rate = 15%

b) Initial Value = 75
Decay Rate = 15%

c) Initial Value = 1250
Growth Rate = 7.5%

d) Initial Value = 12
Growth Rate = 112%

e) Initial Value = 1000
Decay Rate = 12%

f) Initial Value = 56
Decay Rate = 5%

g) Initial Value = 100
Decay Rate = 0.5%

h) Initial Value = 57
Decay Rate = 6.2%

4. For each exponential function, identify the Initial Value and the Growth/Decay Rate. First one is done for you.

a) $f(x) = 1000(0.98)^x$

Initial Value = 1000
Decay Rate = 2%

b) $g(x) = 3200(1.32)^x$

c) $p(t) = 50(0.75)^t$

d) $f(x) = 120(1.23)^x$

e) $A(r) = 1000(1.075)^r$

f) $g(x) = 1200(0.35)^x$

5. Determine the Doubling Time or Half Life for each of the following. Write the appropriate equation and then use your calculator to solve. Round your answer to two decimal places.

a) $y = 200(1.2)^t$

b) $f(x) = 200(0.8)^x$

c) $y = 1500(1.5)^t$

d) $p(t) = 3000(1.45)^t$

e) $f(x) = 3000(0.99)^x$

f) $p(t) = 25000(0.80)^t$

6. The table below shows the value of an investment over time.

Time (years)	0	3	8	12	15	22
Value (\$)	4582	5007	5804	6533	7139	8779

- a) Use your calculator to determine the exponential regression equation that models the set of data above. Clearly indicate what each variable represents.
- b) Based on the equation found in part a, at what percent rate is the value of this investment increasing each year?
- c) Use the TABLE to determine the value of the investment after 3 years. (NOTE: We generally refer to this type of problem as “reading the data” or “*extracting* the data”)
- d) Use the equation from part a to determine the value of the investment after 10 years. (NOTE: We generally refer to this type of problem as “reading between the data” or “*interpolating* the data”)
- e) Use the equation from part a to determine the value of the investment after 30 years. (NOTE: We generally refer to this type of problem as “reading beyond the data” or “*extrapolating* the data”)

8. On April 1, 2000 (Census day), the population of Scottsdale, AZ was 202,625. On April 1, 2010, the population of Scottsdale, AZ was 217,385.
- a) Use this information and your graphing calculator to determine an exponential regression equation that expresses the population, P , of Scottsdale, AZ x years since 2000.
- b) Use the equation found in part a to predict the population of Scottsdale, AZ on April 1, 2013. Show all work, and write your answer in a complete sentence. Round your answer to the nearest whole number.
- c) Use the equation found in part a to predict the year in which the population of Scottsdale AZ will reach 500,000. Show all work, and write your answer in a complete sentence.
- d) Based on the equation found in part a, at what percent rate is Scottsdale's population increasing each year?

9. One 7-oz cup of coffee contains about 130 mg of caffeine. The half-life of caffeine in the body is about 6 hours.
- Use this information and your graphing calculator to determine the exponential regression equation that models the amount, A , of caffeine remaining in your body x hours after drinking one cup of coffee.
 - According to the equation found in part a, how much caffeine remains in the body one hour after drinking one cup of coffee? Show all of your work and write your answer in a complete sentence. Round your answer to one decimal place as needed.
 - According to the equation found in part a, how much caffeine remains in the body eight hours after drinking one cup of coffee? Show all of your work and write your answer in a complete sentence. Round your answer to one decimal place as needed.
 - According to the equation found in part a, at what rate does your body metabolize (eliminate) the caffeine each hour?

10. For each of the following,
- Determine if it is Exponential Growth or Exponential Decay.
 - Write the equation that appropriately models the scenario.
 - If it is growth, determine the doubling time.
 - If it is decay, determine the half life.
 - These are word problems, so make sure your final answer is in a complete sentence.
- a) BoringBob buys a piece of gold that has a value of \$1000. His investment grows at a rate of 5% per year. How long until his investment doubles?
- b) Twidles has 50 rabbits. He usually keeps them separated, but one day he leaves them all free in the back yard (with lots of food and water) and heads off for a 7 day vacation. The rabbits do what rabbits do and the population starts growing at a rate of 10% a day. Will Twidles have 100 rabbits when he gets back?
- c) Jeffta went to a doctor and found out that he had 80 pounds of body fat and needed to lose 40 of that. He goes on a healthy diet and finds that he is losing 2% of his body fat a week. How long will it take Jeffta to lose the 40 pounds?
- d) In 1975, the cost of a Bean Burro from Taco Bell costs 25 cents. Inflation has averaged about 3% a year since then. In what year did it reach 50 cents. In what year should it cost \$1.00? Let t = the number of years since 1975 (i.e. $t = 0$ for 1975, $t = 1$ for 1976 and so on).

11. The table below shows how a Pizza freshly removed from the oven cools over time.

Time (minutes)	0	5	9	12	15	18	22	25	30
Temp (° F)	199.5	184.7	178.1	159.2	141.7	124.6	115.4	104.5	99.3

- a) Use your calculator to determine the exponential regression equation that models the set of data above. Clearly indicate what each variable represents.
- b) Based on the equation found in part a, at what percent rate is the pizza cooling?
- c) Write and answer a “Read the Data” question (Extracting the data).
- d) Write and answer a “Read Between the Data” question (Interpolating the data).
- e) Write and answer a “Read Beyond the Data” question (Extrapolating the data).

Name: _____

Date: _____

Lesson 3b Assessment

1. Consider the functions shown below.

A. $f(x) = (1.023)^x$

B. $w(x) = 320(0.95)^x$

C. $y = 400(1.12)^x$

D. $g(x) = 34.9(1.11)^x$

E. $p(r) = 172(0.99)^r$

F. $k(n) = 8452(0.67)^n$

a) Which functions are increasing? _____

b) Which function is increasing at the fastest rate? _____

What is the growth rate for this function? _____

c) Which function is decreasing at the fastest rate? _____

What is the decay rate for this function? _____

2. Fred and Wilma purchase a home for \$180,000. **Using function notation**, write a formula for the value, V , of the house t years after its purchase, assuming that the value

a) Decreases by \$1,500 per year.

b) Decreases by 2% per year.

c) Increases by \$3,100 per year.

d) Increases by 6% per year.

3. The following data set gives the value of a car over time.

Years since purchase	Value in Dollars
0	21,425
1	18,097
2	15,137
3	12,749
5	8,960
8	5,311

- a) Determine an exponential regression equation for this data set. Clearly indicate what each variable represents.
- b) Use the regression equation from part a to predict the value of the car after 12 years.
- c) Based on the regression equation, at what percent rate is the car's value decreasing each year?

Lesson 4a – Introduction to Logarithms

Logarithms are really exponents in disguise and hopefully you will have that understanding after you complete this lesson. You will start by working with the LOG button on your calculator and then building an understanding of logarithms as exponents. You will learn how to read and interpret logarithms and how to compute with base 10 and other bases as well.

Prior to solving logarithmic equations, you will learn about changing back and forth from logarithmic to exponential forms. The section titled, “Why do we care about Logarithms?” will give you some practical foundation in their use as a scaling tool.

Finally, you will use what you learned about changing forms to solve logarithmic equations. Pay close attention to the idea of exact form vs. approximate form for solutions.

Lesson Objectives:

1. Discuss the concept of LOGARITHMS as exponents
2. Read and interpret LOGARITHMS
3. Compute LOGARITHMS with base 10 (Common Logarithms)
4. Compute LOGARITHMS with bases other than 10
5. Change an equation from LOGARITHMIC FORM to EXPONENTIAL FORM and vice versa
6. Discuss LOGARITHMS as a scaling tool
7. Solve LOGARITHMIC EQUATIONS by changing to EXPONENTIAL FORM
8. Determine EXACT FORM and APPROXIMATE FORM solutions for LOGARITHMIC EQUATIONS

Lesson 4a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 4a

What are Logarithms?

Logarithms are really EXPONENTS in disguise. The following two examples will help explain this idea.

Problem 1 YOU TRY – COMPUTE BASE 10 LOGARITHMS USING YOUR CALCULATOR

Locate the **LOG** button on your calculator. Use it to fill in the missing values in the input/output table. The first and last are done for you. When you use your calculator, remember to close parentheses after your input value.

x	$y = \log(x)$
1	0
10	
100	
1000	
10000	
100000	5

What do the outputs from Problem 1 really represent? Where are the EXPONENTS that were mentioned previously? Let’s continue with the example and see where we end up.

Problem 2 MEDIA EXAMPLE – LOGARITHMS AS EXPONENTS

x	$\log(x)$		$\log_{10}(x) = y$	$10^y = x$
1	0			
10	1			
100	2			
1000	3			
10000	4			
100000	5			

Reading and Interpreting Logarithms

$$\log_b x = y$$

Read this as “Log, to the BASE b, of x, equals y”

This statement is true if and only if

$$b^y = x$$

Meaning:

The logarithm (output of $\log_b x$) is the EXPONENT on the base, b, that will give you input x.

Note: The Problem 2 logarithm is called a COMMON LOGARITHM because the base is understood to be 10. When there is no base value written, you can assume the base = 10.

Let's see how this works with other examples. The base can be almost any number but has some limitations. The base, b , should be bigger than 0 and not equal to 1.

Problem 3	MEDIA EXAMPLE – COMPUTE LOGARITHMS WITH BASES OTHER THAN 10
------------------	--

Compute each of the following logarithms and verify your result with an exponential “because” statement.

a) $\log_2 2^4 =$	because	
b) $\log_2 4 =$	because	
c) $\log_3 27 =$	because	
d) $\log_8 1 =$	because	
e) $\log_5 \sqrt{5} =$	because	
f) $\log_4 4 =$	because	

Reminder: THE NTH ROOT OF a

$\sqrt[n]{a} = a^{1/n}$, the n th root of a . The number a , called the radicand, must be nonnegative if n , called the index, is even. So, for part e) of the problem above this means that $\sqrt{3} = 3^{1/2}$.

Problem 4	WORKED EXAMPLE - COMPUTE LOGARITHMS WITH BASES OTHER THAN 10
------------------	---

Compute each of the following logarithms and verify your result with an exponential “because” statement.

a) $\log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2}$ so $\log_3 \frac{1}{9} = -2$	because	$3^{-2} = \frac{1}{9}$
b) $\log_6 \frac{1}{36} = \log_6 \frac{1}{6^2} = \log_6 6^{-2}$ so $\log_6 \frac{1}{36} = -2$	because	$6^{-2} = \frac{1}{36}$
c) $\log_5 1 = 0$	because	$5^0 = 1$
d) $\log_7 0 = dne$ (does not exist)	because	There is no power of 7 that will give a result of 0.

Problem 5	YOU TRY - COMPUTE LOGARITHMS WITH BASES OTHER THAN 10
------------------	--

Compute each of the following logarithms and verify your result with an exponential “because” statement.

a) $\log_2 64 =$	because	
b) $\log_3 1 =$	because	
c) $\log \frac{1}{1000} =$	because	
d) $\log 0 =$	because	
e) $\log_8 \sqrt{8} =$	because	

Logarithmic and Exponential Form

In order to work effectively with LOGARITHMS, and soon with LOGARITHMIC EQUATIONS, you will need to get very comfortable changing from logarithmic form to exponential form and vice versa.

LOGARITHMIC FORM: $\text{Log}_b x = y$ EXPONENTIAL FORM: $b^y = x$

THESE FORMS ARE EQUIVALENT.

Rewrite the above for practice and brain muscle memory!

$\text{Log}_{\square} \square = \square$ is equivalent to $\square^{\square} = \square$

Note: When you write expressions involving logarithms, be sure the base is a SUBSCRIPT and written just under the writing line for Log. Pay close attention to how things are written and what the spacing and exact locations are.

Problem 6 | YOU TRY – EXPONENTIAL AND LOGARITHMIC FORMS

Complete the table filling in the missing forms for a) and c) using the relationship between Exponential and Logarithmic forms. Refer to parts b) and d) as examples.

	Exponential Form	Logarithmic Form
a)	$3^4 = 81$	
b)	$6^{-2} = \frac{1}{36}$	$\log_6 \frac{1}{36} = -2$
c)		$\log_7 16807 = 5$
d)	$(0.074)^{3t} = 102$	$\log_{0.074} 102 = 3t$

Why do we care about Logarithms?

Logarithms are used in the sciences particularly in biology, astronomy and physics. The Richter scale measurement for earthquakes is based upon logarithms, and logarithms formed the foundation of our early computation tool (pre-calculators) called a Slide Rule.

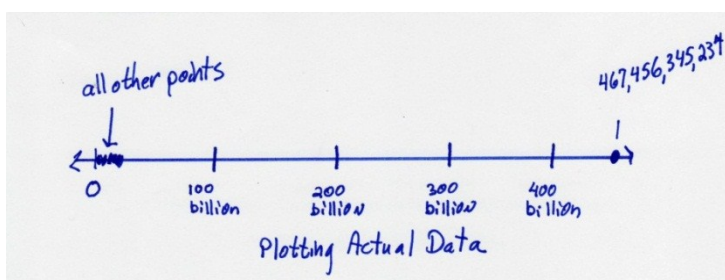
One of the unique properties of Logarithms is their ability to scale numbers of great or small size so that these numbers can be understood and compared. Let’s see how this works with an example.

Problem 7 | WORKED EXAMPLE – USING LOGARITHMS AS A SCALING TOOL

Suppose you are given the following list of numbers and you want to plot them all on the same number line:

Plot 0.00000456, 0.00372, 1.673, 1356, 123,045 and 467,456,345,234.

If we scale to the larger numbers, then the smaller numbers blend together and we can’t differentiate them.



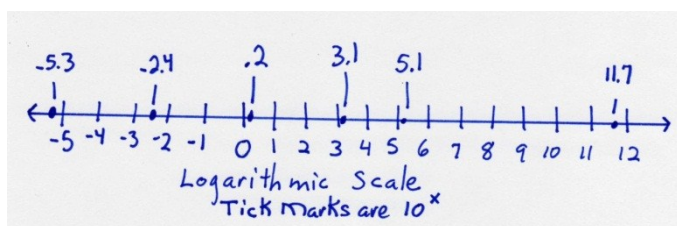
Let’s use logarithms and create a logarithmic scale and see how that works. First, make a table that translates your list of numbers into logarithmic form by taking the “log base 10” or common logarithm of each value.

Original #	0.00000456	0.00372	1.673	1356	123,045	467,456,345,234
Log (#)	-5.3	-2.4	.2	3.1	5.1	11.7

Then, redraw your number line and plot the logarithmic value for each number.

Notice that labeling your scale as a logarithmic scale is VERY important. Otherwise, you may not remember to translate back to the actual data and you may forget that your tick marks are not unit distances.

The new scale gives you an idea of the relative distance between your numbers and allows you to plot all your numbers at the same time. To understand the distance between each tick mark, remember that the tick mark label is the exponent on 10 (base of the logarithm used). So from 1 to 2 is a distance of $10^2 - 10^1 = 100 - 10 = 90$. The distance between 2 and 3 is $10^3 - 10^2$ or $1000 - 100 = 900$, etc...



You will learn a LOT more about logarithmic scaling if you take science classes, as this is just a very brief introduction to the idea.

Solving Logarithmic Equations by Changing to Exponential Form

We will use what we now know about Logarithmic and Exponential forms to help us solve Logarithmic Equations. There is a step-by-step process to solve these types of equations. Try to learn this process and apply it to these types of problems.

Solving Logarithmic Equations – By Changing to Exponential Form

Solving logarithmic equations involves these steps:

1. **ISOLATE** the logarithmic part of the equation
2. Change the equation to **EXPONENTIAL** form
3. **ISOLATE** the variable
4. **CHECK** your result if possible
5. **IDENTIFY** the final result in **EXACT** form then in rounded form as indicated by the problem

Notes:

- To **ISOLATE** means to manipulate the equation using addition, subtraction, multiplication, and division so that the Log part and its input expression are by themselves.
- **EXACT FORM** for an answer means an answer that is not rounded until the last step

Problem 8 | MEDIA EXAMPLE – SOLVING LOGARITHMIC EQUATIONS

Solve $\log_3 x = 2$ for x	Original Problem Statement
	Step 1: ISOLATE the logarithmic part of the equation
	Step 2: Change the equation to EXPONENTIAL form
	Step 3: ISOLATE the variable
	Step 4: CHECK your result if possible
	Step 5: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem

Problem 9 | WORKED EXAMPLE - SOLVING LOGARITHMIC EQUATIONS

Solve $\log_3(x - 1) = 4$ for x	Original Problem Statement
The logarithmic part is already isolated. Move to Step 2.	Step 1: ISOLATE the logarithmic part of the equation
$3^4 = x - 1$ $81 = x - 1$ $81 + 1 = x - 1 + 1$ $82 = x$ $x = \mathbf{82}$	Step 2: Change the equation to EXPONENTIAL form and Step 3: ISOLATE the variable
$\log_3(82-1) = \log_3(81) = 4$ because $3^4 = 81$ (CHECKS)	Step 4: CHECK your result if possible
$x = \mathbf{82}$ (this is exact)	Step 5: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem

Problem 10 | MEDIA EXAMPLE - SOLVING LOGARITHMIC EQUATIONS

Solve $4 + 6\log_2(3x + 2) = 5$ for x	Original Problem Statement
	Step 1: ISOLATE the logarithmic part of the equation
	Step 2: Change the equation to EXPONENTIAL form
	Step 3: ISOLATE the variable
	Step 4: CHECK your result if possible
	Step 5: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem

Problem 11 | YOU TRY - SOLVING LOGARITHMIC EQUATIONS

Solve $\log_2(x - 1) = 5$ for x	Original Problem Statement
	Step 1: ISOLATE the logarithmic part of the equation
	Step 2: Change the equation to EXPONENTIAL form
	Step 3: ISOLATE the variable
	Step 4: CHECK your result if possible
	Step 5: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem

Problem 12 | YOU TRY - SOLVING LOGARITHMIC EQUATIONS

Solve $5 + 4\log_3(7x + 1) = 8$ for x	Original Problem Statement
	Step 1: ISOLATE the logarithmic part of the equation
	Step 2: Change the equation to EXPONENTIAL form
	Step 3: ISOLATE the variable
	Step 4: CHECK your result if possible
	Step 5: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem

What's all the fuss? Exact form? Approximate Form? Why does it matter?

If you wanted to approximate the fraction $\frac{1}{3}$, what would you say? Probably that $\frac{1}{3}$ is about .3, right?

But what does $\frac{1}{3}$ ACTUALLY equal? Well, it equals .3333333333 repeating forever. Any number of decimals that we round to in order to represent $\frac{1}{3}$ is an APPROXIMATION.

The only EXACT representation of $\frac{1}{3}$ is $\frac{1}{3}$.

So what difference does this make? Suppose you wanted to compute $4^{\frac{1}{3}}$. Look at the following computations to as many decimals as we can.

$$4^{\frac{1}{3}} = 4^{(1/3)} \text{ on your calculator} = 1.587401052$$
$$4^3 = 4^{.3} \text{ on your calculator} = 1.515716567$$

The final computation results are not the same but they are pretty close depending on where we would round the final result. Which one is more accurate? The $4^{\frac{1}{3}}$ is more accurate because we used EXACT form for $\frac{1}{3}$.

What happens when the base of the exponential is much larger? Suppose you want to compute $1025^{\frac{1}{3}}$.

$$1025^{\frac{1}{3}} = 1025^{(1/3)} = 10.08264838$$
$$1025^3 = 1025^{(.3)} = 8.002342949$$

These two results are quite a bit different and this type of behavior only gets worse as the numbers you are working with get larger. So, remember, if you are asked to compute a result to EXACT form, do not round any computation in your solution process until the very end.

Name: _____

Date: _____

Lesson 4a Practice Problems

1. Locate the LOG button on your calculator. Use it to fill in the missing values in the input/output table. When you use your calculator, remember to close parentheses after your input value.

	x	Function	y	(x,y)
a)	5	$\log(x)$		
b)	3	$4\log(x)$		
c)	2	$\log(x)^4$		
d)	6	$\log(x)/\log(2)$		

2. Compute each of the following logarithms and verify your result with an exponential “because” statement.

a) $\log_4 16 =$	because	
b) $\log_3 \frac{1}{9} =$	because	
c) $\log_4 2 =$	because	
d) $\log_6 1 =$	because	
e) $\log_2 \sqrt{2} =$	because	
f) $\log_8 8 =$	because	

3. Complete the table filling in the missing forms

	Exponential Form	Logarithmic Form
a)	$3^2 = 9$	
b)	$2^{-3} = \frac{1}{8}$	
c)		$\text{Log}_4 1024 = 5$
d)		$\text{Log}_3 1200 = 2x$

4. Solve each Logarithmic Function for x . Check your answer. IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem.

a) $\log_5 x = 4$

b) $\log_6 x = -3$

c) $2\log_7 x = 8$

d) $4 + 2\log_7 x = 8$

e) $8 - 2\log_7 x = 10$

f) $\log_3(x + 4) = 2$

g) $8\log_3(x - 2) = 48$

h) $2 - 3\log_4(x + 10) = -13$

5. The Richter scale was developed by Charles Richter of the California Institute of Technology in 1935. A single number, called the magnitude, is assigned to quantify the amount of seismic energy released by an earthquake. The magnitude, M , of an earthquake on the Richter scale can be approximated by the formula $M = \log(I)$ where I is the intensity of the earthquake, measured by a seismograph located 8 km from the epicenter (using the Lillie Empirical Formula).

Example 1: If an earthquake has an intensity of 100 then the magnitude would be $M = \log(100)$. Entering it into the calculator, we find that the magnitude of the earthquake is 2.

Example 2: If an earthquake has a magnitude of 4.5 ($M=4.5$), the Intensity would be calculated by solving $4.5 = \log(I)$. We can rewrite this as an exponent. The new formula would be $10^{4.5} = I$ or $I = 31,622.8$.

- a) The intensity of an earthquake with magnitude 6 on the Richter scale is _____ times greater than an earthquake with magnitude 5.
- b) The intensity of an earthquake with magnitude 7 on the Richter scale is _____ times greater than an earthquake with magnitude 5.
- c) On March 27, 1964, Anchorage, Alaska was hit by an earthquake with Magnitude 8.5. Determine the intensity of this earthquake.
- d) Earthquakes that measure less than 3 on the Richter scale are known as microquakes, and are not felt by humans. Determine the intensity of an earthquake with a magnitude of 3.
- e) A major earthquake is the one which registers more than 7 on the Richter scale. Determine the intensity of an earthquake with a magnitude of 7.

6. In Chemistry, the pH of a substance can be determined using the formula $pH = -\log[H^+]$ where H^+ is the hydrogen ion concentration of the substance. Solutions with a pH less than 7 are said to be acidic and solutions with a pH greater than 7 are basic or alkaline. The table below presents the pH values of some common substances.

pH Level	Substance
13	Bleach
12	Soapy Water
11	Ammonia Solution
10	Milk of Magnesia
9	Baking Soda
8	Sea Water
7	Distilled Water
6	Urine
5	Black Coffee
4	Tomato Juice
3	Orange Juice
2	Lemon Juice
1	Gastric Acid

- a) Determine the hydrogen ion (H^+) concentration of distilled water.
- b) Determine the hydrogen ion (H^+) concentration of black coffee.
- c) If the water in your swimming pool has a hydrogen ion concentration of .000001 what would the pH level be? (Just for fun, should you be concerned?)
- d) The hydrogen ion concentration of lemon juice is _____ times greater than the hydrogen ion concentration of orange juice.
- e) **Challenge:** The hydrogen ion concentration of Gastric acid is _____ times greater than the hydrogen ion concentration of Tomato juice.

Name: _____

Date: _____

Lesson 4a Assessment

1. Complete the table below.

Exponential Form	Logarithmic Form
$9^2 = 81$	
$8^{-3} = \frac{1}{512}$	
$\sqrt{9} = 3$	
	$\log_4 16 = 2$
	$\log 1000 = 3$

3. Evaluate each of the following logarithms.

a) $\log_5 1 =$ _____

b) $\log_7 7 =$ _____

c) $\log_3 \left(\frac{1}{3} \right) =$ _____

d) $\log_8 (64) =$ _____

e) $\log_5 \left(\frac{1}{25} \right) =$ _____

f) $\log \sqrt[3]{10} =$ _____

4. Solve the following equations. Show your work. Simplify your answers. Leave your answers in exact form (no decimals).

a) $\log_3 x = 5$

b) $2 + \log_4(2x - 1) = 5$

c) $8 - 2\log_7 x = 10$

d) $3\log(7x - 1) = 6$

Lesson 4b – More on Logarithms

This lesson will begin with a review of exponential and logarithmic forms for equations, as we will switch back and forth from one form to the other when solving equations.

You will learn the all-important “change of base formula” next. This formula allows you to easily compute logarithms that have bases other than 10 and use your calculator to do so.

The lesson will end with an exploration of the graph of logarithmic functions and their characteristics followed by modeling of data sets with Logarithmic Regression.

This lesson ends our tour of logarithms and exponential functions as we will next be moving on to quadratic functions.

Lesson Objectives

By the end of this lesson, you should be able to:

1. Discuss the concept of LOGARITHMS as EXPONENTS
2. Compute LOGARITHMS of bases other than 10
3. Compute LOGARITHMS using the CHANGE OF BASE formula and a calculator
4. Solve EXPONENTIAL EQUATIONS algebraically and graphically
5. Graph $f(x) = \log_b x$ and identify important characteristics
6. Determine if a given data set can be well modeled by a LOGARITHMIC FUNCTION
7. Use LOGARITHMIC REGRESSION (natural logarithm) to model a given data set

Lesson 4b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 4b

The information below is a review from Lesson 4a.

Logarithmic and Exponential Forms for equations

LOGARITHMIC FORM: $\text{Log}_b x = y$ EXPONENTIAL FORM: $b^y = x$

THESE FORMS ARE EQUIVALENT.

Problem 1 | YOU TRY – COMPUTING LOGARITHMS OF BASES OTHER THAN 10

Rewrite the above for practice and brain muscle memory! Use the information to help you compute the logarithms below.

Log $\begin{matrix} \square \\ \square \end{matrix} = \square$ is equivalent to $\begin{matrix} \square \\ \square \end{matrix} = \square$

a) $\log_2 256 =$	because	
b) $\log_5 625 =$	because	
c) $\log_3 \frac{1}{27} =$	because	
d) $\log_{27} 3 =$	because	

Using Calculators to Compute with Bases other than 10

Now that we know something about working with logarithms, let's see how our calculator can help us with more complicated examples.

Problem 2 | MEDIA EXAMPLE – INTRODUCING CHANGE OF BASE FORMULA

Let's try to compute $\log_2 19$. To start, let's estimate values for this number. Try to find the two consecutive (one right after the other) whole numbers that $\log_2 19$ lives between.

$$\underline{\hspace{2cm}} < \log_2 19 < \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} < \log_2 19 < \underline{\hspace{2cm}}$$

So, now we have a good estimate for $\log_2 19$ let's see how our calculator can help us find a better approximation for the number.

To compute $\log_2 19$ in your calculator, use the following steps: $\text{Log}\>19\>/\text{Log}2\>\text{ENTER}$ and round to three decimals to get:

$$\log_2 19 = \underline{\hspace{2cm}}$$

Do we believe this is a good approximation for $\log_2 19$? How would we check?

$\underline{\hspace{2cm}}$

So, our estimation for $\log_2 19$ is good and we can say $\log_2 19 = \underline{\hspace{2cm}}$ with certainty.

How did we do that again? We said that $\log_2 19 = \frac{\log(19)}{\log(2)}$. How can we do that for any problem?

Change of Base Formula – Converting with Common Logarithms (base 10)

$$\log_b x = \frac{\log(x)}{\log(b)}$$

Problem 3	YOU TRY – COMPUTE LOGARITHMS USING CHANGE OF BASE FORMULA
------------------	--

Use the Change of Base formula given on the previous page, and your calculator, to compute each of the following. The first one is done for you.

Compute	Rewrite using Change of Base	Final Result (3 decimal places) – Be sure to use () with each logarithm separately
a) $\log_3 8$	$\frac{\log(8)}{\log(3)}$	1.893
b) $\log_5 41$		
c) $\log_8 12$		
d) $\log_{1.5} 32$		
e) $12.8 + \log_3 25$		

Solving Exponential Equations Algebraically and Graphically

We will use what we now know about Logarithmic and Exponential forms and Change of Base formula to help us solve Exponential Equations. There is a step-by-step process to solve these types of equations.

Solving Exponential Equations – Algebraically and Graphically

Solving exponential equations involves these steps:

- **ISOLATE** the exponential part of the equation
- Change the equation to **LOGARITHMIC** form
- **ISOLATE** the variable
- **IDENTIFY** the final result in **EXACT** form then in rounded form as indicated by the problem. You may need to use Change of Base here to compute your logarithm.
- **CHECK** your result by using the graphing (intersection) method to solve the original problem

Notes:

- To **ISOLATE** means to manipulate the equation using addition, subtraction, multiplication, and division so that the exponential part and its input expression are by themselves.
- **EXACT FORM** for an answer means an answer that is not rounded until the last step.

Problem 4 | MEDIA EXAMPLE – SOLVE EXPONENTIAL EQUATIONS

<p>Solve $3^x = 25$ for x Round the final result to three decimal places.</p>	<p>Original Problem Statement</p>
	<p>Step 1: ISOLATE the exponential part of the equation</p>
	<p>Step 2: Change the equation to LOGARITHMIC form</p>
	<p>Step 3: ISOLATE the variable</p>
	<p>Step 4: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem. You may need to use Change of Base here to compute your logarithm.</p>
	<p>Step 5: CHECK your result by using the graphing (intersection) method to solve the original problem</p>

Problem 5	MEDIA EXAMPLE – SOLVE EXPONENTIAL EQUATIONS
------------------	--

<p style="text-align: center;">Solve $11.36(1.080)^t = 180$ for t Round the final result to three decimal places.</p>	<p>Original Problem Statement</p>
	<p>Step 1: ISOLATE the exponential part of the equation</p>
	<p>Step 2: Change the equation to LOGARITHMIC form</p>
	<p>Step 3: ISOLATE the variable</p>
	<p>Step 4: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem. You may need to use Change of Base here to compute your logarithm.</p>
	<p>Step 5: CHECK your result by using the graphing (intersection) method to solve the original problem</p>

Problem 6 | YOU TRY – SOLVE EXPONENTIAL EQUATIONS

<p>Solve for $12.5 + 3^x = 17.8$ Round the final result to three decimal places.</p>	<p>Original Problem Statement</p>
	<p>Step 1: ISOLATE the exponential part of the equation <i>(Hint: Start by subtracting 12.5 from both sides).</i></p>
	<p>Step 2: Change the equation to LOGARITHMIC form</p>
	<p>Step 3: ISOLATE the variable</p>
	<p>Step 4: IDENTIFY the final result in EXACT form then in rounded form as indicated by the problem. You may need to use Change of Base here to compute your logarithm.</p>
	<p>Step 5: CHECK your result by using the graphing (intersection) method to solve the original problem</p>

Graphing and Characteristics of the Logarithmic Function

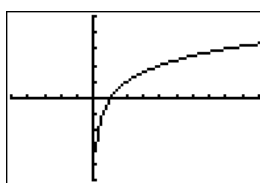
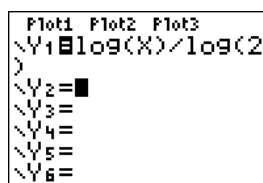
The Change of Base Formula can be used to graph Logarithmic Functions. In the following examples, we will look at the graphs of two Logarithmic Functions and analyze the characteristics of each.

Problem 7 | WORKED EXAMPLE – GRAPHING LOGARITHMIC FUNCTIONS

Given the function $f(x) = \log_2 x$, graph the function using your calculator and identify the characteristics listed below. Use window x: [-5..10] and y: [-5..5].

Graphed function: To enter the function into the calculator, we need to rewrite it using the Change of Base Formula, enter that equation into Y₁, and then Graph.

$$f(x) = \log_2 x = \frac{\log x}{\log 2}$$



X	Y1
-2	ERROR
-1	ERROR
0	ERROR
1	0
2	1
4	1.585

Characteristics of the function:

Domain: $x > 0$, Interval Notation: $(0, \infty)$

The graph comes close to, but never crosses the y-axis. Any value of x that is less than or equal to 0 ($x \leq 0$) produces an error. Any value of x greater than 0, to infinity is valid. To the right is a snapshot of the table from the calculator to help illustrate this point.

Range: All Real Numbers, Interval Notation $(-\infty, \infty)$

The graph has y values from negative infinity to infinity. As the value of x gets closer and closer to zero, the value of y continues to decrease (See the table to the right). As the value of x gets larger, the value of y continues to increase. It slows, but it never stops increasing.

X	Y1
4	-1.322
5	-1
6	-.737
7	-.5146
8	-.3219
9	-.152
10	0

y-intercept: Does Not Exist (DNE). The graph comes close to, but never crosses the y-axis.

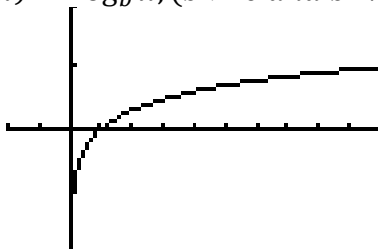
x-intercept: (1,0) The graph crosses the x-axis when $y = 0$. This can be checked by looking at both the graph and the table above as well as by entering the equation into the calculator $(\log(1)/\log(2))$

Left to Right behavior: The value of y quickly increases from $-\infty$ to 0 as the value of x increases from just above 0 to 1 (Remember that x cannot equal 0). As the value of x increases from 1 to ∞ , the value of y continues to increase (also to ∞), but at a slower and slower rate.

Problem 8 | WORKED EXAMPLE – GRAPHING LOGARITHMIC FUNCTIONS

The Logarithmic Function in Problem 7 is of the form $f(x) = \log_b x, (b > 0 \text{ and } b \neq 1)$. All Logarithmic Functions of this form share key characteristics. In this example, we look at a typical graph of this type of function and list the key characteristics in the table below.

$$f(x) = \log_b x, (b > 0 \text{ and } b \neq 1)$$



Domain	$x > 0$ (all positive real numbers)
Range	All real numbers
x-intercept	(1, 0)
y-intercept	Does not cross the y-axis so no y-intercept Y-axis is a guiding line for the graph
Left to Right Behavior	Y-values (outputs) increasing but more and more slowly (at a decreasing rate)
Values of x for which $f(x) > 0$	$x > 1$
Values of x for which $f(x) < 0$	$0 < x < 1$
Values of x for which $f(x) = 0$	$x = 1$
Values of x for which $f(x) = 1$	$x = b$ because $\log_b b = 1$

Problem 9 | YOU TRY – GRAPHING LOGARITHMIC FUNCTIONS

Graph $f(x) = \log_6 x$ on your graphing calculator. Use window x: [0..10] and y: [-2..2]. Use Change of Base to rewrite your function before graphing. Draw an accurate graph in the space below and fill in the table.

$f(x) = \log_6 x =$ _____ (rewrite using Change of Base)

Place your graph in this box	Fill in this information
	a) Domain of y:
	b) Range of y:
	c) x-intercept:
	d) If $f(x) = 1, x =$ _____.

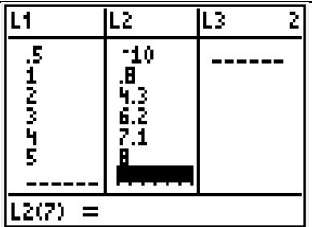

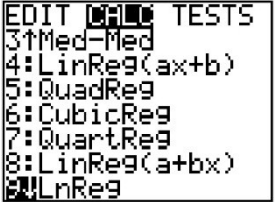
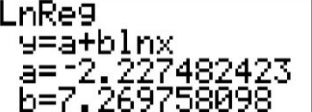
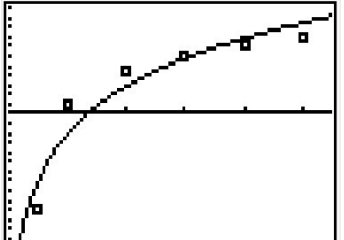
Modeling Data with Logarithmic Regression

Just as we did with Linear and Exponential Functions, we can model appropriate data using regression. The steps are very similar to the other processes we have used. We will use a slightly different type of logarithm for this regression denoted by LN or Natural Logarithm. You will learn more about this in later classes.

Problem 10 | WORKED EXAMPLE – LOGARITHMIC REGRESSION

Determine if the data set below can be modeled by a logarithmic function. If so, find that function rounding all decimals to 4 places.

x	.5	1	2	3	4	5
y	-10	.8	4.3	6.2	7.1	8

<p><i>Step 1: Insert data into L1, L2 list using Stat>Edit.</i></p> <p>Your data should look the same as in the table on the right.</p>	
<p><i>Step 2: Turn on your Plot (Go to Y= and highlight Plot 1) then select Zoom9 to work with a good window for your data. Your graph should look the same as the table on the right.</i></p> <p>The data are increasing but at a decreasing rate and generally have the shape of a logarithmic graph. We should be able to model the data effectively with a logarithmic equation.</p>	
<p><i>Step 3: Go to Stat>Calc>9:LnReg to find the Logarithmic Regression model for the data.</i></p>	
<p><i>Step 4: Press Enter twice to form the equation information on your screen. Write the equation as</i> $y = -2.2274 + 7.2698 \ln x$</p>	
<p><i>Step 5: Enter the equation into your Y= list and press Graph to show the graph of the data and the model.</i></p> <p>Looks like the model is a pretty good approximation of our data set.</p>	

Name: _____

Date: _____

Lesson 4b Practice Problems

1. Calculator Practice. Use your calculator to solve each of the following. Write down the exact key strokes you used.

Compute	Calculator Steps (TI 83/84)	Final Result (3 decimal places)
a) $2^{1/3}$		
b) $2^{3/2}$		
c) $\frac{2^{1/2}}{2^{2/3}}$		
d) $\sqrt[3]{2^4}$		
e) $(\sqrt[3]{2})^4$		
f) $\log 4$		
g) $2 \log 120$		
h) $3 + 2 \log 240$		

2. Calculate each of the following logarithms using the Change of Base Formula. Show your work. Round your answer to two decimal places.

a) $\log_3 9 =$

b) $\log_{1.5} 12 =$

c) $2 \log_4 10 =$

d) $\log_8 2 =$

e) $\log_2 8 + \log 100 =$

f) $\log_6 12.5 + \log_3 27 =$

3. Rewrite each exponent as a log. Then use the change of base rule and your calculator to find the answer. Show your work. Round your answers to two decimal places.

An example of how your work should look is shown to the right.

Example:

$$5^x = 125$$

$$\log_5 125 = x$$

$$\frac{\log 125}{\log 5} = x$$

$$3 = x$$

a) $8^x = 42$

b) $3^x = 100$

c) $4^x = .5$

d) $10^x = 4025$

e) $\left(\frac{1}{4}\right)^x = 256$

f) $20^x = 1$

4. Graph $f(x) = \log_2 x$ on your graphing calculator. Use window x: [0..10] and y: [-2..2]. Use Change of Base to rewrite your function before graphing. Draw an accurate graph in the space below and fill in the table.

$$f(x) = \log_2 x = \text{_____} \text{ (rewrite using Change of Base)}$$

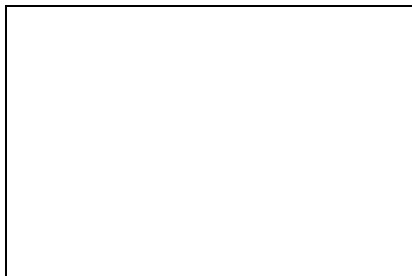
Place your graph in this box	Fill in this information
	a) Domain of y: b) Range of y: c) x-intercept: d) $f(2) =$ e) If $f(x) = 1$, $x =$ _____.

5. Graph $f(x) = 2\log_4 x$ on your graphing calculator. Use window x: [0..10] and y: [-2..2]. Use Change of Base to rewrite your function before graphing. Draw an accurate graph in the space below and fill in the table.

$$f(x) = 2\log_4 x = \text{_____} \text{ (rewrite using Change of Base)}$$

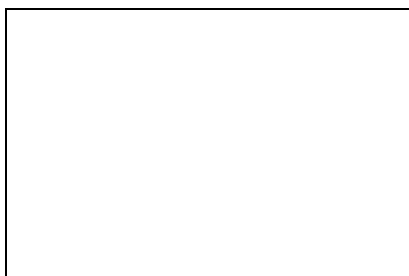
Place your graph in this box	Fill in this information
	a) Domain of y: b) Range of y: c) x-intercept: d) $f(2) =$ e) If $f(x) = 1$, $x =$ _____.

6. Consider the function $g(x) = \log_5 x$. Graph $g(x)$ on your graphing calculator. Use window x : $[0..10]$ and y : $[-2..2]$. In the space below, draw what you see on your calculator screen.



- a) What is the domain of $g(x)$? _____
- b) What is the range of $g(x)$? _____
- c) For what values of x is $g(x)$ positive? _____
- d) For what values of x is $g(x)$ negative? _____
- e) For what values of x is $g(x)$ increasing? _____
- f) What is the vertical intercept? _____
- g) What is the horizontal intercept? _____
- h) Give the **equation** of the vertical asymptote for $g(x)$. _____
- i) For what value of x is $g(x) = 1$? _____
- j) Determine $g(12)$. Round your answer to three decimal places.

7. Consider the function $g(x) = 3\log_2 x$. Graph $g(x)$ on your graphing calculator. Use window $x: [0..10]$ and $y: [-10..10]$. In the space below, draw what you see on your calculator screen.



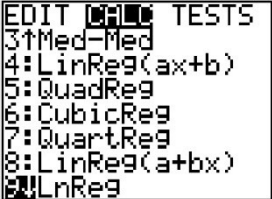
- a) What is the domain of $g(x)$? _____
- b) What is the range of $g(x)$? _____
- c) For what values of x is $g(x)$ positive? _____
- d) For what values of x is $g(x)$ negative? _____
- e) For what values of x is $g(x)$ increasing? _____
- f) What is the vertical intercept? _____
- g) What is the horizontal intercept? _____
- h) Give the **equation** of the vertical asymptote for $g(x)$. _____
- i) For what value of x is $g(x) = 1$? _____
- j) Determine $g(12)$. Round your answer to three decimal places.

8. Modeling Data with Logarithmic Regression

Just as we did with Linear and Exponential Functions, we can model appropriate data using regression. The steps are very similar to the other processes we have used.

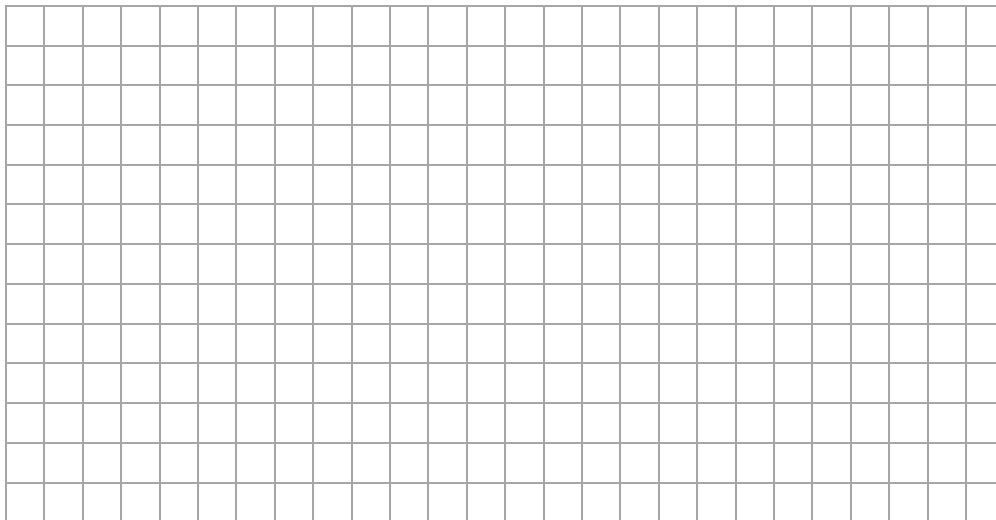
Determine if the data set below can be modeled by a logarithmic function. If so, find that function rounding all decimals to 4 places.

x	.5	1	2	3	4	5
y	-27	2	32	50	61	70

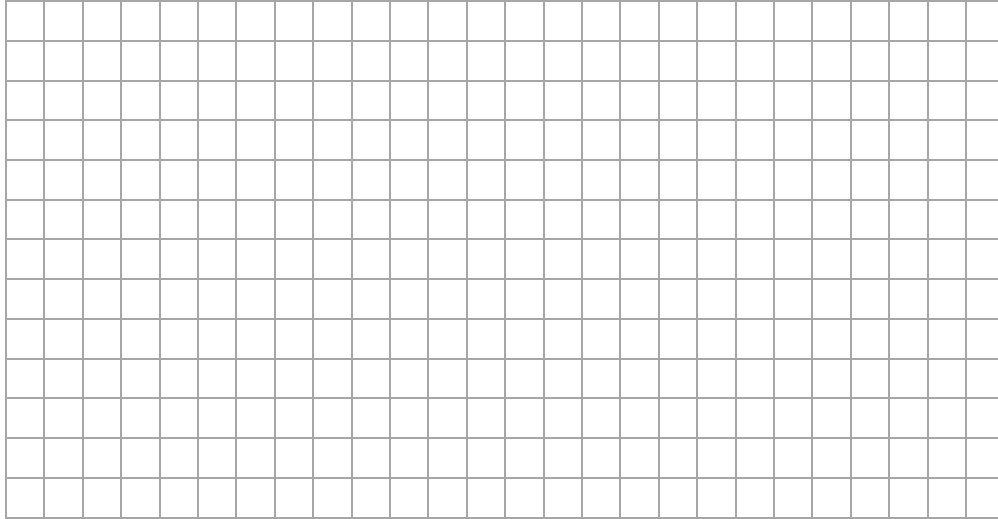
<p><i>Step 1: Insert data into L1, L2 list using Stat>Edit.</i></p>	
<p><i>Step 2: Turn on your Plot (Go to Y= and highlight Plot 1) then select Zoom9 to work with a good window for your data. Draw your graph on the right.</i></p>	
<p><i>Step 3: Go to Stat>Calc>9:LnReg to find the Logarithmic Regression model for the data.</i></p>	
<p><i>Step 4: Press Enter twice to form the equation information on your screen. Using the formula Write the equation using $Y = a + b\ln X$</i></p>	
<p><i>Step 5: Enter the equation into your Y= list and press Graph to show the graph of the data and the model. Draw the graph.</i></p>	

9. Linear, Exponential or Logarithmic? For each of the following sets of data, determine if it is best modeled with a Linear, Exponential or Logarithmic Equation.
- Enter the data into your calculator.
 - Create a Scatter Plot for the data
 - Decide which Regression to use, Linear, Exponential or Logarithmic
 - Write the final equation.
 - Enter the equation into your calculator
 - Graph the results with labels (On graph paper). Include both the Scatter Plot and the Graph
 - For each of the sets of data, write and solve a “Read Between the Data (Interpolating the Data)” or a “Read Beyond the Data (Extrapolating the Data)” Question.

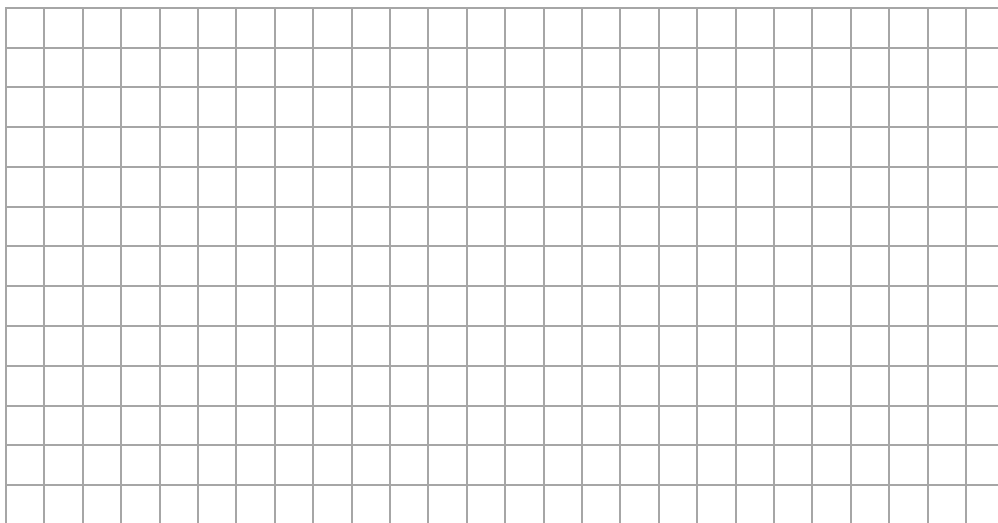
Age of Tree (Years)	1	2	3	4	5	6	7	8	9
Height (Feet)	6	9.5	13	15	16.5	17.5	18.5	19	19.5



Brain Weight (lbs)	1.7	3.3	36.33	60	100	187.1	250
Body Weight (lbs)	6.3	25.6	119.5	81	157	419	490



Year	1815	1825	1835	1845	1855	1865	1875	1885	1895
Population (Millions)	8.3	11	14.7	19.7	26.7	35.2	44.4	55.9	68.9



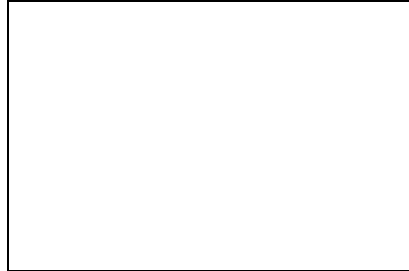
Name: _____

Date: _____

Lesson 4b Assessment

1. Consider the function $g(x) = \log_3 x$

- a) Graph $g(x)$ on your graphing calculator. Use window $x: [0..10]$ and $y: [-2..2]$. In the space below, draw what you see on your calculator screen.



- b) What is the domain of $g(x)$? _____
- c) What is the range of $g(x)$? _____
- d) For what values of x is $g(x)$ positive? _____
- e) For what values of x is $g(x)$ negative? _____
- f) For what values of x is $g(x)$ increasing? _____
- g) What is the vertical intercept? _____
- h) What is the horizontal intercept? _____
- i) Give the **equation** of the vertical asymptote for $g(x)$. _____
- j) For what value of x is $g(x) = 1$? _____
- k) Determine $g(42)$. Round your answer to three decimal places.

2. Use a graphing calculator to estimate the value of the following expressions. Round your answers to three decimal places.

a) $\log_5 37$

b) $\log_3 108$

3. Solve for x . Where applicable, give both the exact answer and the decimal approximation rounded to three decimal places. Show your work.

a) $5 + \log_2(x - 8) = 5$

b) $1000(1.12)^x = 2000$

Lesson 5a – Introduction to Quadratic Functions

We are leaving exponential and logarithmic functions behind and entering an entirely different world. As you work through this lesson, you will learn to identify quadratic functions and their graphs (called parabolas). You will learn the important parts of the parabola including the direction of opening, the vertex, y-intercept, and axis of symmetry.

You will use graphs of quadratic functions to solve equations and, finally, you will learn how to recognize all the important characteristics of quadratic functions in the context of a specific application. Even if a problem does not ask you to graph the given quadratic function or equation, doing so is always a good idea so that you can get a visual feel for the problem at hand.

Lesson Objectives

By the end of this lesson, you should be able to:

1. Identify QUADRATIC FUNCTIONS and list their characteristics
2. Graph a QUADRATIC FUNCTION and determine direction of opening, vertex, axis of symmetry, y- intercept, x-intercepts.
3. Solve QUADRATIC EQUATIONS by graphing
4. Solve applications involving QUADRATIC FUNCTIONS

Lesson 5a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 5a

A QUADRATIC FUNCTION is a function of the form

$$f(x) = ax^2 + bx + c$$

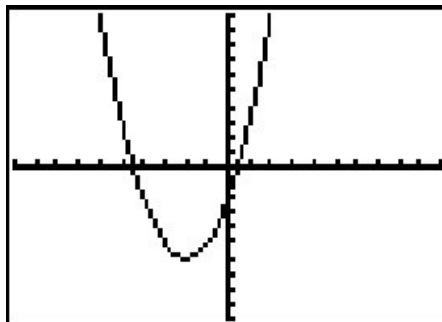
Characteristics Include:

- Three distinct terms each with its own coefficient:
 - An x^2 term with coefficient a
 - An x term with coefficient b
 - A constant term, c
 - Note: If any term is missing, the coefficient of that term is 0
- The graph of this function is called a “parabola”, is shaped like a “U”, and opens either up or down
- a determines which direction the parabola opens ($a > 0$ opens up, $a < 0$ opens down)
- c is the y- (or vertical) intercept with coordinates $(0, c)$

Problem 1 | WORKED EXAMPLE – GRAPH QUADRATIC FUNCTIONS

Given the Quadratic Function $f(x) = x^2 + 4x - 2$, complete the table and generate a graph of the function.

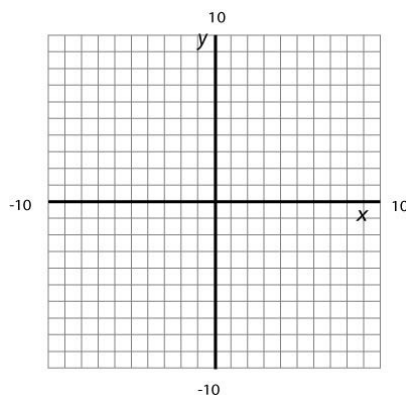
Identify the coefficients a, b, c	$a = 1, b = 4, c = -2$
Which direction does the parabola open? why?	$a = 1$ which is greater than 0 so parabola opens up
What is the y-intercept?	$c = -2$ so y-intercept = $(0, -2)$



Problem 2 | MEDIA EXAMPLE – GRAPH QUADRATIC FUNCTIONS

Given the Quadratic Function $f(x) = -3x^2 - 2x$, complete the table and generate a graph of the function.

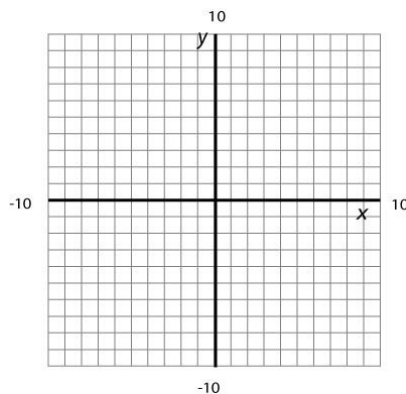
Identify the coefficients a, b, c	
Which direction does the parabola open? why?	
What is the y-intercept?	



Problem 3 | YOU TRY – GRAPH QUADRATIC FUNCTIONS

Given the Quadratic Function $f(x) = 2x^2 - 5$, complete the table and generate a graph of the function.

Identify the coefficients a, b, c	
Which direction does the parabola open? why?	
What is the y-intercept? Plot and label on the graph.	



VERTEX and the AXIS of SYMMETRY for a QUADRATIC FUNCTION

Given a quadratic function,
 $f(x) = ax^2 + bx + c$

The VERTEX is the lowest or highest point (ordered pair) of the parabola

- To find the x-coordinate, identify coefficients a and b then compute $-\frac{b}{2a}$
- To find the y-coordinate, plug the x-coordinate into the function (i.e. evaluate $f(-\frac{b}{2a})$)
- The Vertex is written as an ordered pair (x,y). Another way to look at this is:

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

The AXIS OF SYMMETRY is the vertical line that divides the parabola in half.

- Equation $x = -\frac{b}{2a}$

Problem 4 | WORKED EXAMPLE – QUADRATIC FUNCTIONS: VERTEX/AXIS OF SYMMETRY

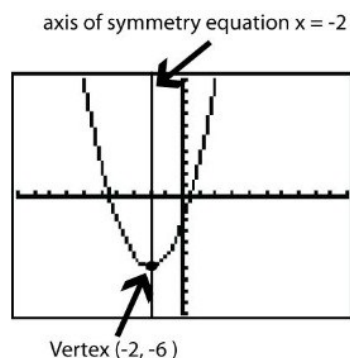
Given the Quadratic Function $f(x) = x^2 + 4x - 2$, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

Identify the coefficients a, b, c : $a = 1, b = 4, c = -2$

Determine the coordinates of the Vertex:

<p>x-coordinate:</p> $x = \frac{-b}{2a}$ $x = \frac{-(4)}{2(1)}$ $x = -2$	<p>y-coordinate:</p> $f(x) = x^2 + 4x - 2$ $f(-2) = (-2)^2 + 4(-2) - 2$ $f(-2) = 4 - 8 - 2$ $f(-2) = -6$ $y = -6$
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Vertex ordered pair: (-2, -6)



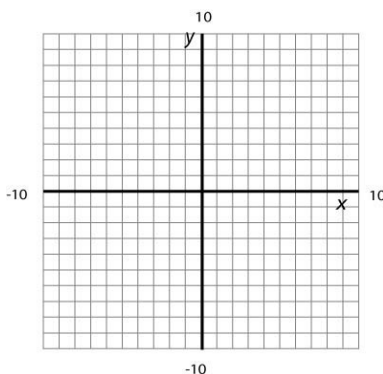
Identify the Axis of Symmetry Equation:

Equation $x = -\frac{b}{2a} = -2$ so $x = -2$ is the Axis of Symmetry.

Problem 5	MEDIA EXAMPLE – QUADRATIC FUNCTIONS: VERTEX/AXIS OF SYMMETRY
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Given the Quadratic Function $f(x) = -3x^2 - 2x$, complete the table, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

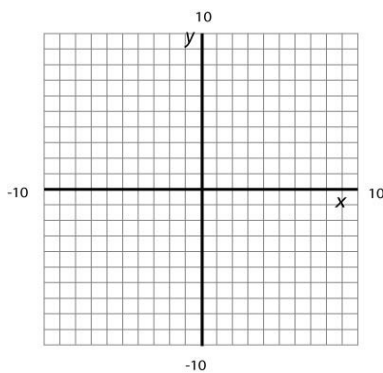
Identify the coefficients a, b, c	
Determine the coordinates of the Vertex.	
Identify the Axis of Symmetry Equation.	



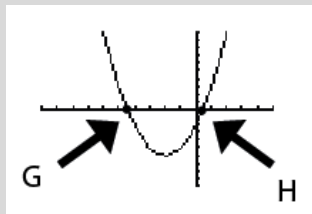
Problem 6	YOU TRY – QUADRATIC FUNCTIONS: VERTEX/AXIS OF SYMMETRY
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Given the Quadratic Function $f(x) = 2x^2 - 5$, complete the table, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

Identify the coefficients a, b, c	
Determine the coordinates of the Vertex. Plot and label on the graph.	
Identify the Axis of Symmetry Equation.	



Finding X-Intercepts (Horizontal Intercepts) of a Quadratic Function



The quadratic function, $f(x) = ax^2 + bx + c$, will have x-intercepts when the graph crosses the x-axis (i.e. when $f(x) = 0$). These points are marked on the graph above as G and H. To find the coordinates of these points, what we are really doing is solving the equation $ax^2 + bx + c = 0$. At this point, we will use the following general calculator process. In the next lesson, we will learn other methods for solving these equations.

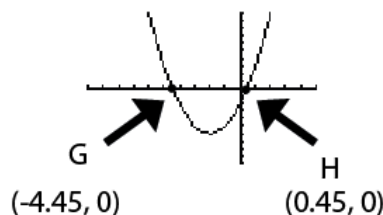
Calculator Process to solve $ax^2 + bx + c = 0$

1. Press Y= then enter $f(x)$ into Y1
2. Enter 0 into Y2
3. Use the graphing/intersection method once to determine G and again to determine H.
4. You will have to use the calculator arrow keys to move your cursor when finding H.

Problem 7	WORKED EXAMPLE – FINDING X-INTERCEPTS OF A QUADRATIC FUNCTION
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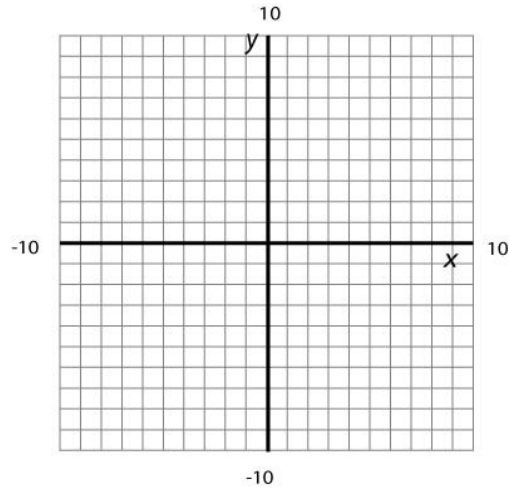
Find the x-intercepts of $f(x) = x^2 + 4x - 2$ and plot/label them on the graph.

1. Press Y= then enter $x^2 + 4x - 2$ into Y1
2. Enter 0 into Y2
3. Use the graphing/intersection method once to determine G as $(-4.45, 0)$. You may have to move your cursor close to G during the “First Curve?” part.
4. Use the graphing/intersection method again to determine H as $(0.45, 0)$. You may have to move your cursor close to H during the “First Curve?” part.



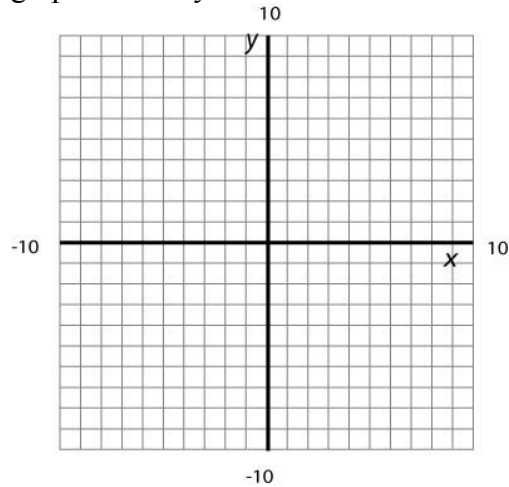
Problem 8	MEDIA EXAMPLE – FINDING X-INTERCEPTS OF A QUADRATIC FUNCTION
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Given the Quadratic Function $f(x) = -3x^2 - 2x$, find the x-intercepts and plot/label them on the graph. Round to 2 decimals.



Problem 9	YOU TRY – FINDING X-INTERCEPTS OF A QUADRATIC FUNCTION
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Given the Quadratic Function $f(x) = 2x^2 - 5$, find the x-intercepts and plot/label them on the graph. Round your values to two decimals.



Solve Quadratic Equations Graphically

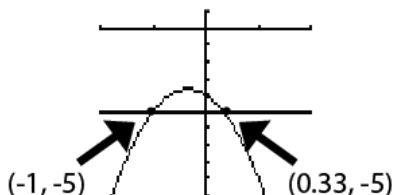
A quadratic equation of the form $ax^2+bx+c = d$ can be solved in the following way using your graphing calculator:

1. Go to Y=
2. Let $Y1= ax^2+bx+c$
3. Let $Y2 = d$
4. Graph the two equations. You may need to adjust your window to be sure the intersection(s) is/are visible.
5. For each intersection, use $2^{nd}>Calc>Intersect$. Follow on-screen directions to designate each graph then determine intersection (hitting Enter each time).
6. Solution(s) to the equation are the intersecting x-values

NOTE: The Intersection method will provide us only with approximate solutions to a quadratic equation when decimal solutions are obtained. To find EXACT solution values, you will need to use the Quadratic Formula. This will be covered in the next lesson.

Problem 10	WORKED EXAMPLE – SOLVE QUADRATIC EQUATIONS GRAPHICALLY
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Solve the equation $-3x^2 - 2x - 4 = -5$ by graphing.

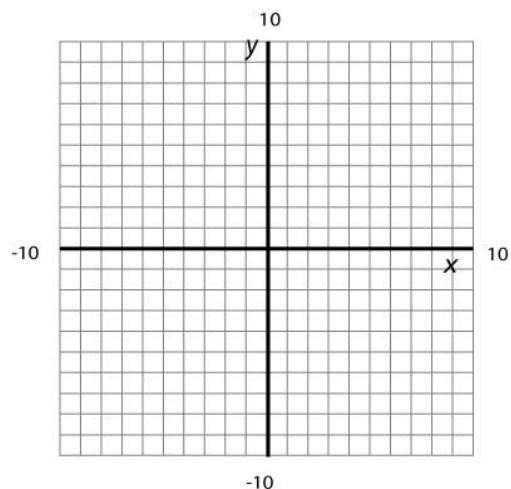


There are two intersections. Follow the process above to find the intersections $(-1, -5)$ and $(0.33, -5)$. Solutions to the equation are $x = -1$ and $x = 0.33$.

Problem 11	MEDIA EXAMPLE – SOLVE QUADRATIC EQUATIONS GRAPHICALLY
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Solve $x^2 - 10x + 1 = 4$.

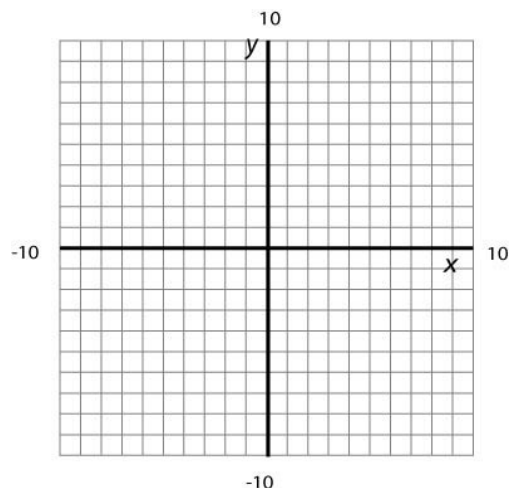
Plot and label the graphs and intersection points that are part of your solution process. Identify the final solutions clearly. Round to 2 decimals.



Problem 12	YOU TRY – SOLVE QUADRATIC EQUATIONS GRAPHICALLY
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Solve $2x^2 - 5 = 6$.

Plot and label the graphs and intersection points that are part of your solution process. Identify the final solutions clearly.



APPLICATIONS OF QUADRATIC FUNCTIONS

A large number of quadratic applications involve launching objects into the sky (arrows, baseballs, rockets, etc...) or throwing things off buildings or spanning a distance with an arched shape. While the specifics of each problem are certainly different, the information below will guide you as you decipher the different parts.

HOW TO SOLVE QUADRATIC APPLICATION PROBLEMS

1. Draw an accurate graph of the function using first quadrant values only. Label the x-axis with the input quantity and units. Label the y-axis with the output quantity and units.
2. Identify, plot, and label the y-intercept.
3. Identify, plot, and label the vertex.
4. Identify, plot, and label the positive x-intercept (usually, there is only one x-intercept that we care about...if both are needed for some reason, then plot them both and include negative x-values in your graph for part 1).
5. Once you have done steps 1 – 4, THEN read the specific questions you are asked to solve.

Questions that involve the y-intercept $(0, c)$:

- How high was the object at time $t = 0$? (c)
- What was the starting height of the object? (c)

Questions that involve the vertex $(\frac{-b}{2a}, f(\frac{-b}{2a}))$:

- How high was the object at its highest point? $f(\frac{-b}{2a})$
- What was the max height of the object? $f(\frac{-b}{2a})$
- How long did it take the object to get to its max height? $\frac{-b}{2a}$
- What is the practical range of this function? $0 \leq f(x) \leq f(\frac{-b}{2a})$

Questions that involve (usually) the positive x-intercept $(x_2, 0)$:

- When did the object hit the ground? (x_2)
- What is the practical domain of this function? $0 \leq x \leq x_2$
- How long did it take the object to hit the ground? (x_2)
- How far was the object from the center? (x_2)

Problem 13 | WORKED EXAMPLE –APPLICATIONS OF QUADRATIC FUNCTIONS

The function $h(t) = -16t^2 + 80t + 130$, where $h(t)$ is height in feet, models the height of an arrow shot into the sky as a function of time (seconds).

Before even LOOKING at the specific questions asked, find the following items and plot/label the graph.

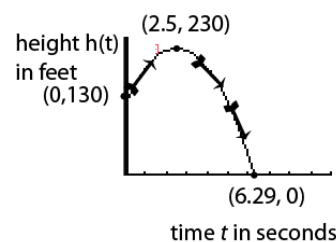
1. Draw an accurate graph of the function using first quadrant values only. Label the x-axis with the input quantity and units. Label the y-axis with the output quantity and units. *In this case, I am going to use $x[0, 10]$, $y[0, 250]$. You may have to play around with the window to get a good graph.*
2. Identify, plot, and label the y-intercept. *The y-intercept is $(0, 130)$ since $c = 130$.*
3. Identify, plot, and label the vertex.

$$\text{The x-coordinate of the vertex is } x = -\frac{b}{2a} = -\frac{80}{2(-16)} = 2.5.$$

The y-coordinate is

$$f\left(-\frac{b}{2a}\right) = f(2.5) = -16(2.5)^2 + 80(2.5) + 130 = 230$$

4. Identify, plot, and label the positive x-intercept – *using the process discussed in earlier examples, we want to solve $-16t^2 + 80t + 130 = 0$. Using the intersect method, the positive x-intercept is $(6.29, 0)$.*

**QUESTIONS TO ANSWER NOW:**

- a) After how many seconds does the arrow reach its highest point?

The x-coordinate of the vertex is 2.5. So, the arrow reaches its highest point after 2.5 seconds.

- b) How high is the arrow at its highest point?

The y-coordinate of the vertex is 230. So, the arrow is 230 ft. at its highest point.

- c) After how many seconds does the arrow hit the ground?

The positive x-intercept is $x = 6.29$. The arrow will hit the ground after 6.29 seconds.

- d) What is the practical domain of this function?

Time starts at 0 seconds and goes until the arrow hits the ground. So, practical domain is $0 \leq t \leq 6.29$

- e) What is the practical range of this function?

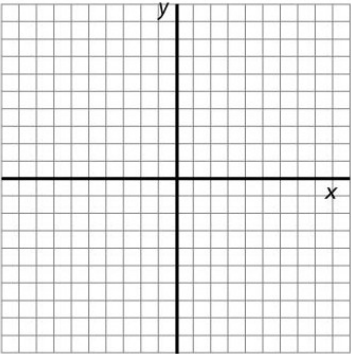
The arrow passes through all height values from 0 (when it hits the ground) to its max height of 230 ft. So, practical range is $0 \leq h(t) \leq 230$

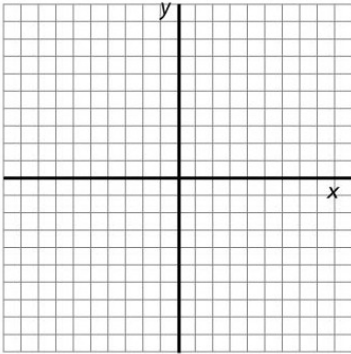
- f) What does the y-intercept represent?

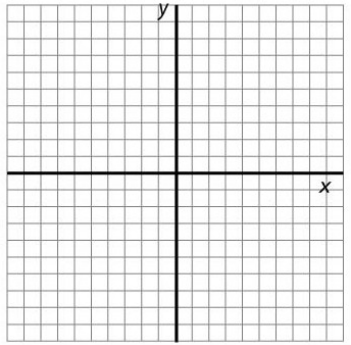
The y-intercept represents the height of the arrow at time $t = 0$. Thus, the arrow starts at 130 feet off the ground.

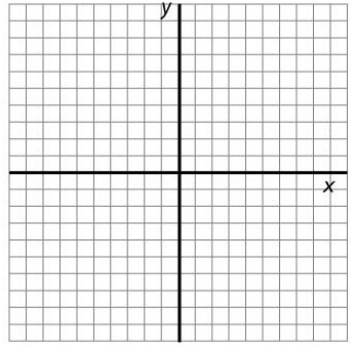
Lesson 5a Practice Problems

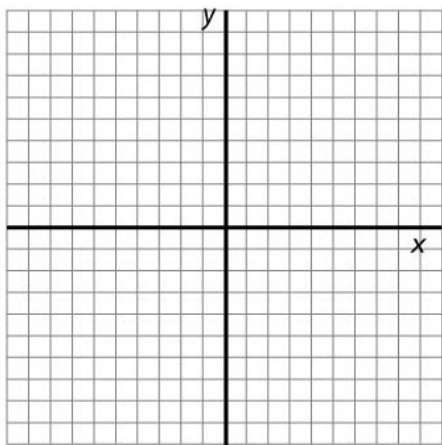
1. For each of the following quadratic functions:
- Graph the function on your calculator. Draw the graph neatly below.
 - Identify the coefficients, a,b,c
 - Determine if the parabola opens up or down and state why.
 - Identify the y-intercept.
 - Label the y-intercept on the graph. Make sure it is an ordered pair.

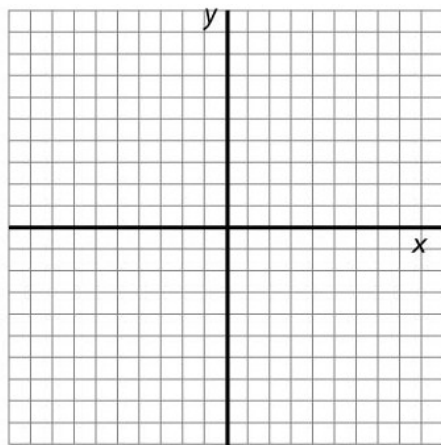
a) $f(x) = 2x^2 - 4x - 4$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

b) $f(x) = -x^2 + 6x - 4$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

c) $f(x) = 2x^2 - 6x + 4$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

d) $f(x) = x^2 - 3x$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

e) $f(x) = \frac{x^2}{2} - 3$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

f) $f(x) = -x^2 + x + \frac{2}{3}$	
Identify the coefficients a, b, c	
Which direction does the parabola open?	
What is the y-intercept?	
	

2. For each of the following quadratic functions (Show your work):
- Determine the line of symmetry. Make sure to write it as an equation.
 - Calculate the Vertex. Make sure to write it as an ordered pair.

a) $f(x) = -2x^2 + 2x - 3$

b) $g(x) = \frac{x^2}{2} - 3x + 2$

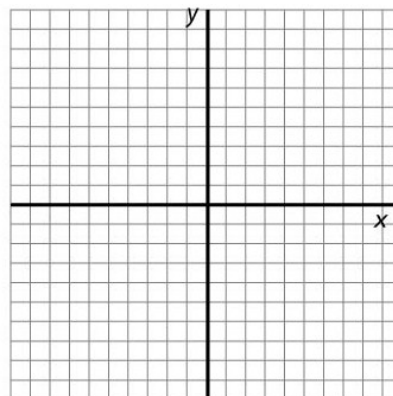
c) $f(x) = -x^2 + 3$

d) $p(t) = 4x^2 + 2x$

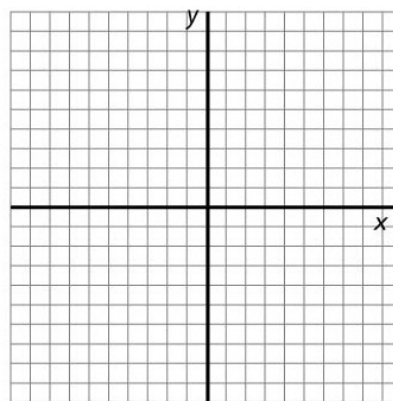
3. For each quadratic function:

- Graph the function on your calculator. Draw the graph neatly below.
- Label the y-intercept.
- Calculate the vertex.
- Label the vertex on the graph.
- Find the x-intercepts of each function using your calculator (If they exist).
- Label the x-intercepts on the graph (If they exist).

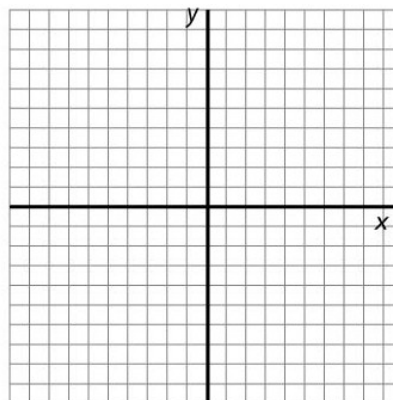
a) $f(x) = -2x^2 + 6x + 3$



b) $f(x) = \frac{3}{4}x^2 - 2x$

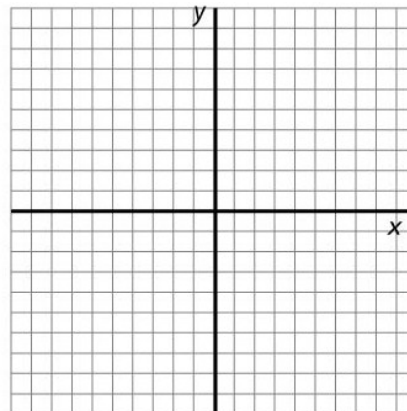


c) $f(x) = 5x^2 + 4$

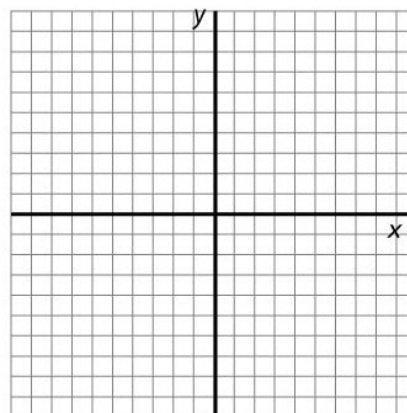


4. Solve each equation using your calculator. Draw the graph and plot/label the point of intersection.

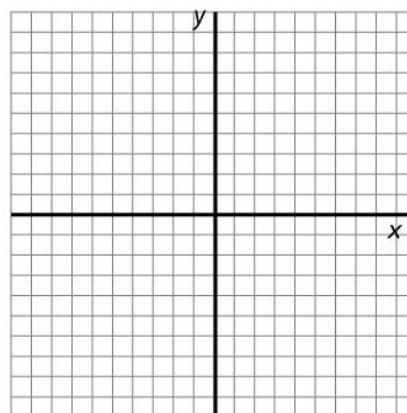
a) $-x^2 + 6x - 4 = -10$



b) $\frac{3}{2}x^2 - 6x + 6 = 10$



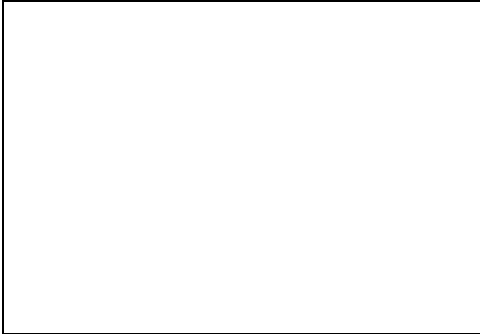
c) $5x^2 + \frac{x}{2} - 5 = 8$



6. A company's revenue earned from selling x items is given by the function $R(x) = 680x$, and cost is given by $C(x) = 10000 + 2x^2$.
- Write a function, $P(x)$, that represents the company's *profit* from selling x items.
 - Identify the vertical intercept of $P(x)$. Write it as an ordered pair and interpret its meaning in a complete sentence.
 - How many items must be sold in order to maximize the profit?
 - What is the maximum profit?
 - How many items does this company need to sell in order to break even?

7. An arrow is shot straight up into the air. The function $H(t) = -16t^2 + 90t + 6$ gives the height (in feet) of an arrow after t seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.
- How long does it take for the arrow to reach its maximum height? Show your work. Write your answer in a complete sentence.
 - Determine the maximum height of the arrow. Show your work. Write your answer in a complete sentence.
 - How long does it take for the arrow to hit the ground? Show your work. Write your answer in a complete sentence.
 - Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
 - Determine the practical domain of $H(t)$. Use proper notation.
 - Determine the practical range of $H(t)$. Use proper notation.

- g. Use your graphing calculator to create a good graph of $H(t)$. Use the practical domain and range to determine an appropriate viewing window. In the space below, sketch what you see on your calculator screen, and indicate the window you used.



Xmin: _____

Xmax: _____

Ymin: _____

Ymax: _____

- h. Use your graphing calculator to solve the equation $H(t) = 80$. Write a sentence explaining the meaning of your answer in the context of the arrow.

Name: _____

Date: _____

Lesson 5a Assessment

1. Fill out the following table. Intercepts must be written as ordered pairs. Always use proper notation. Round to two decimal places.

	$f(x) = 2x^2 - 4x - 30$	$g(x) = 5 - x^2$	$y = 5x^2 - 4x + 17$
Opens Upward or Downward?			
Vertical Intercept			
Horizontal Intercept(s)			
Vertex			
Domain			
Range			
Axis of Symmetry (Equation)			

2. The function $h(t) = -16t^2 + 88t$ gives the height (in feet) of a golf ball after t seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.
- Determine the maximum height of the ball. Show your work. Write your answer in a complete sentence.
 - How long does it take for the ball to hit the ground? Show your work. Write your answer in a complete sentence.
 - Identify the vertical intercept. Write it as an ordered pair and interpret its meaning in a complete sentence.
 - Determine the practical domain of $h(t)$. Use proper notation.
 - Determine the practical range of $h(t)$. Use proper notation.

Lesson 5b – Solving Quadratic Equations

In this lesson, we will continue our work with Quadratics in this lesson and will learn several methods for solving quadratic equations.

The first section will introduce you to the types of solutions that can occur. You may remember in L5a that we worked with x-intercepts of a parabola. Turns out that if the given quadratic equation is in standard form, then those x-intercepts correspond to solutions for the equation.

Graphing is the first method you will work with to solve quadratic equations followed by factoring and then the quadratic formula. You will get a tiny taste of something called Complex Numbers and then will finish up by putting all the solution methods together.

Pay special attention to the problems you are working with and details such as signs and coefficients of variable terms. Extra attention to detail will pay off in this lesson.

Lesson Objectives

By the end of this lesson, you should be able to:

1. Determine the number and type of solutions to a QUADRATIC EQUATION by graphing
2. Identify solutions to QUADRATIC EQUATIONS in standard form as x-intercepts of the parabola
3. Solve QUADRATIC EQUATIONS by factoring (and check by graphing)
4. Solve QUADRATIC EQUATIONS using the QUADRATIC FORMULA
5. Identify members of the set of COMPLEX NUMBERS
6. Perform operations with COMPLEX NUMBERS

Lesson 5b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Mini-Lesson 5b

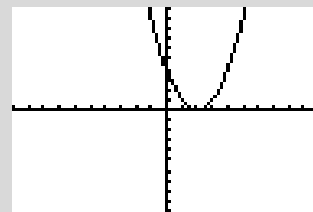
Number and Types of Solutions to Quadratic Equations in Standard Form
--

There are three possible cases for the number of solutions to a quadratic equation in standard form.

Note that Standard Form just means the equation is set = 0 (see next page for more formal definition).

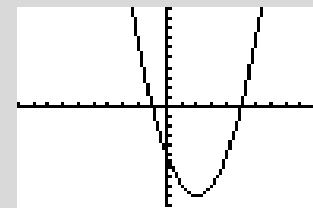
CASE 1:

- Parabola: Touches the x-axis in just one location (i.e. only the vertex touches the x-axis)
- One, repeated, real number solution
- Example: $(x - 2)^2 = 0$



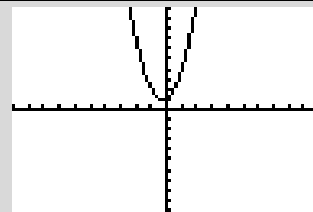
CASE 2:

- Parabola: Crosses the x-axis in two unique locations.
- Two, unique, real number solutions
- Example: $x^2 - 4x - 5 = 0$



CASE 3:

- Parabola: Does NOT cross the x-axis
- No real number solutions but two, complex number solutions (explained later in the lesson)
- Example: $2x^2 + x + 1 = 0$



REMINDER: A QUADRATIC FUNCTION is a function of the form

$$f(x) = ax^2 + bx + c$$

A QUADRATIC EQUATION in STANDARD FORM is an equation of the form

$$ax^2 + bx + c = 0$$

Problem 1 | MEDIA EXAMPLE – HOW MANY AND WHAT KIND OF SOLUTIONS

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into standard form. Draw an accurate sketch of the parabola indicating the window you used. IF your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the x-intercepts of the parabola. Label the x-intercepts on your graph.

a) $x^2 - 10x + 25 = 0$

b) $-2x^2 + 8x - 3 = 0$

c) $3x^2 - 2x = -5$

How are Solutions to a Quadratic Equation connected to X-Intercepts of the parabola?

If the quadratic equation $ax^2 + bx + c = 0$ has real number solutions x_1 and x_2 , then the x-intercepts of $f(x) = ax^2 + bx + c$ are $(x_1, 0)$ and $(x_2, 0)$.

Note that if a parabola does not cross the x-axis, then its solutions lie in the complex number system and we say that it has no real x-intercepts.

Problem 2 | YOU TRY – HOW MANY AND WHAT KIND OF SOLUTIONS

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into standard form. Draw an accurate sketch of the parabola indicating the window you used. IF your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the x-intercepts of the parabola. Label the x-intercepts on your graph.

a) $-x^2 - 6x - 9 = 0$

b) $3x^2 + 5x + 20 = 0$

c) $2x^2 + 5x = 7$

Solving Quadratic Equations by Factoring

So far, we have only used our graphing calculators to solve quadratic equations utilizing the Intersection process. There are other methods to solve quadratic equations. The first method we will discuss is the method of FACTORING. Before we jump into this process, you need to have some concept of what it means to FACTOR using numbers that are more familiar.

Factoring Whole Numbers

To FACTOR the number 60, you could write down a variety of responses some of which are below:

- $60 = 1 * 60$ (not very interesting but true)
- $60 = 2 * 30$
- $60 = 3 * 20$
- $60 = 4 * 3 * 5$

All of these are called FACTORIZATIONS of 60, meaning to write 60 as a product of some of the numbers that divide it evenly.

The most basic factorization of 60 is as a product of its prime factors (remember that prime numbers are only divisible by themselves and 1). The PRIME FACTORIZATION of 60 is:

$$60 = 2 * 2 * 3 * 5$$

There is only one PRIME FACTORIZATION of 60 so we can now say that 60 is COMPLETELY FACTORED when we write it as $60 = 2 * 2 * 3 * 5$.

When we factor polynomial expressions, we use a similar process. For example, to factor the expression $24x^2$, we would first find the prime factorization of 24 and then factor x^2 .

$$24 = 2 * 2 * 2 * 3 \quad \text{and} \quad x^2 = x * x$$

Putting these factorizations together, we obtain the following:

$$24x^2 = 2 * 2 * 2 * 3 * x * x$$

Let's see how the information above helps us to factor more complicated polynomial expressions and ultimately leads us to a second solution method for quadratic equations.

Factoring Quadratic Expressions and Solving Quadratic Equations by Factoring (Factoring using Greatest Common Factor Method)

Let's use the information on the previous page to help us FACTOR $3x^2 + 6x$. The building blocks of $3x^2 + 6x$ are the terms $3x^2$ and $6x$. Each is written in FACTORED FORM below.

$$3x^2 = 3 * x * x \quad \text{and} \quad 6x = 3 * 2 * x$$

Let's rearrange these factorizations just slightly as follows:

$$3x^2 = (3 * x) * x \quad \text{and} \quad 6x = (3 * x) * 2$$

We can see that $(3 * x) = 3x$ is a common FACTOR to both terms. In fact, $3x$ is the GREATEST COMMON FACTOR to both terms.

Let's rewrite the full expression with the terms in factored form and see how that helps us factor the expression:

$$\begin{aligned} 3x^2 + 6x &= (3 * x) * x + (3 * x) * 2 \\ &= (3x) * x + (3x) * 2 \\ &= (3x)(x + 2) \end{aligned}$$

Always check your factorization by multiplying the final result. $(3x)(x + 2) = 3x^2 + 6x$ CHECKS

$$3x^2 + 6x = (3x)(x + 2) \text{ is in COMPLETELY FACTORED FORM}$$

How can we use this factored form to solve quadratic equations such as $3x^2 + 6x = 0$?

To solve $3x^2 + 6x = 0$, FACTOR the left side:

$$(3x)(x + 2) = 0$$

Set each linear factor to 0:

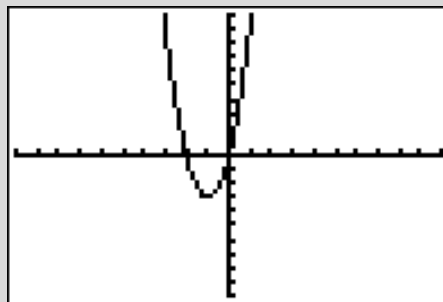
$$3x = 0 \quad \text{OR} \quad x + 2 = 0$$

Solve each linear equation to get

$$x = 0 \quad \text{OR} \quad x = -2$$

The solutions to $3x^2 + 6x = 0$ are $x = 0$ or $x = -2$.

If you graph the parabola $3x^2 + 6x$, I see that it crosses the x-axis at $(0, 0)$ and $(-2, 0)$ which confirms the solutions found above.



Problem 3	WORKED EXAMPLE – SOLVE QUADRATIC EQUATIONS BY FACTORING (GCF)
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Use the method discussed on the previous page, Factoring using the GCF, to solve each of the quadratic equations below. Verify your result by graphing and using the Intersection method.

a) Solve by factoring: $5x^2 - 10x = 0$

One way you can recognize that GCF is a good method to try is if you are given 2 terms only (called a BINOMIAL). This will be more important when we have other factoring methods to try and quadratics with more terms.

Step 1: Make sure the quadratic is in standard form (check!).

Step 2: Check if there is a common factor, other than 1, for each term (yes...5x is common to both terms)

Step 3: Write the left side in Completely Factored Form

$$5x^2 - 10x = 0$$

$$(5x)(x - 2) = 0$$

Step 4: Set each linear factor to 0 and solve for x:

$$5x = 0 \text{ OR } x - 2 = 0$$

$$x = 0 \text{ OR } x = 2$$

Step 5: Verify result by graphing (Let $Y1 = 5x^2 - 10x$, $Y2 = 0$, zoom:6 for standard window then 2nd>Calc>Intersect (two separate times) to verify solutions are $x = 0$ and $x = 2$).

Step 6: Write final solutions (usually separated by a comma): $x = 0, 2$

b) Solve by factoring: $-2x^2 = 8x$

Step 1: Make sure the quadratic is in standard form (need to rewrite as $-2x^2 - 8x = 0$)

Step 2: Check if there is a common factor, other than 1, for each term (yes...-2x is common to both terms – note that you can also use 2x but easier to pull the negative out with the 2x)

Step 3: Write the left side in Completely Factored Form

$$-2x^2 - 8x = 0$$

$$(-2x)(x + 4) = 0$$

Step 4: Set each linear factor to 0 and solve for x:

$$-2x = 0 \text{ OR } x + 4 = 0$$

$$x = 0 \text{ OR } x = -4$$

Step 5: Verify result by graphing (Let $Y1 = -2x^2 - 8x$, $Y2 = 0$, zoom:6 for standard window then 2nd>Calc>Intersect (two separate times) to verify solutions are $x = 0$ and $x = -4$).

Step 6: Write final solutions (usually separated by a comma): $x = 0, -4$

Problem 4	MEDIA EXAMPLE—SOLVE QUADRATIC EQUATIONS BY TRIAL/ERROR FACTORING
------------------	---

Use the Trial and Error Factoring Method to solve each of the quadratic equations below. Verify your result by graphing the quadratic part of the equation and looking at where it crosses the x-axis.

a) Solve $x^2 + 3x + 2 = 0$

b) Solve $x^2 - x - 6 = 0$

c) Solve $2x^2 - 5x - 3 = 0$

Problem 5	WORKED EXAMPLE—SOLVE QUADRATIC EQUATIONS BY TRIAL/ERROR FACTORING
------------------	--

Use the Trial and Error Factoring Method to solve each of the quadratic equations below. Verify your result by graphing the quadratic part of the equation and looking at where it crosses the x-axis.

a) Solve $x^2 + x - 6 = 0$

Step 1: Make sure the equation is in standard form (check!).

Step 2: Can we use GCF method? If no common factor other than one then no (can't use here).

Step 3: Try to factor the left side by Trial and Error

$$\text{Factor } x^2 + x - 6 = (x + 3)(x - 2)$$

Check by Foiling to be sure your answer is correct.

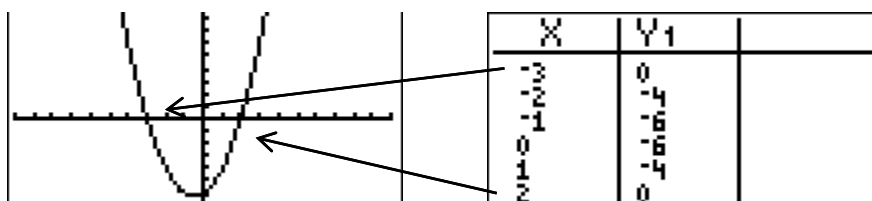
$$\begin{aligned} \text{Check: } (x + 3)(x - 2) &= x^2 - 2x + 3x - 6 \\ &= x^2 + x - 6 \quad (\text{checks!}) \end{aligned}$$

Step 4: Write the factored form of the quadratic then set each factor to 0 and solve for x.

$$(x + 3)(x - 2) = 0 \text{ so } x + 3 = 0 \text{ or } x - 2 = 0$$

$x = -3$ or $x = 2$ are the solutions to $x^2 + x - 6 = 0$. Can also write as $x = -3, 2$

Note: Graph $x^2 + x - 6$ and verify it crosses the x-axis at -3 and at 2.



b) Solve $x^2 - 4x - 32 = 0$

Step 1: In standard form (check!).

Step 2: No GCF other than 1 (check!).

Step 3: Try to factor the left side by Trial and Error

$$\text{Factor: } x^2 - 4x - 32 = (x - 8)(x + 4)$$

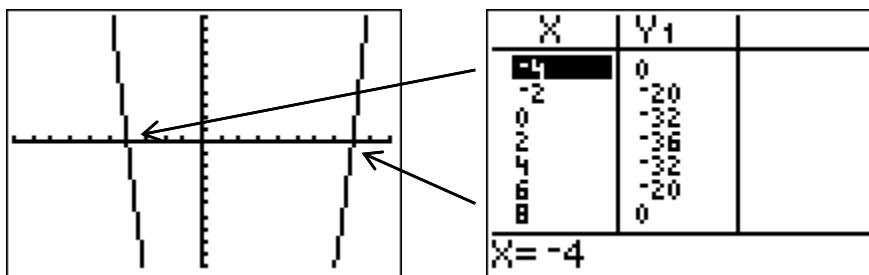
Check by Foiling to be sure your answer is correct.

$$\begin{aligned} \text{Check: } (x - 8)(x + 4) &= x^2 + 4x - 8x - 32 \\ &= x^2 - 4x - 32 \quad (\text{checks!}) \end{aligned}$$

Step 4: Write the factored form of the quadratic then set each factor to 0 and solve for x.

$$\begin{aligned} (x - 8)(x + 4) = 0 &\text{ so } x - 8 = 0 \text{ or } x + 4 = 0. \\ \text{Solutions to } x^2 - 4x - 32 = 0 &\text{ are } x = 8, -4. \end{aligned}$$

Note: Verify by graphing.



c) Solve $2x^2 + 7x = 15$

Step 1: Convert the equation to Standard Form

$$\begin{aligned} 2x^2 + 7x &= 15 \\ 2x^2 + 7x - 15 &= 15 - 15 && \text{Subtract 15 from each side} \\ 2x^2 + 7x - 15 &= 0 && \text{Now in Standard Form} \end{aligned}$$

Step 2: No GCF other than 1 (check!).

Step 3: Try to factor the left side by Trial and Error

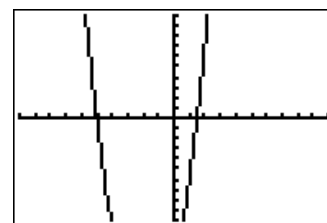
$$\text{Factor: } 2x^2 + 7x - 15 = (2x - 3)(x + 5)$$

Check by Foiling.

$$\begin{aligned} (2x - 3)(x + 5) &= 2x^2 + 10x - 3x - 15 \\ &= 2x^2 + 7x - 15 \end{aligned}$$

Step 4: Write the factored form of the quadratic then set each factor to 0 and solve for x.

$$\begin{aligned} (2x - 3)(x + 5) = 0 &\text{ so } 2x - 3 = 0 \text{ or } x + 5 = 0. \\ \text{Solutions to } 2x^2 + 7x = 15 &\text{ are } x = 3/2, -5. \\ \text{Verify by graphing.} \end{aligned}$$



Problem 6 | **YOU TRY – SOLVING QUADRATIC EQUATIONS: GCF and Trial/Error**

Use an appropriate factoring method to solve each of the quadratic equations below. Verify your result by graphing and using the Intersection method.

a) Solve $2x^2 + 6x = 0$ [Hint: Use GCF]

b) Solve $x^2 + 3x - 10 = 0$

c) Solve $2x^2 - 3x = 2$

Solving Quadratic Equations – Using the Quadratic Formula

Our final method for solving quadratic equations involves use of something called the Quadratic Formula. This is a brute force method that can sometimes help you if you are stuck on factoring.

Solving Quadratic Equations Using the Quadratic Formula

1. Place the quadratic equation in standard form (i.e. set = 0).
2. Identify the coefficients a , b , c
3. Substitute these values into the quadratic formula below
4. Simplify your result completely

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Do you wonder where this formula came from? Well, you can actually derive this formula directly from the quadratic equation in standard form $ax^2 + bx + c = 0$ using a factoring method called COMPLETING THE SQUARE. You will not be asked to use COMPLETING THE SQUARE in this class, but go through the information below and try to follow each step.

How to Derive the Quadratic Formula From $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad [\text{Subtract } c \text{ from both sides then divide all by } a]$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \quad [\text{Take the coefficient of } x, \text{ divide it by } 2, \text{ square it, and add to both sides}]$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \quad [\text{Factor the left side. On the right side, get a common denominator of } 4a^2]$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad [\text{Combine the right side to one fraction then take square root of both sides}]$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{Simplify the square roots}]$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad [\text{Solve for } x]$$

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} \quad [\text{Combine into one fraction to obtain the final form for the Quadratic Formula}]$$

Problem 7	WORKED EXAMPLE–SOLVE QUADRATIC EQUATIONS USING QUADRATIC FORMULA
------------------	---

Solve the quadratic equation by using the Quadratic Formula. Verify your result by graphing and using the Intersection method.

Solve $3x^2 - 2 = -x$ using the quadratic formula.

1. Set to zero and write in standard form $3x^2 + x - 2 = 0$
2. Identify $a = 3$, $b = 1$, and $c = -2$

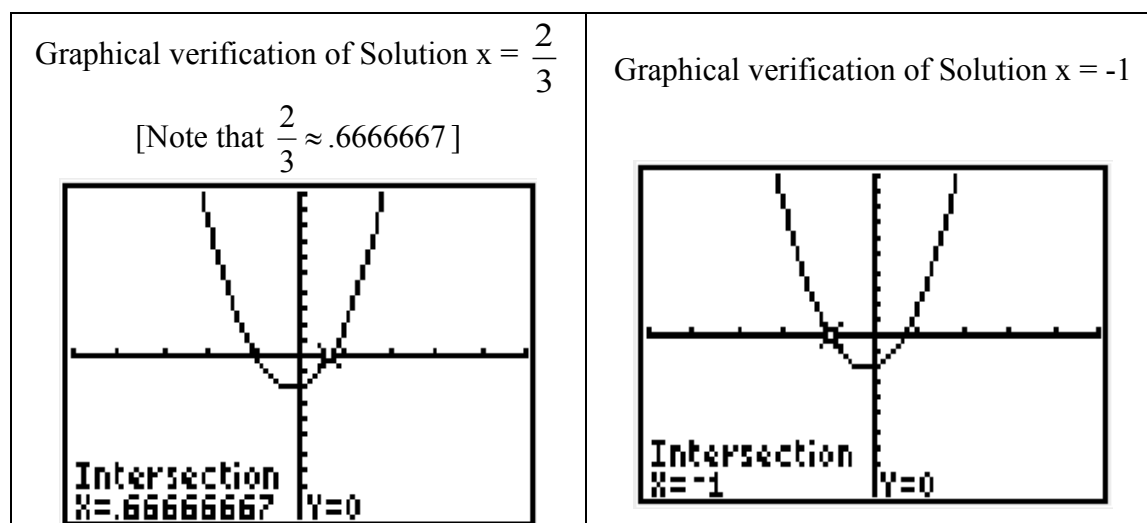
$$3. x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)} = \frac{-1 \pm \sqrt{1 - (-24)}}{6} = \frac{-1 \pm \sqrt{1 + 24}}{6} = \frac{-1 \pm \sqrt{25}}{6}$$

4. Make computations for x_1 and x_2 as below and note the complete simplification process:

$$\text{So, } x_1 = \frac{-1 + \sqrt{25}}{6} = \frac{-1 + 5}{6} = \frac{4}{6} = \frac{2}{3}$$

$$x_2 = \frac{-1 - \sqrt{25}}{6} = \frac{-1 - 5}{6} = \frac{-6}{6} = -1$$

Final solution $x = \frac{2}{3}$, $x = -1$ (be sure to verify graphically...you may also need to obtain a decimal approximation for a given value depending on how you are asked to leave your final answer)



You can see by the graphs above that this equation is an example of the “Case 2” possibility of two, unique real number solutions for a given quadratic equation.

Problem 8	MEDIA EXAMPLE – SOLVE QUADRATIC EQUATIONS USING QUADRATIC FORMULA
------------------	--

Solve each quadratic equation by using the Quadratic Formula. Verify your result by graphing and using the Intersection method.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) Solve $-x^2 + 3x + 10 = 0$

b) Solve $2x^2 - 4x = 3$

Complex Numbers – a HIGH LEVEL overview

Suppose we are asked to solve the quadratic equation $x^2 = -1$. Well, right away you should think that this looks a little weird. If I take any real number times itself, the result is always positive. Therefore, there is no REAL number x such that $x^2 = -1$. [Note: See explanation of Number Systems on the next page]

Hmmm...well, let's approach this using the Quadratic Formula and see what happens.

To solve $x^2 = -1$, I need to write in standard form as $x^2 + 1 = 0$. Thus, $a = 1$ and $b = 0$ and $c = 1$.

Plugging these in to the quadratic formula, I get the following:

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm \sqrt{-4}}{2} = \frac{\pm \sqrt{4(-1)}}{2} = \frac{\pm \sqrt{4}\sqrt{-1}}{2} = \frac{\pm 2\sqrt{-1}}{2} = \pm\sqrt{-1}$$

Well, again, the number $\sqrt{-1}$ does not live in the real number system nor does the number $-\sqrt{-1}$ yet these are the two solutions to our equation $x^2 + 1 = 0$.

The way mathematicians have handled this problem is to define a number system that is an extension of the real number system. This system is called the Complex Number System and it has, as its base defining characteristic, that equations such as $x^2 + 1 = 0$ can be solved in this system. To do so, a special definition is used and that is the definition that:

$$i = \sqrt{-1}$$

With this definition, then, the solutions to $x^2 + 1 = 0$ are just $x = i$ and $x = -i$ which is a lot simpler than the notation with negative under the radical.

When Will We See These Kinds of Solutions?

We will see solutions that involve the complex number "i" when we solve quadratic equations that never cross the x-axis. You will see several examples to follow that will help you get a feel for these kinds of problems.

Complex Numbers

Complex numbers are an extension of the real number system. Standard form for a complex number is

$$a + bi$$

where a & b are real numbers, $i = \sqrt{-1}$ and $i^2 = -1$

To work with i on your calculator, press MODE then change REAL to a+bi by using your arrow keys.

The i button is on the bottom row. You can use this button for the problems on the next page when simplifying and performing operations with i .

COMPLEX NUMBER SYSTEM

Complex Numbers – all numbers of the form $a + bi$ where a, b are real numbers and $i = \sqrt{-1}$

Examples: $3 + 4i, 2 + (-3)i, 0 + 2i, 3 + 0i$

Real Numbers – all the numbers on the REAL NUMBER LINE – include all RATIONAL NUMBERS and IRRATIONAL NUMBERS

Rational Numbers –

- ratios of integers
- decimals that terminate or repeat
- Examples: $0.50 = \frac{1}{2}, -.75 = -\frac{3}{4},$
 $0.43 = \frac{43}{100}, 0.33 = \frac{33}{100}$

Irrational Numbers

Examples: $\pi, e, \sqrt{5}, \sqrt{47}, \sqrt{11}$

- Decimal representations for these numbers never terminate and never repeat

Integers – Zero, Counting Numbers and their negatives
 $\{\dots-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, \dots\}$

Whole Numbers – Counting Numbers and Zero
 $\{0, 1, 2, 3, 4, 5, 6, 7, \dots\}$

Counting Numbers
 $\{1, 2, 3, 4, 5, 6, 7, \dots\}$

Problem 9	WORKED EXAMPLE – OPERATIONS WITH COMPLEX NUMBERS
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Simplify each of the following and write in the form $a + bi$

$$\begin{aligned} \text{a) } \sqrt{-16} &= \sqrt{-1}\sqrt{16} \\ &= 4\sqrt{-1} \\ &= 0 + 4i \end{aligned}$$

b) Note: This is SUBTRACTION and not FOIL.

$$\begin{aligned} (3 - 2i) - (4 + i) &= 3 - 2i - 4 - i \\ &= -1 - 3i \end{aligned}$$

$$\begin{aligned} \text{c) } (2 + i)(4 - 2i) &= 8 - 4i + 4i - 2i^2 \\ &= 8 - 2i^2 \\ &= 8 - 2(-1) \\ &= 8 + 2 \\ &= 10 + 0i \end{aligned}$$

Note: This one is FOIL. You can also just enter $(2 + i)(4 - 2i)$ on your calculator if you put your calculator in $a + bi$ mode. To work with i on your calculator, press MODE then change REAL to $a+bi$ by using your arrow keys. The i button is on the bottom row.

Problem 10	MEDIA EXAMPLE – OPERATIONS WITH COMPLEX NUMBERS
-------------------	--

Simplify each of the following and write in the form $a + bi$. To work with i on your calculator, press MODE then change REAL to $a+bi$ by using your arrow keys. The i button is on the bottom row.

$$\text{a) } (2 - 3i) - (4 + 2i) =$$

$$\text{b) } 4i(5 - 2i) =$$

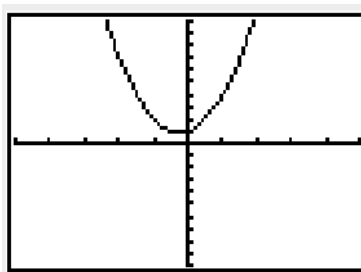
$$\text{c) } (2 - i)(1 + i) =$$

Work through the following to see how to deal with equations that can only be solved in the Complex Number System.

Problem 11	WORKED EXAMPLE – SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
-------------------	--

Solve $2x^2 + x + 1 = 0$ for x . Leave results in the form of a complex number, $a+bi$.

First, graph the two equations as Y1 and Y2 in your calculator and view the number of times the graph crosses the x-axis. The graph below shows that the graph of $y = 2x^2 + x + 1$ does not cross the x-axis at all. This is an example of our Case 3 possibility and will result in no Real Number solutions but two unique Complex Number Solutions.



To find the solutions, make sure the equation is in standard form (check).

Identify the coefficients $a = 2$, $b = 1$, $c = 1$.

Insert these into the quadratic formula and simplify as follows:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)} = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4}$$

Break this into two solutions and use the $a+bi$ form to get

$$\begin{aligned} x_1 &= \frac{-1 + \sqrt{-7}}{4} && x_2 = \frac{-1 - \sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{-7}}{4} && \text{and} && = -\frac{1}{4} - \frac{\sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{i\sqrt{7}}{4} && && = -\frac{1}{4} - \frac{i\sqrt{7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{7}}{4}i && && = -\frac{1}{4} - \frac{\sqrt{7}}{4}i \end{aligned}$$

The final solutions are $x_1 = -\frac{1}{4} + \frac{\sqrt{7}}{4}i$, $x_2 = -\frac{1}{4} - \frac{\sqrt{7}}{4}i$

Remember that $\sqrt{-1} = i$ so $\sqrt{-7} = i\sqrt{7}$

Work through the following problem to put the solution methods of graphing, factoring and quadratic formula together while working with the same equation.

Problem 12	YOU TRY – SOLVING QUADRATIC EQUATIONS
-------------------	--

Given the quadratic equation $x^2 + 3x - 10 = 0$, solve using the processes indicated below.

a) Solve by graphing (use your calculator and the Intersection process). Draw a good graph, label the x-intercepts, and list your final solutions using proper notation.

b) Solve by factoring. Show all steps as previously seen in this lesson. Clearly identify your final solutions.

c) Solve using the Quadratic Formula. Clearly identify your final solutions.

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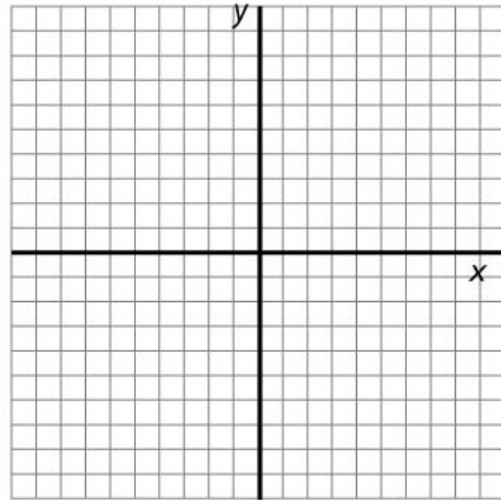
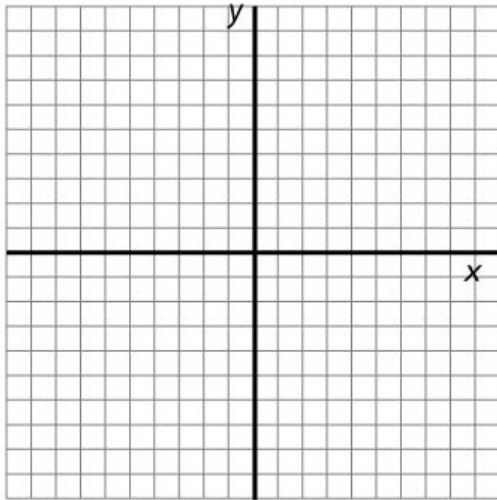
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Lesson 5b Practice Problems

1. Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below.
 - a) Begin by putting the equations into standard form.
 - b) Draw an accurate sketch of the parabola indicating the window you used.
 - c) IF your solutions are real number solutions, use the graphing INTERSECT method to find them.
 - d) Use proper notation to write the solutions and the x-intercepts of the parabola.
 - e) Label the x-intercepts on your graph.

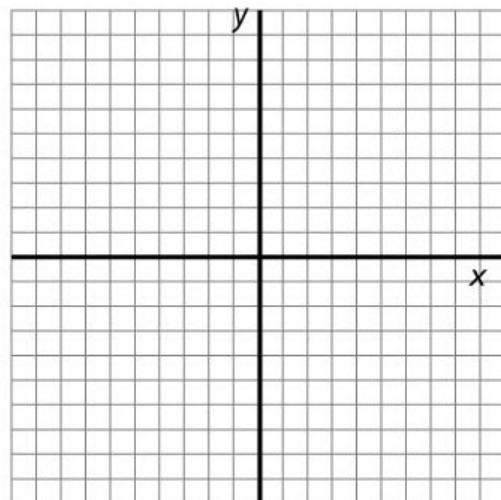
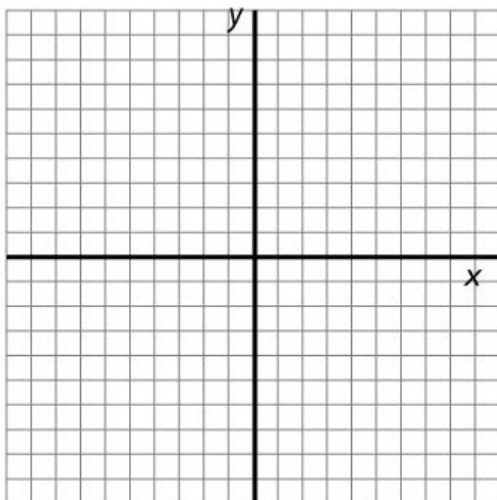
a) $x^2 - 6x + 9 = 0$

b) $5x^2 + 4x - 5 = 0$



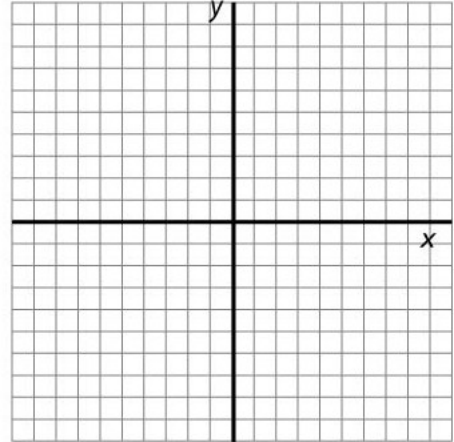
c) $2x^2 - 4x = 3$

d) $3x^2 + 6x + 4 = 0$

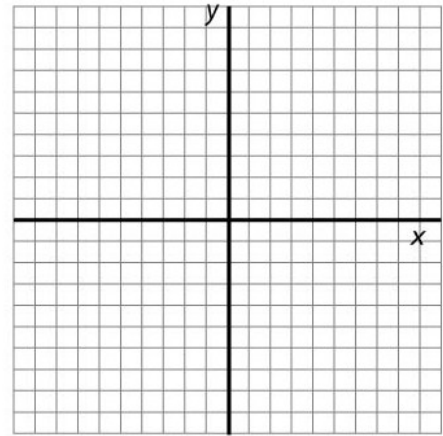


2. Solving each of the following Quadratic Equations by Factoring (GCF). Verify your answer by graphing. Be sure to write your final solutions using proper notation and to label the x-intercepts on your graph.

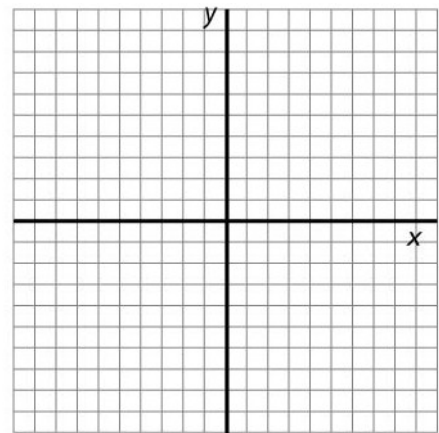
a) $4x^2 - 8x = 0$



b) $9x^2 - 6x = 0$

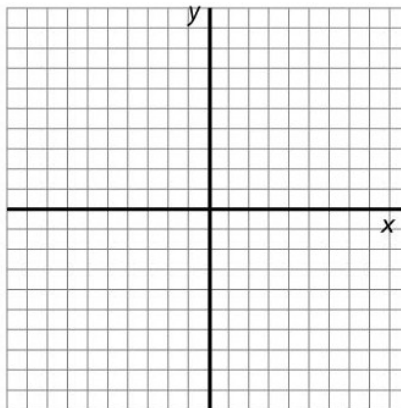


c) $2x^2 = 4x$

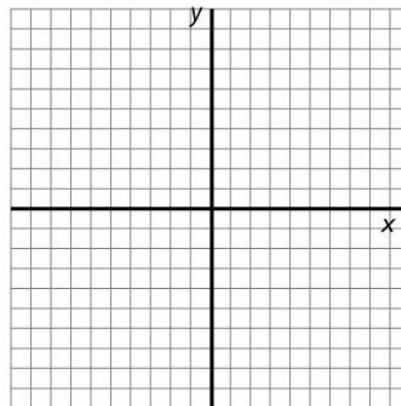


3. Use the Trial and Error Factoring Method to solve each of the quadratic equations below. Verify your answer by graphing. Be sure to write your final solutions using proper notation and to label the x-intercepts on your graph.

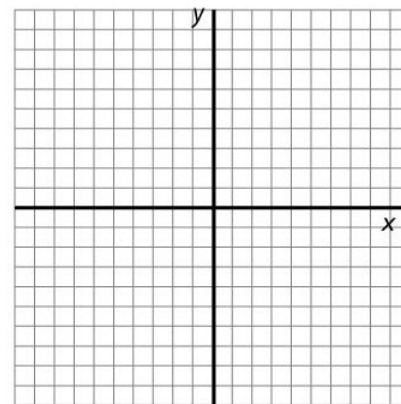
a) $x^2 + 8x + 12 = 0$



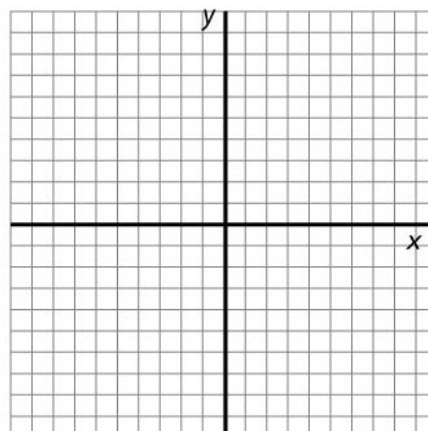
b) $x^2 + 42 = x$



c) $x^2 - 4x = 5$

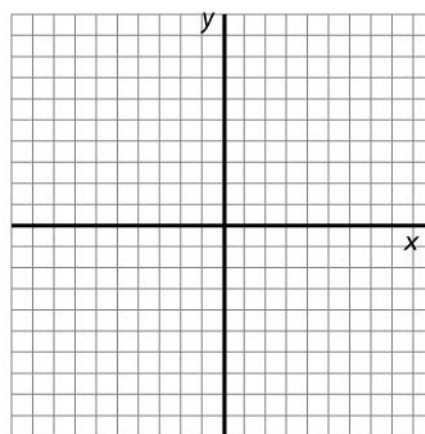


d) $x^2 - 36 = 0$

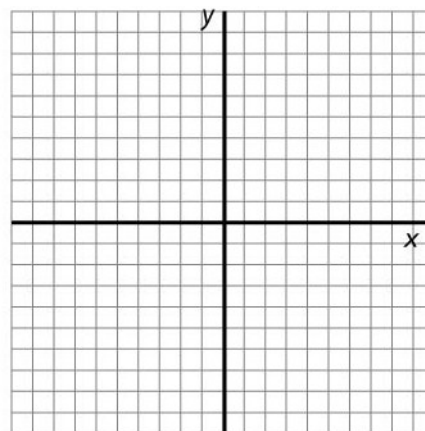


4. Use an appropriate factoring method to solve each of the quadratic equations below. Verify your answer by graphing. Be sure to write your final solutions using proper notation and to label the x-intercepts on your graph.

a) $9x^2 + 15x = 0$



b) $x^2 + 10x - 24 = 0$



5. Solve each quadratic equation by using the Quadratic Formula.

- Place your given quadratic equation in standard form.
- Identify the coefficients a, b, c
- Substitute these values into the quadratic formula below
- Simplify your result completely then check your solution graphically and write your final solutions
- Label the x-intercepts on your graph.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) $2x^2 - 2x - 4 = 0$ (This one is a fill in the blank)

$$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$$

$$x = \frac{() \pm \sqrt{() - ()}}{()}$$

$$x = \frac{() \pm \sqrt{()}}{()}$$

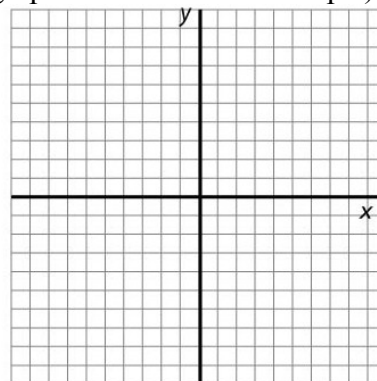
$$x_1 = \frac{() + \sqrt{()}}{()} \text{ and } x_2 = \frac{() - \sqrt{()}}{()}$$

$$x_1 = \frac{() + ()}{()} \text{ and } x_2 = \frac{() - ()}{()}$$

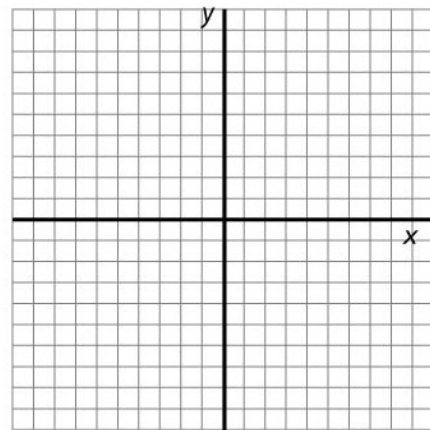
$$x_1 = \frac{()}{()} \text{ and } x_2 = \frac{()}{()}$$

$$x_1 = 2 \text{ and } x_2 = -1$$

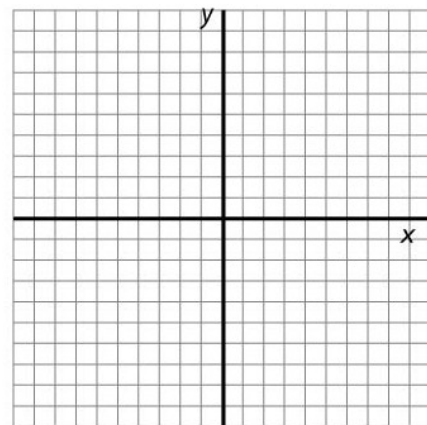
Final solution $x = -1, 2$ (Be sure to verify graphically. Draw the graph and label the intercepts)



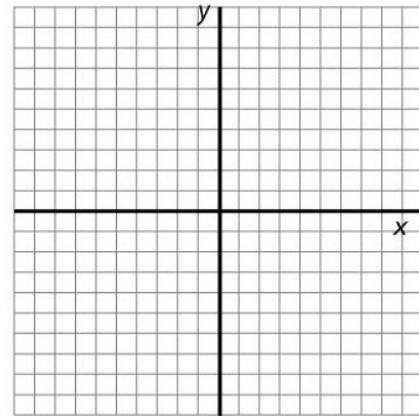
b) $2x^2 - 5x = 4$



c) $4x^2 - 2x = 6$



d) $6x^2 - 4x = 1$



6. Simplify each of the following and write in the form $a + bi$. To work with i on your calculator, press MODE then change REAL to $a+bi$ by using your arrow keys. The i button is on the bottom row.

a) $\sqrt{-49} =$

b) $\sqrt{-11} =$

c) $(4 - 2i) - (6 + 8i) =$

d) $3i(2 - 4i) =$

e) $(3 - i)(2 + i) =$

f) $(4 - 8i) - 3(4 + 4i) =$

7. Solve the quadratic equations, in the complex number system. Leave your final solution in the complex form $a \pm bi$.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) $\frac{1}{2}x^2 + 5x + 17 = 0$ (This one is a fill in the blank)

$$x = \frac{-(\) \pm \sqrt{(\)^2 - 4(\)(\)}}{2(\)}$$

$$x = \frac{(\) \pm \sqrt{(\) - (\)}}{(\)}$$

$$x = \frac{(\) \pm \sqrt{(\)}}{(\)}$$

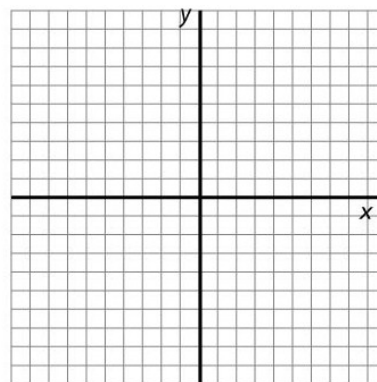
$$x = \frac{(\) \pm i\sqrt{(\)}}{(\)}$$

$$x_1 = \frac{(\) + (\)i}{(\)} \text{ and } x_2 = \frac{(\) - (\)i}{(\)}$$

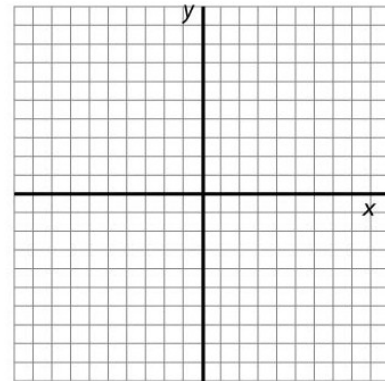
$$x_1 = \frac{(\)}{(\)} + \frac{(\)i}{(\)} \text{ and } x_2 = \frac{(\)}{(\)} - \frac{(\)i}{(\)}$$

$$x_1 = -5 + 3i \text{ and } x_2 = -5 - 3i$$

Final solution: $x = -5 + 3i, -5 - 3i$ (Be sure to draw the graph and label the y intercept)



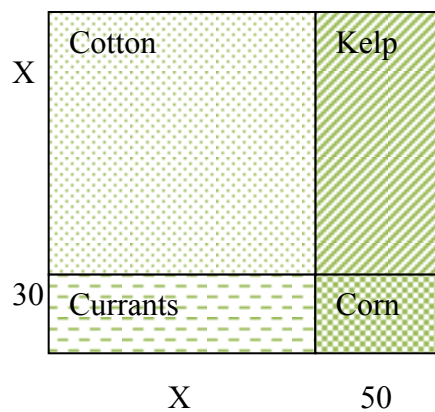
b) $x^2 + 2x + 5 = 0$



8. Farmer Treeman wants to plant four crops on his land, Cotton, Corn, Kelp and Currants. He has 40,000 square feet for planting. He needs the length and width of the property to be as shown in the picture below (measured in feet). He determines the equation for the area of his property is $x^2 + 80x + 1500 = 40000$

a. What will the length and width of the property need to be?

b. Determine the area of each section of the land.



9. The function $h(t) = -16t^2 + 96t + 10$, where $h(t)$ is height (in feet) of an arrow shot into the sky as a function of time, in seconds. Round answers to two decimal places as needed.
- a) How high above the ground was the arrow when it was shot? Write your answer in a complete sentence and include all appropriate units. Show your work.
- b) After how many seconds does the arrow hit the ground? Write your answer in a complete sentence and include all appropriate units. Show your work.
- c) What is the maximum height of the arrow? Write your answer in a complete sentence and include all appropriate units. Show your work.
- d) Determine $h(t) = 100$, and interpret your answer in a complete sentence. Show your work.

Name: _____

Date: _____

Lesson 5b Assessment

1. Factor each of the following quadratic expressions. Leave your answers in factored form.

a) $x^2 - 6x + 8$

b) $x^2 + x - 2$

c) $15x^2 - 3x$

d) $x^2 - 9$

2. Simplify each of the following and write in the form $a + bi$.

a) $\sqrt{-9} =$

b) $(3 - 5i)(2 + 8i) =$

3. Solve the following equations *algebraically* (Factoring or Quadratic Formula). You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Write complex solutions in the form $x = a + bi$ and $x = a - bi$. Use your calculator to *check* your answers.

a) $3x^2 + 2x + 3 = 8$

b) $x^2 + 9x + 11 = x - 5$

c) $x^2 + 3x + 7 = 2$

Lesson 6a – Exponents and Rational Functions

In this lesson, we put quadratics aside for the most part (not entirely) in this lesson and move to a study of exponents and rational functions. The rules of exponents should be familiar to you from a previous course and from lesson 2b but they are presented here again for your review and practice.

After exponents, you will embark on a study of rational functions. These may be unlike any function you have ever seen. Rational functions look different because they are in pieces but understand that the image presented is that of a single function.

In this lesson, you will graph rational functions and solve rational equations both graphically and algebraically. You will finish the lesson with an application of rational functions.

Lesson Objectives

By the end of this lesson, you should be able to:

1. Use PROPERTIES OF EXPONENTS to simplify expressions
2. Identify key characteristics of graphs of RATIONAL FUNCTIONS
3. Solve RATIONAL FUNCTIONS by graphing
4. Solve RATIONAL FUNCTIONS algebraically
5. Solve applications of RATIONAL FUNCTIONS

Lesson 6a Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 6a

Properties of Exponents

MULTIPLICATION PROPERTY OF EXPONENTS

Let m, n be rational numbers. To multiply powers of the same base, keep the base & add the exponents:

$$a^m a^n = a^{m+n}$$

EXAMPLES:

a) $(x^2)(x^4) = x^{2+4} = x^6$

b) $(-3x^5)(4x^2) = (-3)(4)x^{5+2} = -12x^7$

c) $(-2a^5)(8b^3)(3a^2b) = (-2)(8)(3)a^{5+2} b^{3+1} = -48a^7b^4$

DIVISION PROPERTY OF EXPONENTS

Let m, n be rational numbers. To divide powers of the same base, keep the base & subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n} \text{ where } a \neq 0$$

EXAMPLES:

a) $\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$

b) $\frac{x^{15}}{x^9} = x^{15-9} = x^6$

c) $\frac{15t^6}{3t^2} = \left(\frac{15}{3}\right)\left(\frac{t^6}{t^2}\right) = 5t^{6-2} = 5t^4$

d) $\frac{-4a^7}{2a^5} = -2a^2$

e) $\frac{x^9}{y^5}$ cannot be simplified as the bases x and y are different

ZERO EXPONENT $a^0 = 1$ if $a \neq 0$

EXAMPLES:

a) $16^0 = 1$

b) $\left(\frac{3}{X}\right)^0 = 1, X \neq 0$

c) $(4x)^0 = 1, x \neq 0$

d) $4x^0 = 4(1) = 4, x \neq 0$

e) $5(x+3)^0 = 5(1) = 5, x \neq -3$

NEGATIVE EXPONENTS

If $a \neq 0$ and n is a rational number, then $a^{-n} = \frac{1}{a^n}$

EXAMPLES: Rewrite the expressions using only positive exponents.

a) $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

b) $(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{2^3 x^3} = \frac{1}{8x^3}$

c) $x^{-1} = \frac{1}{x}$

d) $\frac{1}{x^{-4}} = x^4$

e) $3y^{-2} = \frac{3}{y^2}$

f) $(x^{-3})(x^{-5}) = x^{(-3)+(-5)} = x^{-8} = \frac{1}{x^8}$

POWER TO A POWER

If a is a real # and m and n are rational #'s, then $(a^m)^n = a^{mn}$

EXAMPLES:

a) $(3^2)^3 = 3^6 = 729$

b) $(x^3)^5 = x^{15}$

c) $(a^{-2})^3 = a^{-6} = \frac{1}{a^6}$

POWER OF A PRODUCT

If a and b are real #'s and n is a rational #, then

$$(ab)^n = a^n b^n$$

EXAMPLE:

a) $(x^2 y^3)^4 = (x^2)^4 (y^3)^4 = x^8 y^{12}$

b) $(-2a^5)^3 = (-2)^3 (a^5)^3 = -8a^{15}$

c) $(3x^{-3})^4 = 3^4 (x^{-3})^4 = 81x^{-12} = \frac{81}{x^{12}}$

Problem 1	MEDIA EXAMPLE–SIMPLIFY EXPRESSIONS USING PROPS OF EXPONENTS
------------------	--

Determine the following products:

a) $(-8x^4y^7)(5x^3y^2) =$

b) $(2x + 3)(x - 7)$

c) $(4x + 3)^2 =$

d) $(4x + 2)(x^2 - 4x + 3) =$

e) $(-3x)^2(xy)(2x)^3$

Problem 2	MEDIA EXAMPLE-SIMPLIFY EXPRESSIONS USING PROPS OF EXPONENTS
------------------	--

Simplify each of the following using properties of exponents.

Division with Exponents

a) $\frac{8^9}{8^4} =$

b) $\frac{-6t^{13}}{2t^7} =$

The Zero Exponent:

c) $7^0 =$

d) $-4x^0 =$

e) $-3(x^2 + 4)^0 =$

Negative Exponents:

f) $5^{-3} =$

g) $\frac{-2x^{-3}}{zy^{-4}} =$

h) $-6x^2y^{-5} =$

Putting it all together: Simplify each of the following using properties of exponents. Remember to write all final results with positive exponents only.

i) $\frac{-8x^3y^{-2}z^{-1}}{4x^5y^{-3}}$

Problem 3 | **YOU TRY - SIMPLIFY EXPRESSIONS USING PROPS OF EXPONENTS**

Simplify each of the following using properties of exponents. Remember to write all final results with positive exponents only.

a) $(-2a)^4(7a^5)(6ab)^2$

b) $(4x - y)(3x + y)$

c) $y^4x^3(5x)^{-2}$

d) $\frac{60x^{-13}y^{10}}{10x^{10}y^{-7}}$

e) $(x + 2)(3x^2 - x + 1)$

RATIONAL FUNCTIONS – KEY CHARACTERISTICS

A RATIONAL FUNCTION is a function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials and $q(x)$ does not equal zero (remember that division by zero is undefined). Rational functions have similar shapes depending on the degree of the polynomials $p(x)$ and $q(x)$. However, the shapes are different enough that only the general characteristics are listed here and not a general graph:

VERTICAL ASYMPTOTES

- $f(x)$ will have VERTICAL ASYMPTOTES (can be more than one) at all input values where $q(x) = 0$. These asymptotes are vertical guiding lines for the graph of $f(x)$ and $f(x)$ will never cross over these lines.
- To find the Vertical Asymptotes for $f(x)$, set $q(x) = 0$ and solve for x . For each x value that you find, $x =$ that value is the equation of your vertical asymptote. Draw a dotted, vertical line on your graph that represents this equation.

DOMAIN

- The x 's you found when solving $q(x) = 0$ are the values that are NOT part of the domain of $f(x)$.
- To write your domain, suppose that your function denominator is $x - a$. When you solve $x - a = 0$, you get $x = a$. Therefore, your domain would be "all real numbers x not equal to a ".

HORIZONTAL ASYMPTOTES

- *If $f(x)$ has a HORIZONTAL ASYMPTOTE, it can be determined by the ratio of the highest degree terms in $p(x)$ and $q(x)$. This asymptote is a guiding line for large values of the function (big positive x 's and big negative x 's).
- To find the Horizontal Asymptote, make a fraction with only the highest degree term in $p(x)$ in the numerator and the highest degree term in $q(x)$ as the denominator. Reduce this fraction completely. If the fraction reduces to a number, then $y =$ that number is your horizontal asymptote equation. If the fraction reduces to $\frac{\text{number}}{x}$, then $y = 0$ is your horizontal asymptote equation. [Remember the degree of a polynomial is the highest power of the polynomial]

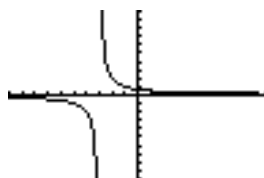
* $f(x)$ will have a HORIZONTAL ASYMPTOTE only if the degree of $q(x) \geq$ degree of $p(x)$.

NOTE: The information above is VERY general and probably very confusing at this point since no examples were provided. Go carefully through the next two examples and the information will make more sense.

Problem 4	WORKED EXAMPLE – KEY CHARACTERISTICS OF RATIONAL FUNCTIONS
------------------	---

Graph $f(x) = \frac{2}{x+3}$ and determine the horizontal and vertical asymptotes and the domain.

To graph, let $Y1 = \frac{2}{x+3}$. Input $Y1=2/(x+3)$ into your Y= list and note the use of ().



To find any VERTICAL ASYMPTOTES, set the denominator of to 0 and solve for x.
 $x + 3 = 0$, therefore $x = -3$. The equation of the vertical asymptote is $x = -3$.

To find the DOMAIN, because $x = -3$ makes the denominator of $f(x)$ equal zero, this value is not part of the domain. All other inputs are allowable. So, the domain for $f(x)$ is “all real numbers except -3”.

To find the HORIZONTAL ASYMPTOTE, make a fraction of the highest power term in the numerator (2) and the highest power term in the denominator (x). Reduce. Here is what the fraction looks like.

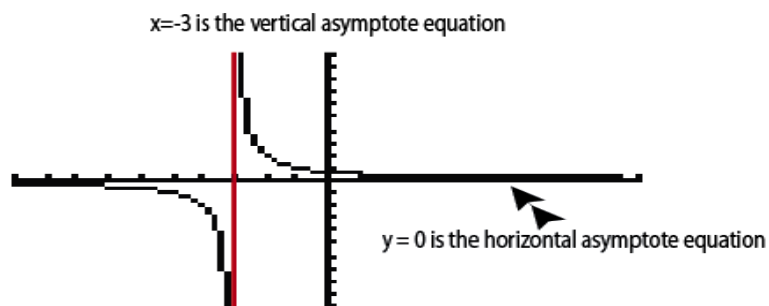
$$\frac{2}{x}$$

What you are trying to find out is, what is the value of this function as x gets really big (positive) and really big (negative)? To answer this question, we need to apply a little abstract thinking.

ABSTRACT THINKING

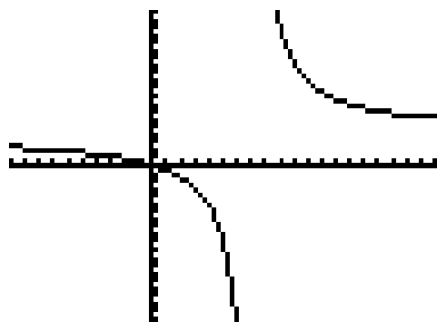
- In your mind, think of the very biggest positive number you can think of. What happens when you divide 2 by that number? Well, the result is going to be very, very small...effectively zero if your number is big enough. So, $y = 0$ is your horizontal asymptote equation as the same thing works for the biggest negative number you can think of.

Putting all these things together gives the following graph with asymptotes labeled:

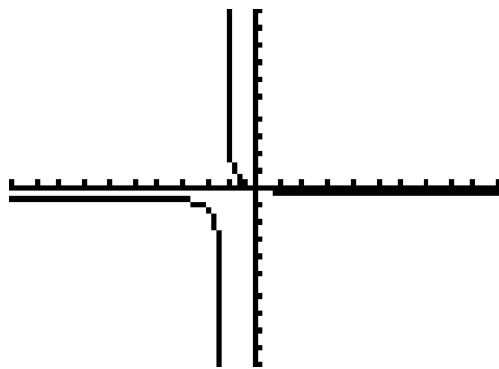


Problem 5	MEDIA EXAMPLE – KEY CHARACTERISTICS OF RATIONAL FUNCTIONS
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a) Graph $f(x) = \frac{4x}{x-7}$ and determine the horizontal and vertical asymptotes and the domain.



b) Graph $f(x) = \frac{-3x}{7x+9}$ and determine the horizontal and vertical asymptotes and the domain.



c) **Quick Example:** Find the Horizontal Asymptote for $f(x) = \frac{x-1}{x+5}$.

The ratio of highest degree terms in the numerator/denominator is $y = \frac{x}{x} = 1$ so the Horizontal Asymptote for this function is $y = 1$.

SOLVE RATIONAL EQUATIONS BY GRAPHING

To solve Rational Equations by graphing:

- Let $y_1 =$ one side of the equation
- Let $y_2 =$ other side of the equation
- Determine an appropriate window to see important parts of the graph
- Use the intersection method
- You may have more than one solution
- The x -value(s) of the intersection are your solutions

Problem 6 | WORKED EXAMPLE – SOLVE RATIONAL EQUATIONS BY GRAPHING

Solve $5x = 4 + \frac{3}{x-4}$

Let $Y_1 = 5x$

Let $Y_2 = 4 + 3/(x-4)$ Note use of ()

Graph on window $x: [-10..10], y: [-10..30]$

If you use standard window you do not see the upper intersection.



You will need to perform the intersection process two separate times on your calculator. One time, you should get $x = 0.62$ (the left intersection) and the second time you should get $x = 4.18$. Be sure to move your cursor far enough (it has to go all the way across the vertical asymptote) to read the second intersection. Solutions, then, are $x = 0.62, 4.18$

Problem 7 | MEDIA EXAMPLE – SOLVE RATIONAL EQUATIONS BY GRAPHING

Solve $3 = 1 + \frac{3x}{x-1}$ by graphing. Round answer(s) to two decimals as needed.

SOLVE RATIONAL EQUATIONS ALGEBRAICALLY

To solve rational equations algebraically (also called symbolically):

- Identify the common denominator for all fractions in the equation.
- Clear the fractions by multiplying both sides of the equation by the common denominator
- Take note of the values of x that make the common denominator zero. These x -values cannot be used as solutions to the equation since we cannot divide by 0.
- Solve for x .
- Check your work by plugging the value(s) back into the equation or by graphing.

Problem 8 | WORKED EXAMPLE – SOLVE RATIONAL EQUATIONS ALGEBRAICALLY

Solve $5x = 4 + \frac{3}{x-4}$ algebraically. Round solutions to two decimal places.

- Common denominator for all sides is $x-4$. Multiply both sides of the equation by $(x-4)$ and solve for x to get the following:

$$(5x)(x-4) = \left(4 + \frac{3}{x-4}\right)(x-4)$$

$$5x^2 - 20x = 4(x-4) + \frac{3}{x-4}(x-4)$$

$$5x^2 - 20x = 4x - 16 + 3$$

$$5x^2 - 20x = 4x - 13$$

$$5x^2 - 20x - 4x + 13 = 0$$

$$5x^2 - 24x + 13 = 0$$

Notice that we now have a quadratic equation, which can be solved using the methods of last chapter. Because we are asked to solve our original problem algebraically, let's continue that process and not resort to a graphical solution. We will use the Quadratic Formula with $a=5$, $b=-24$, and $c=13$ to get:

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(13)}}{2(5)} = \frac{24 \pm \sqrt{576 - 260}}{10} = \frac{24 \pm \sqrt{316}}{10}$$

Because we want rounded solutions, I do NOT need to continue reducing my fraction solutions above but can compute the following directly:

$$x = \frac{24 + \sqrt{316}}{10} \approx 4.18, \quad x = \frac{24 - \sqrt{316}}{10} \approx .62$$

These solutions match what we found in the graphing example previously.

To check, plug the values back into the original equation (one at a time) or use the graphing method.

Problem 9	MEDIA EXAMPLE – SOLVING RATIONAL EQUATIONS ALGEBRAICALLY
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Solve $3 = 1 + \frac{3x}{x-1}$ algebraically. Round answer(s) to two decimals as needed.

Problem 10	YOU TRY – SOLVING RATIONAL EQUATIONS GRAPHICALLY/ALGEBRAICALLY
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Solve $4 = \frac{5}{x-2}$ algebraically and graphically. Round answer(s) to two decimals as needed.
Show complete work in this space.

Name: _____

Date: _____

Lesson 6a Practice Problems

1. Simplify each of the following using properties of exponents.

a) $\frac{3^4}{3^6} =$

b) $\frac{-4t^{14}}{8t^2} =$

c) $y^0 t^2 =$

d) $4x(x + 8) =$

e) $(x+2)^2 =$

f) $\frac{2x^3}{-3z^{-2}y^{-4}} =$

g) $4x^{-3}y^{-5} =$

i) $\frac{-8x^6y^{-3}z^{-2}}{2x^5y^{-3}} =$

j) $\frac{6x^3 + 4x^2}{2x} =$

k) $\frac{6x + 4}{2x^2} =$

2. Determine the domain and the vertical asymptote for each of the following

a) $f(x) = \frac{4x}{6-2x}$

b) $f(x) = \frac{8x+2}{3x-9}$

c) $g(x) = \frac{4x+3}{3x+6}$

d) $p(t) = \frac{t}{12t-6}$

3. Determine the horizontal asymptote for each of the following. If it doesn't exist, write DNE.

a) $f(x) = \frac{4x}{6-2x}$

b) $f(x) = \frac{9x+2}{3x^2-9}$

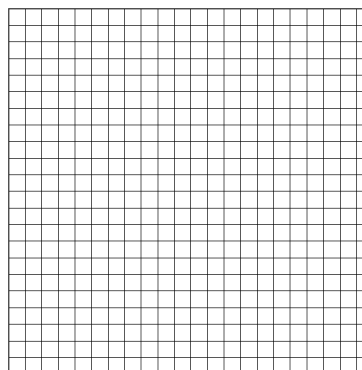
c) $g(x) = \frac{3}{4x+3}$

d) $p(t) = \frac{t^2}{12t-6}$

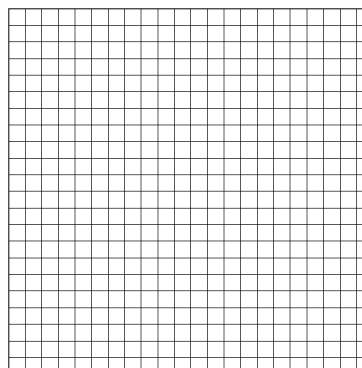
4. For each of the following functions

- Graph the function using your calculator
- Determine and graph the horizontal asymptote
- Determine and graph the vertical asymptote
- Determine the domain

a) $f(x) = \frac{4x}{2-x}$

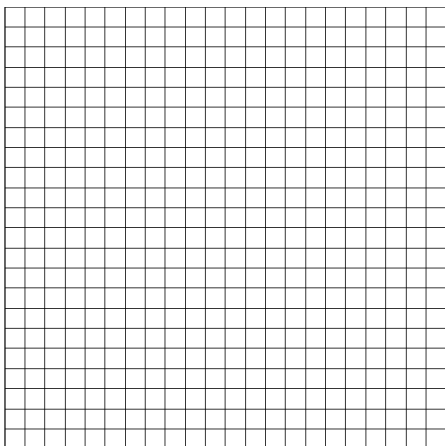


b) $f(x) = \frac{-3}{2x+3}$

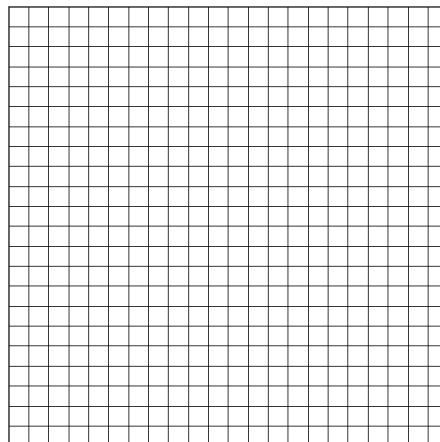


5. Solve each of the following equations by graphing. Round answer(s) to two decimals as needed.

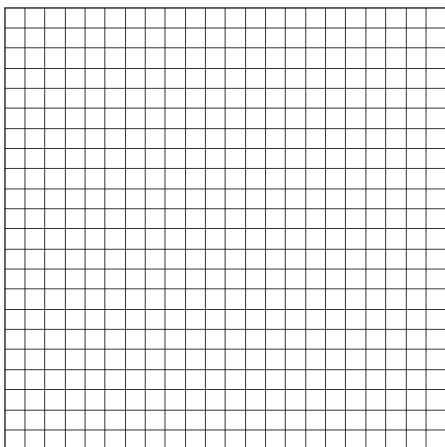
a) $4 = \frac{3x}{x-1}$



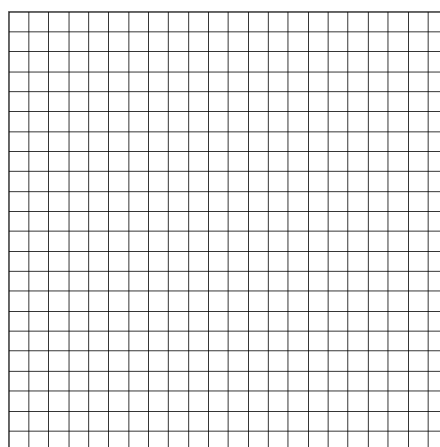
b) $3 = 2 + \frac{5}{x-2}$



c) $x + 2 = \frac{2}{x^2-4}$



d) $3 = x + \frac{1}{x}$



6. Solve each of the following rational equations algebraically (also called symbolically). Check your work by plugging the value(s) back into the equation or by graphing.

a) $4 = \frac{3}{x-6}$

b) $\frac{4}{x+4} = \frac{6}{x-2}$

c) $\frac{4-2x}{3} = \frac{3x+2}{4}$

d) $6 = 2 + \frac{3}{x-5}$

e) $x + 4 = \frac{-4}{x}$

f) $\frac{-1}{x-3} = \frac{x+3}{5}$

7. Mr. Ewald decides to make and sell Left Handed Smoke Shifters as a side business. The fixed cost to run his business is \$250 per month and the cost to produce each Smoke Shifter averages \$8. The Smoke Shifters will sell for \$19.95.

The function $A(x) = \frac{8x + 250}{x}$ gives the *average* cost (in dollars) per hat when x hats are produced.

- a) Determine $A(1)$, and write a sentence explaining the meaning of your answer.

- b) Complete the table below.

x	1	2	3	4	5
$A(x)$					

- c) Determine $A(10)$, and write a sentence explaining the meaning of your answer.

- d) How many Smoke Shifters must be produced in order to reduce the average cost to \$15 per hat?

- e) Give the *equation* of the horizontal asymptote of $A(x)$, and write a sentence explaining its significance in this situation.

8. Harkins Theaters offers \$1.50 soft drink refills every time you bring your 2013 Harkins Loyalty Cup to the theater. You can purchase the Loyalty Cup for \$5.

The function $C(x) = \frac{1.5x + 5}{x}$ gives the average cost (in dollars) per refill with the Loyalty Cup, where x is the number of soft drink refills purchased.

- a) Determine $C(1)$, and write a sentence explaining the meaning of your answer.

- b) Complete the table below.

x	1	2	3	4	5
$C(x)$					

- c) How many refills must you purchase in order to reduce the average cost to \$2 per refill?

- d) Give the *equation* of the horizontal asymptote of $C(x)$, and write a sentence explaining its significance in this situation.

9. An application of a rational function is $T = \frac{AB}{A+B}$, which gives the time, T , it takes for two workers to complete a particular task where A & B represent the time it would take for each individual worker to complete the identical task.
- a) The monthly reports are due and the department manager has decided to try and accelerate the completion of the report by assigning a team of two workers the task. Mary has completed the monthly reports in an average of 6 hours over the past few months. John, who had worked in Mary's capacity prior to a change in assignments, used to complete the task in 8 hours. How quickly will Mary and John finish the report if they work together? In this problem, **Mary = A = 6** and **John = B = 8**.
- b) The very next month, both Mary and John are out sick. The manager knows that Skip can do the reports in 12 hours working alone, but the manager needs the work done in 4 hours. He needs to choose a staff member to work with Skip. How quickly would the other staff member have to complete the report working alone if the two hope to complete the report in 4 hours?
Let $A = 12$ (Skip's time)
Let $T = 4$
- c) Sara takes 3 hours longer to paint a floor than it takes Katie. When they work together, it takes them 2 hours. How long would each take to do the job alone?
Sara = A = (X+3) and **Kate = B = X**

Name: _____

Date: _____

Lesson 6a Assessment

1. Simplify completely. Show all steps, and box your answers. Answers should include only positive exponents.

a) $5(3n)^0 =$

b) $\frac{2}{3}\left(\frac{3}{4}\right)^2 =$

c) $4a^2(3a^2 - 2a - 5) =$

d) $(2x - 4)^2 =$

e) $(5b^{-3})^2 =$

2. Consider the function $g(x) = \frac{2x - 4}{x + 5}$

a) What is the domain? _____

b) Give the *equation* of the vertical asymptote for $g(x)$. _____

c) Give the *equation* of the horizontal asymptote for $g(x)$. _____

d) What is the vertical intercept? _____

What is the horizontal intercept? _____

e) For what value of x is $g(x) = 3$? Show your work.

f) Determine $g(42)$. Show your work. Round your answer to three decimal places.

Lesson 6b – Rational Exponents & Radical Functions

In this lesson, we will continue our review of Properties of Exponents and will learn some new properties including those dealing with Rational and Radical Roots.

Our function type for this lesson is Radical Functions and you will learn their characteristics, their graphs, and you will solve their equations both graphically and algebraically.

This lesson puts to a close our study of functions that has led us through linear, exponential, logarithmic, quadratic, rational, and now radical functions.

Lesson Objectives

By the end of this lesson, you should be able to:

1. Compute using RATIONAL or RADICAL EXPONENTS
2. Convert expressions from RATIONAL EXPONENT form to RADICAL EXPONENT form and back
3. Simplify expressions using properties of EXPONENTS
4. Identify key characteristics of graphs of RADICAL FUNCTIONS
5. Solve RADICAL FUNCTIONS by graphing
6. Solve RADICAL FUNCTIONS algebraically

Lesson 6b Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Name: _____

Date: _____

Mini-Lesson 6b

Rational/Radical Roots

SQUARE ROOTS

The square root of a is written as \sqrt{a} . If $\sqrt{a} = b$ then $b^2 = a$.

NOTATION:

The notation \sqrt{a} , is RADICAL NOTATION for the square root of a . The notation $a^{\frac{1}{2}}$ is RATIONAL NOTATION for the square root of a .

On your TI 83/84 calculator, you can use the $\sqrt{\quad}$ symbol to compute square roots.

EXAMPLES:

- a) $\sqrt{25} = 5$ because $5^2 = 25$
- b) $(\sqrt{144})^2 = 144$ (Note that the square and square root “undo” each other)
- c) $25^{1/2} = \sqrt{25} = 5$
- e) $(-64)^{1/2}$ is not a real number because there is no number, squared, that will give -64

CUBE ROOTS

The cube root of a is written as $\sqrt[3]{a}$. If $\sqrt[3]{a} = b$ then $b^3 = a$.

NOTATION:

The notation $\sqrt[3]{a}$, is RADICAL NOTATION for the cubed root of a . The notation $a^{\frac{1}{3}}$ is RATIONAL NOTATION for the cubed root of a .

EXAMPLES:

- a) $\sqrt[3]{81} = 9$ because $9^3 = 81$
- b) $\sqrt[3]{-64} = -4$ because $(-4)^3 = -64$ Note that cube roots can be negative.

On your TI 83/84 calculator, you can use the MATH>4: $\sqrt[3]{\quad}$ (sequence to compute cube roots.

THE NTH ROOT OF a

$\sqrt[n]{a} = a^{1/n}$, the nth root of a .

On your TI 83/84 calculator, you can use the MATH>5: $\sqrt[n]{}$ sequence to compute nth roots. You can also use the ^ key to raise to fractional powers. You are recommended to put any fractional power in ().

EXAMPLES:

- a) $\sqrt[4]{256} = 4$
- b) $(-2187)^{1/7} = -3$
- c) $-15^{1/3} \approx -2.47$
- d) $\sqrt[6]{-324}$ is not a real number

RATIONAL EXPONENTS

$$a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

EXAMPLES:

- a) $(81)^{3/4} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = 27$
- b) $(-125)^{2/3} = \left(\sqrt[3]{-125}\right)^2 = (-5)^2 = 25$
- c) $32^{-2/5} = \frac{1}{32^{2/5}} = \frac{1}{\left(\sqrt[5]{32}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$

Problem 1**MEDIA EXAMPLE – COMPUTE WITH RATIONAL/RADICAL EXPONENTS**

a) $\sqrt{49} =$

b) $\sqrt[3]{8} =$

c) $\sqrt{-49} =$

d) $\sqrt[3]{-8} =$

e) $-25^{3/2}$

f) $(-25)^{3/2}$

Problem 2 | MEDIA EXAMPLE – SIMPLIFY USING PROPERTIES OF EXPONENTS

Simplify each of the following using properties of exponents.

a) $x^{1/2}x^{2/3} =$

b) $\frac{x^3}{x^{1/3}} =$

c) $\frac{10x^2y^5}{2x^{-3}}$

d) $\frac{5^{-1}z}{x^{-1}z^{-2}}$

e) $-3(x^0 - 4y^0)$

f) $(4x^2y^3)(3x^{-3}y^{-2})$

g) $\frac{(4x^{-2}y^{-3})(5x^3y^{-2})}{6x^2y^{-3}z^{-3}}$

Problem 3 | YOU TRY – COMPUTE WITH RATIONAL/RADICAL EXPONENTS

Compute each of the following showing as much work as possible. Check results using your calculator.

a) $\sqrt{36}$

b) $\sqrt[3]{-64}$

c) $16^{3/2}$

d) $(-25)^{1/2}$

e) $(\sqrt[3]{27})^4$

Problem 4 | YOU TRY – SIMPLIFY USING PROPERTIES OF EXPONENTS

Simplify each of the following using properties of exponents. Remember to write all final results with positive exponents only. Show as much work as possible.

a) $x^{1/4}x^{3/7} =$

b) $\frac{x^2}{x^{1/4}} =$

c) $(-2x^3y^{-4})(3x^{-4}y^6) =$

d) $\frac{(3x^{-3}y^{-2})(4x^2y^{-1})}{6x^4y^2z^{-5}} =$

RADICAL FUNCTIONS – KEY CHARACTERISTICS

Radical Functions

A basic radical (square root) function is a function of the form

$$f(x) = \sqrt{p(x)}$$

where $p(x)$ is a polynomial and $p(x) \geq 0$ (remember that we cannot take the square root of negative numbers in the real number system).

DOMAIN

To determine the domain of $f(x)$, you want to find the values of x such that $p(x) \geq 0$.

X-INTERCEPT

To determine the x-intercept, solve the equation $p(x) = 0$.

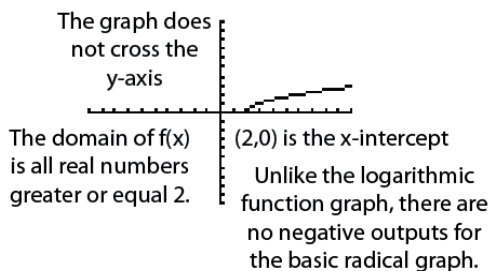
Y-INTERCEPT

To determine the y-intercept, determine the value for $f(0)$. This means, plug 0 in for x and compute the result. You will not always have a y-intercept.

Problem 5 | WORKED EXAMPLE – KEY CHARACTERISTICS OF RADICAL FUNCTIONS

Graph $f(x) = \sqrt{x-2}$ and determine y-intercept, x-intercept, and domain of $f(x)$.

To graph, input into Y1 the following: $2^{\text{nd}} > X^2$ then $x-2$) so that $Y1 = \sqrt{(x-2)}$. Graph on the standard window (Zoom 6) to get the graph below:

DOMAIN

Solve $x - 2 \geq 0$ to get $x \geq 2$. Therefore the domain is $x \geq 2$.

X-INTERCEPT

Solve $x - 2 = 0$ to get $x = 2$. The x-intercept is (2,0)

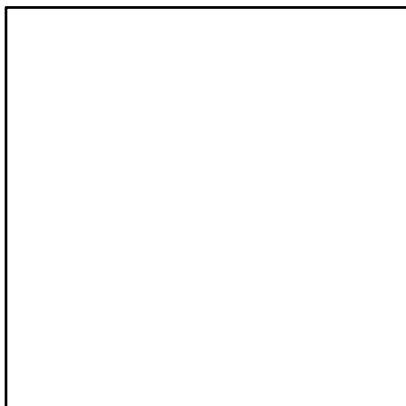
Y-INTERCEPT

Determine $f(0) = \sqrt{0-2} = \sqrt{-2}$ which is not a real number, so there is no y-intercept.

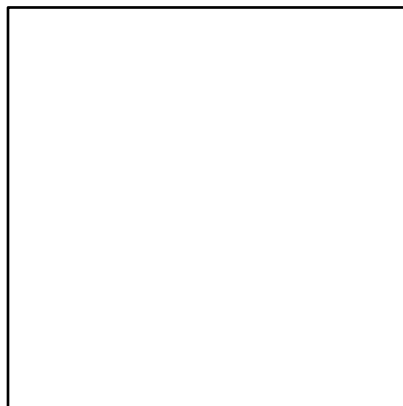
Problem 6 | MEDIA EXAMPLE – KEY CHARACTERISTICS OF RADICAL FUNCTIONS

For each of the following, determine the domain, x -intercept, and y -intercept, then sketch an accurate graph of each.

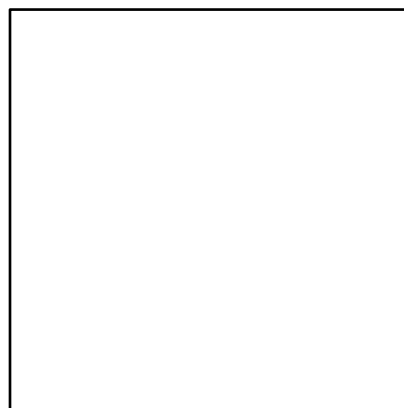
a) $f(x) = \sqrt{4-x}$

Domain of $f(x)$: x -intercept: y -intercept:

b) $g(x) = \sqrt{x-4}$

Domain of $g(x)$: x -intercept: y -intercept:**Problem 7** | YOU TRY – KEY CHARACTERISTICS OF RADICAL FUNCTIONS

Given the function $f(x) = \sqrt{12-4x}$, determine the domain, the x -intercept, the y -intercept (if it exists), and draw an accurate graph of the function.

Domain of $f(x)$: x -intercept: y -intercept:

SOLVE RADICAL EQUATIONS BY GRAPHING

- Let $y_1 =$ one side of the equation
- Let $y_2 =$ other side of the equation
- Determine an appropriate window to see important parts of the graph
- Use the intersection method
- The x -value(s) of the intersection are your solutions
- Note: If your graphs do not cross, then there is no intersection and no solution to the equation.

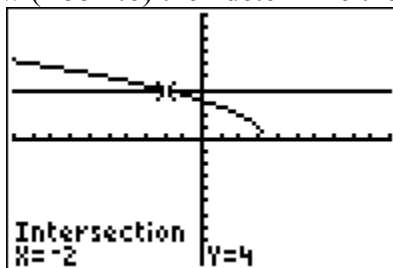
Problem 8 WORKED EXAMPLE – SOLVE RADICAL EQUATIONS BY GRAPHING

Solve the equation $\sqrt{10 - 3x} = 4$ graphically.

Let $Y_1 = \sqrt{10 - 3x}$

Let $Y_2 = 4$

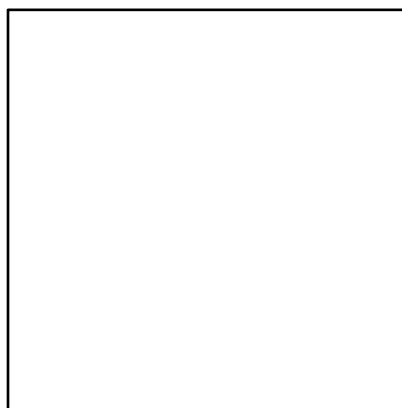
Graph on the standard window (Zoom:6) then determine the intersection (seen below).



Your solution is the x -value of the intersection which in this case is $x = -2$.

Problem 9 MEDIA EXAMPLE – SOLVE RADICAL EQUATIONS BY GRAPHING

Solve the equation $\sqrt{2x+1} - 5 = 0$ graphically.



To solve SIMPLE radical equations algebraically (also called symbolically):

- Isolate the radical part of the equation on one side and anything else on the other
- Sometimes you will have radicals on both sides. That is ok.
- Raise both sides of the equation to a power that will “undo” the radical (2 power to get rid of square root, 3 power to get rid of cubed root, etc...)
- Solve for x.
- Be sure x is part of the domain for the radical part of your equation by checking your result in the original.
- Check your work using the graphing method if possible.

Problem 10	WORKED EXAMPLE – SOLVE RADICAL EQUATIONS ALGEBRAICALLY
-------------------	---

Solve the equation $\sqrt{10 - 3x} = 4$ algebraically.

First, square both sides to remove the square root.

$$\begin{aligned}\sqrt{10 - 3x} &= 4 \\ (\sqrt{10 - 3x})^2 &= 4^2 \\ 10 - 3x &= 16\end{aligned}$$

Next, isolate x.

$$\begin{aligned}10 - 3x &= 16 \\ -3x &= 6 \\ x &= -2\end{aligned}$$

VERY IMPORTANT! Check $x = -2$ in the original equation to be sure it works! *Not all solutions obtained using the process above will check properly in your equation.* If an x does not check, then it is not a solution.

$$\begin{aligned}\sqrt{10 - 3(-2)} &= 4 \\ \sqrt{10 + 6} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4\end{aligned}$$

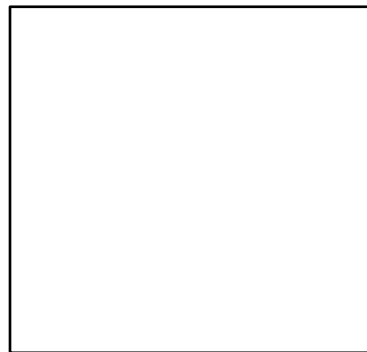
$x = -2$ is the solution to this equation.

Problem 11	MEDIA EXAMPLE – SOLVE RADICAL EQUATIONS ALGEBRAICALLY
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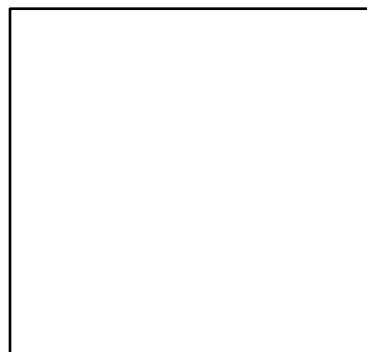
Solve $41 + 5\sqrt{2x - 4} = 11$ algebraically.

Problem 12	YOU TRY – SOLVE RADICAL EQUATIONS GRAPHICALLY/ALGEBRAICALLY
-------------------	--

a) Solve the equation algebraically and graphically: $3\sqrt{4-x} - 7 = 20$. Be sure to check your final result!



b) Solve the equation algebraically and graphically: $2\sqrt{5x} + 24 = 4$. Be sure to check your final result!



Problem 13	WORKED EXAMPLE – SOLVE RADICAL EQUATIONS – MORE ADVANCED
-------------------	---

Solve the equation algebraically and check graphically: $\sqrt{x+6} = x$. Be sure to check your final result!

Since the radical is isolated, square both sides to remove the square root. Then, isolate x .

$$\begin{aligned}\sqrt{x+6} &= x \\ (\sqrt{x+6})^2 &= x^2 \\ x+6 &= x^2 \\ 0 &= x^2 - x - 6 \\ x^2 - x - 6 &= 0\end{aligned}$$

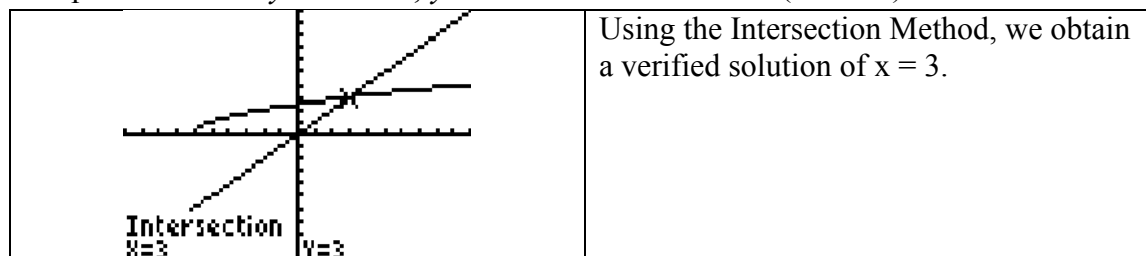
What we now have is a quadratic equation that we can solve using the methods of Lesson 5. The easiest and fastest way to work with this problem is through factoring. You can also use the Quadratic Formula or graphing.

$$\begin{aligned}x^2 - x - 6 &= 0 \\ (x+2)(x-3) &= 0 \\ x+2 = 0 \text{ or } x-3 &= 0 \\ x = -2 \text{ or } x = 3\end{aligned}$$

CHECK:

When $x = -2$ $\sqrt{-2+6} = -2?$ $\sqrt{4} = -2?$ $2 \neq -2$ $x = -2$ does not check so is not a solution.	When $x = 3$ $\sqrt{3+6} = 3?$ $\sqrt{9} = 3?$ $3 = 3$ $x = 3$ checks so is a solution.
--	---

Graphical Check: $y_1 = \sqrt{x+6}$, $y_2 = x$ Window: Standard (Zoom:6)



Name: _____

Date: _____

Lesson 6b Practice Problems

1. Complete the table below. Each expression should be written in radical notation, written with rational exponents and evaluated using the calculator. The first one is done for you.

Written in radical notation	Written using rational exponents	Evaluated using the calculator (Rounded to two decimal places)
$\sqrt[3]{9^2}$	$9^{2/3}$	$9^{(2/3)} = 4.33$
$\sqrt[5]{20^2}$		
$\sqrt[3]{2^4}$		
$-\sqrt[4]{7^2}$		
$\sqrt[3]{(-7)^2}$		
	$3^{5/2}$	
	$11^{3/7}$	
	$-4^{1/2}$	
	$(-2)^{2/3}$	

2. Solve the following Rational and Radical Roots by hand. Check using your calculator

a) $\sqrt{81} =$

b) $\sqrt[5]{32} =$

c) $(-64)^{1/3} =$

d) $49^{3/2} =$

e) $\sqrt[4]{(-4)^2} =$

f) $-9^{5/2} =$

3. Simplify each of the following expressions.

a) $x^{1/4}x^{1/2} =$

b) $\frac{x^{3/5}}{x^{1/5}} =$

c) $(y^{3/4})^{1/3} =$

d) $\left(\frac{z^{1/2}}{z^{2/3}}\right)^{3/2} =$

e) $\sqrt{4x}\sqrt{9x^3} =$

f) $\sqrt[3]{27x^6} =$

g) $\sqrt[3]{y^2}\sqrt[3]{y} =$

h) $\sqrt[3]{8x^0} =$

4. Determine the domain for each of the following functions.

a) $f(x) = \sqrt{4x-6}$

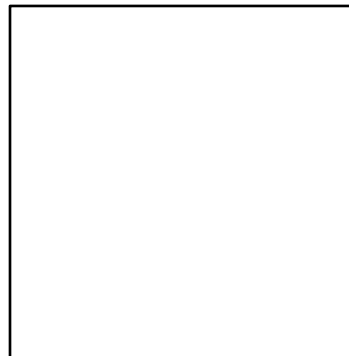
b) $f(x) = 2\sqrt{4x+2}$

c) $g(x) = 4 + \sqrt{2x-8}$

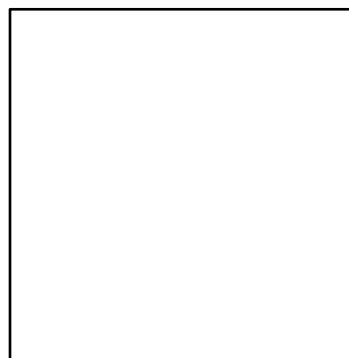
d) $g(x) = \sqrt{12-6x}$

5. Draw a graph of each of the following functions. Then determine the domain and the x-intercept.

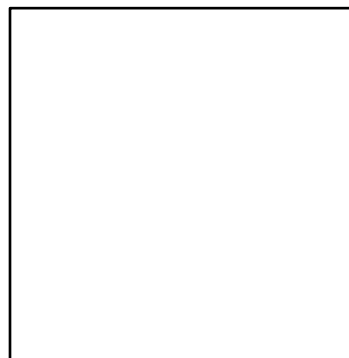
a) $f(x) = \sqrt{4x - 5}$,



b) $f(x) = \sqrt{6x + 4}$

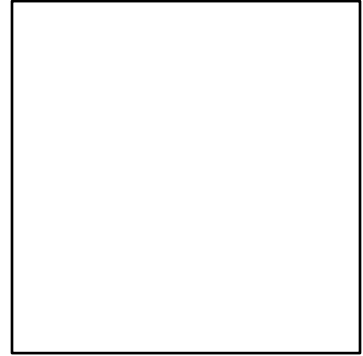


c) $f(x) = \sqrt{-2x - 6}$

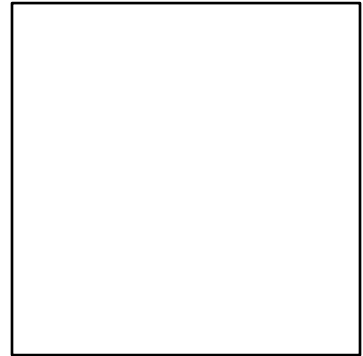


6. Solve each of the following algebraically. Check your answer graphically. You must show both the algebraic and graphical solution.

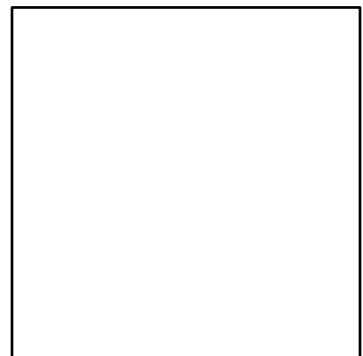
a) $4\sqrt{x+2} = 16$



b) $\sqrt{2x+1} - 5 = 0$



c) $4\sqrt{x-6} = 12$



7. A person's Body Mass Index is calculated with the formula: $BMI = \left(\frac{Weight}{Height^2} \right) 703$ where Weight is in pounds and Height is in inches. Rewrite the equation, solving for Height.

Height =

Use the equation to determine the height of the following people. Each has a BMI of 30.
(Hint, enter the formula in Y1 and look at the table)

Name	Weight	Height
Sarilee	120	
Bubbalee	300	
Martilou	160	
Davalee	188	
Megamike	380	
Peggiroo	99	

Name: _____

Date: _____

Lesson 6b Assessment

1. Evaluate the following using your graphing calculator. If there is no real solution, write "N". Round answers to three decimal places if necessary.

$$\sqrt[6]{42} = \underline{\hspace{2cm}} \quad \sqrt[3]{-512} = \underline{\hspace{2cm}} \quad \sqrt[4]{-625} = \underline{\hspace{2cm}}$$

2. Determine the domain of each of the following.

a) $f(x) = \sqrt{6-2x}$

b) $g(x) = 7-2\sqrt{x+1}$

3. Solve the following equations algebraically. Show all steps. Use your graphing calculator to check your answers.

a) $10\sqrt{2x+1} = 30$

b) $9 + \sqrt{x-4} = 8$

c) $\sqrt{2x+15} - x = 0$

d) $\sqrt{5-x} = x+1$

Appendix A: You Try Answers

Lesson 1a

You Try Problem 8:

- a) $C(x) = 3.25x + 30.00$
- b) 0, 25
- c) 30, 111.25
- d) 30, 46.25, 78.75, 95
- e) 78.75
- f) 0
- g) $x = 0$
- h) Answers vary

You Try Problem 12:

For Function $F(x)$

- Domain: $-7 \leq x \leq 4$
- Range: $-6 \leq F(x) \leq 7$
- $F(0) = 2$
- When $F(x) = 0$, $x = -2$

For Function $G(x)$

- Domain: $-7 \leq x \leq 7$
- Range: $-6 \leq G(x) \leq 5$
- $G(0) = 5$
- When $G(x) = 0$, $x = -3, 3, 5.5$

You Try Problem 14:

- $f(2) = -10$, $f(-3) = 5$
- When $f(x) = 7$, $x = -11/3$, when $f(x) = -12$, $x = 8/3$

You Try Problem 15:

- a) t = time in years
- b) $V(t)$ = value in \$
- c) 1200, 600, 0
- d) Graph should include labels for plotted points and axes. Points should be connected. Graph should not extend beyond the starting/ending points from the table.
- e) 8 years old
- f) \$1000
- g) $0 \leq t \leq 12$ or $[0, 12]$
- h) $0 \leq V(t) \leq 1200$ or $[0, 1200]$

Lesson 1b

You Try Problem 6:

x-intercept (6, 0), y-intercept (0, 9), Answers vary on other ordered pairs, graphs for parts a) b) should be the same

You Try Problem 9:

- a) decreasing
- b) (0, 4)
- c) (4, 0)
- d) slope = -1
- e) $y = -x + 4$

You Try Problem 10:

- a) $y = -3x + 7$
- b) (0, 7)
- c) $(\frac{7}{3}, 0)$ or (2.33, 0)

You Try Problem 11:

- a) equation $x = 2$, slope is undefined, no y-intercept, x-intercept (2, 0)
- b) equation $y = -3$, slope $m = 0$, y-intercept (0, -3), no x-intercept

Lesson 2a

You Try Problem 1

- a) VI (0, 3861)
- b) $E(2) = 3706.6$ feet
- c) $E(4) = 3552.2$ feet
- d) Decreasing
- e) $3397.8 \leq E(t) \leq 3861$ or [3397.8, 3861]

You Try Problem 5

- a) -6.3 lbs per week
- b) -5 lbs per week
- c) average pounds lost per week over the given time period
- d) NO – AROC not constant

You Try Problem 7

- a) $y = 0.279x + 27.281$, $C(t) = 0.279t + 27.281$
- b) $C(3) = 28.118$ million trees
- c) $C(9) = 29.792$ million trees
- d) $m = 0.279$; The slope is the rate of change of tree sales in millions of trees per year

You Try Problem 8

- a) sketch of data points
- b) $y = .322x + 24.155$
- c) 26.24; (6, 26.24)
- d) 26.087; (6, 26.087)
- e) given data set is not exactly linear so model is an approximation or best fit
- f) $x = 12.376$; (12.376, 28.14)

Lesson 2b

You Try Problem 1

follow the given example

You Try Problem 2

- $g(t) = 3t - 4$; $g(4) = 8$; when $g(t) = -1$, $t = 1$
- $f(x) = x - 5$; $f(-4) = -9$; when $f(x) = 10$, $x = 15$
- $g(x) = 2x + 4$; $g(3) = 10$; when $g(x) = 5$, $x = \frac{1}{2}$

You Try Problem 5

- a) 11
- b) $-x^2 + x + 3$

You Try Problem 8

- a) $9x^2 - 4$
- b) $2x^5 - 7x^3 + 10x^2 - 4x + 5$
- c) $4x - 1$ d) $-24x^4$

You Try Problem 11

- a) $-5x + \frac{4}{5}x^3$
- b) $-5x^3 + 4 - \frac{2}{x^3}$

You Try Problem 14

Not all answers provided for this problem

- a) 0
- c) -4
- e) -1
- g) 0 or -2
- i) -3
- k) -4

You Try Problem 16

Not all answers provided for this problem

- c) $P(x) = 0.90x - 450$
- d) and e) 500 chocolates to break even

Lesson 3a

You Try Problem 1

- a) input
- b) output, y
- c) line, m, b
- d) >
- e) <
- f) constant

You Try Problem 6

- a) Common Ratio is about 1.5 so data are best modeled by exponential function
- b) $y = 5.0(1.5)^x$

You Try Problem 8

- $f(x) = 125(1.25)^x$
125, 1.25, all real numbers, $f(x) > 0$, none, (0, 125), $y = 0$, increasing

- $g(x) = 125(0.75)^x$
125, 0.75, all real numbers, $g(x) > 0$, none, (0, 125), $y = 0$, decreasing

You Try Problem 11

- a) $125(0.75)^x = 300$
- b) $y_1 = 125(0.75)^x$ $Y_2 = 300$
- c) -10, 10, 0, 350
- d) -3.04
- e) (-3.04, 300)
- f) Show graph on appropriate window, label y_1 , y_2 , intersection

You Try Problem 13

- a) $125(0.75)^x = 50$
- b) $y_1 = 125(0.75)^x$ $Y_2 = 50$
- c) -10, 10, 0, 100
- d) 3.19
- e) (3.19, 50)
- f) Show graph on appropriate window, label y_1 , y_2 , intersection

You Try – Problem 15

- a) $x = 31.41$
- b) $x = 3.16$
- c) $x = -38.96$

Lesson 3b

You Try Problem 3

- a) 52000, .03, 1.03, $52000(1.03)^t$
- b) \$60282.25
- c) $104000 = 52000(1.03)^t$, 23 years

You Try Problem 6

- a) 462,768, .014, .986, $P(t)=462,768(0.986)^t$
- b) 245,372
- c) $231,384 = 462,768(0.986)^t$, t is approximately 49 years

You Try Problem 7

- $P(t) = 145(1.03)^t$
- $V(t) = 314(0.035)^t$
- $P(t) = 412(1.1324)^t$
- $I(t) = 546(0.98)^t$
- $A(t) = 3(0.78)^t$
- $P(t) = 3061(3.05)^t$

You Try Problem 9

- a) $y = 82.9(.837)^x$
- b) 16.3%
- c) (20, 2.36)
- d) (6.10, 28) and show graph
- e) $41.45 = 82.9(0.837)^x$, half-life 3.9

Lesson 4a

You Try Problem 1

1, 2, 3, 4

You Try Problem 5 (be sure to include the “because” statements for each one)

- a) 6
- b) 0 because $5^0 = 1$,
- c) -3
- d) Does not exist
- e) $\frac{1}{2}$

You Try Problem 6

- b) $\log_3 81 = 4$
- c) $7^5 = 16807$

You Try Problem 11

 $x = 33$ (exact) and no rounding needed

You Try Problem 12

exact $x = \frac{3^{0.75} - 1}{7}$, rounded to three decimals 0.183 (note that check not possible in this lesson unless you use the exact value which is very challenging)

Lesson 4b

You Try Problem 1

- a) 8
- b) 4
- c) -3
- d) $\frac{1}{3}$

You Try Problem 3

- b) 2.307
- c) 1.195
- d) 8.548
- e) 15.730

You Try Problem 6

$x = \log_3 5.3$ (exact),
 $x = 1.518$ (rounded)

You Try Problem 9

- a) $x > 0$
- b) All real numbers
- c) (1,0) d) $x = 6$

Lesson 5a

You Try Problem 3

$a=2, b=0, c=-5$
 parabola opens up
 y-int = (0, -5)

You Try Problem 6

$a=2, b=0, c=-5$
 Vertex (0, -5)
 Axis of symmetry equation: $x = 0$

You Try Problem 9

x-intercepts $(-1.58, 0)$ & $(1.58, 0)$

You Try Problem 12

Solutions $x = -2.35, x=2.35$

Lesson 5b

You Try Problem 2

- a) 1 repeated real solution, $x = -3$
- b) no real solution
- c) 2 real solutions, $x = -3.5, 1$

You Try Problem 6

- a) $x = 0, -3$
- b) $x = 2, -5$
- c) $x = -1/2, 2$

You Try Problem 12

a, b, and c
All solutions are $x=2, -5$

Lesson 6a

You Try Problem 3

- a) $4032a^{11}b^2$
- b) $12x^2 + xy - y^2$
- c) $\frac{xy^4}{25}$
- d) $\frac{6y^{17}}{x^{23}}$
- e) $3x^3 + 5x^2 - x + 2$

You Try Problem 10

 $x = 3.25$

You Try Problem 11

- a) $T(r) = \frac{354.5}{r}$
- c) $T(60)$ is about 6 hours.
- d) $r = 35.45$ mph

Lesson 6b

You Try Problem 3

- a) 6
- b) -4
- c) 64
- d) not a real number
- e) 81

You Try Problem 4

- a) $x^{19/28}$
- b) $x^{7/4}$
- c) $\frac{-6y^2}{x}$
- d) $\frac{2z^5}{x^5y^5}$

You Try Problem 7

Domain $x \leq 3$

x-intercept (3, 0)

y-intercept (0, 3.5)

Graph should include vertical and horizontal intercepts plotted and labeled as well as accurate representation of the graph.

You Try Problem 12

- a) $x = -77$
- b) no solution